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Taking advantage of entanglement in B factories to measure the weak phase gamma

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$B^0 - \bar{B}^0$ entanglement offers a conceptual alternative to the single charged B-decay asymmetry for the measurement of the direct CP-violating ϕ_3/γ phase. With $f = J/\psi K_L$, $J/\psi K_S$ and $g = (\pi\pi)^0, (\rho_L\rho_L)^0$ the 16 time-ordered double-decay rate intensities to (f, g) depend on the relative phase between the f- and g-decay amplitudes given by γ at tree level. Several constraining consistencies appear. An intrinsic accuracy of the method at the level of $\pm 1°$ could be achievable at Belle-II with an improved determination of the penguin amplitude to g channels from existing facilities. Other non-CP eigenstates are also analysed as $g = D^-\pi^+, D^-\rho^+$, in those cases one has to consider together the (f, g) channel with $(f, \bar{g})$. We also explore $B_s^0 - \bar{B}_s^0$ entangled states.

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1. Introduction

The quark charged current couplings are proportional to the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]

$$V=\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}=\begin{pmatrix} |V_{ud}| & |V_{us}|e^{i\chi'} & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} \qquad (1)$$

The rephasing invariant quartet $Im\left(V_{ij}V_{il}^{*}V_{kl}V_{kj}^{*}\right)$ for for $i\neq k, j\neq l$ is universal [2-3] (Chau, Keung, Jarlskog…) up to a sign, therefore one can show, that

$$\beta,\gamma\sim O(1) \;\; ; \;\; \beta_s\sim\lambda^2 \;\; ; \;\; \chi'\sim\lambda^4 \qquad (2)$$

where $|V_{us}|\sim\lambda\sim 0.22$ is the Wolfenstein CKM expansion parameter. Because γ is

$$\gamma=\phi_3=\ arg\,(-V_{ud}V_{cb}V_{ub}^{*}V_{cd}^{*}\,) \qquad (3)$$

it turns to be the unique non-small phase that can appear at tree level in B decays. In fact, a minimal ingredient to measure γ is, for example, the interference of the decays $B^-\to D^0+K^-$ and $B^-\to\bar{D}^0+K^-$

$$\begin{aligned} A(b\to c\bar{u}s) &= A(B^-\to\bar{D}^0K^-)\propto V_{ub}V_{cs}^{*} \\ A(b\to u\bar{c}s) &= (B^-\to D^0K^-)\propto V_{cb}V_{us}^{*} \end{aligned} \qquad (4)$$

Interference that is only possible if both D^0 and $\bar{D}^0$ decay to a common final state f:

$$\begin{aligned} A^- &= A\left(B^-\to D_{\hookrightarrow f}K^-\right)=A\left(B^-\to D_{\nrightarrow f}^{\perp}K^-\right)= \\ &= A(D^0\to f)A(B^-\to D^0K^-)+A(\bar{D}^0\to f)A(B^-\to\bar{D}^0K^-) \end{aligned} \qquad (5)$$

where the D state that does not decay to f $D_{\nrightarrow f}$, and its orthogonal $D_{\nrightarrow f}^{\perp}$ are [15-16-17]

$$\begin{aligned} |D_{\nrightarrow f}\rangle &= \bar{c}_f|D^0\rangle-c_f|\bar{D}^0\rangle \\ |D_{\nrightarrow f}^{\perp}\rangle &= c_f^{*}|D^0\rangle+\bar{c}_f^{*}|\bar{D}^0\rangle \end{aligned} \qquad (6)$$

with $|c_f|^2+|\bar{c}_f|^2=1$ and, for $f=K^+K^-$ and having

$$A(\bar{D}^0\to K^+K^-)\;\propto V_{us}^{*}V_{cs}\;;\;A(D^0\to K^+K^-)\;\propto V_{us}\,V_{cs}^{*} \qquad (7)$$

one gets

$$\frac{c_f}{\bar{c}_f}=\frac{A(D^0\to f)}{A(\bar{D}^0\to f)}=\frac{V_{us}V_{cs}^{*}}{V_{us}^{*}V_{cs}} \qquad (8)$$

In such a way that

$$\begin{aligned} A^- &= aV_{cb}V_{cs}^{*}+bV_{ub}V_{us}^{*} \\ A^+ &= aV_{cs}V_{cb}^{*}+bV_{us}V_{ub}^{*} \end{aligned} \qquad (9)$$

Giving rise to the CP the violating difference of decay rates

$$\begin{aligned} |A^-|^2-|A^+|^2 &\propto Im(ab^{*})Im(V_{us}V_{cb}V_{ub}^{*}V_{cs}^{*}) \\ Im(V_{us}V_{cb}V_{ub}^{*}V_{cs}^{*}) &\propto \sin(\chi'+\gamma)\sim sin(\gamma) \end{aligned} \qquad (10)$$

where the piece $Im(ab^{*})$ encodes the need for relative strong final state interactions. Behind this comparison are the different ways to measured γ [4] proposed by Gronau, London, Wyler (GLW), Atwood, Dunietz, Soni (ADS), Giri, Grossman, Soffer, Zupan (GGSZ) and many more [5-7].

2. Using entanglement for γ at $t=0$

The entangled $B^0-\bar{B}^0$ system produced at Belle II from the decay of the $\Upsilon(4s)$ is

$$|\Psi_0\rangle=\frac{1}{\sqrt{2}}\left(|B_d^0\rangle|\bar{B}_d^0\rangle-|\bar{B}_d^0\rangle|B_d^0\rangle\right) \qquad (11)$$

The decay amplitude to the states f and g (simultaneously) is

$$\langle f,g|\boldsymbol{T}|\Psi_0\rangle = \frac{1}{\sqrt{2}}\left(A_f\bar{A}_g - \bar{A}_f A_g\right) \qquad (12)$$

with the usual notation $A_f = \langle f|\boldsymbol{T}|B_d^0\rangle$; $\bar{A}_f = \langle f|\boldsymbol{T}|\bar{B}_d^0\rangle$ the double decay rate (DDR) at $t = 0$ can be written as

$$|\langle f,g|\boldsymbol{T}|\Psi_0\rangle|^2 = \frac{1}{2}\left|A_f A_g\right|^2\left|\frac{\bar{A}_g}{A_g} - \frac{\bar{A}_f}{A_f}\right|^2 \qquad (13)$$

This expression clearly shows the appearance of interference in the DDR at $t = 0$ without the need of strong phases. If we take, for example, $f = J/\psi K_{S.L}$ and $g = (\pi\pi)_{I=2}$ we have

$$\frac{\bar{A}_f}{A_f} = \pm 1 \ ; \quad \frac{\bar{A}_g}{A_g} = e^{-2i\gamma} \qquad (14)$$

And therefore, we get

$$\begin{aligned} |\langle J/\psi K_{S,}, (\pi\pi)_{I=2}|\boldsymbol{T}|\Psi_0\rangle|^2 &\propto cos^2\gamma \\ |\langle J/\psi K_{L}, (\pi\pi)_{I=2}|\boldsymbol{T}|\Psi_0\rangle|^2 &\propto sin^2\gamma \end{aligned} \qquad (15)$$

The use of the EPR [8] correlation to study CP violation was proposed by Wolfenstein, Gavela et al, Falk and Petrov and Alvarez and Bernabeu [9-12] among others for several decay channels in the B factories. The method for γ consists in the observation of the coherent double decay of $\Upsilon(4s)\,(1^{--})$ to the CP eigenstates (f,g), with $f = J/\psi K_S(0^{-+}), J/\psi K_L(0^{--})$ and $g = h^+h^-, h^0h^0(0^{++})$ and $h = \pi, \rho_L$. In such a way that

$$\begin{aligned} \Upsilon(4s) &\to (J/\psi K_S)_B (hh)_B \ ; \quad \text{is CP allowed} \\ \Upsilon(4s) &\to (J/\psi K_L)_B (hh)_B \ ; \quad \text{is CP forbidden} \end{aligned} \qquad (16)$$

The necessary interference between amplitudes [13] containing the $V_{cd}V_{cb}^*$ and $V_{ud}V_{ub}^*$ sides of the unitarity triangle is automatic in the double decay amplitude from the two terms of the entangled $B^0 - \bar{B}^0$ system in eq (11)

$$\langle f,t_0; g,t_0+t|\mathrm{T}|\Psi_0\rangle = \frac{e^{-i(\mu_H+\mu_L)t_0}}{2\sqrt{2}pq}\left(e^{-i\mu_H t}A_L^f A_H^g - e^{-i\mu_L t}A_H^f A_L^g\right) \qquad (17)$$

with $B_H = pB^0 + q\bar{B}^0$, $B_L = pB^0 - q\bar{B}^0$ the eigenstate of the system, $\mu_{H,L}$ the eigenvalues and $A_{H,L}^f = \langle f|\boldsymbol{T}|B_{H,L}\rangle$ where as usual the single particle decay amplitudes are $A_f = \langle f|\boldsymbol{T}|B^0\rangle$; $\bar{A}_f = \langle f|\mathrm{T}|\bar{B}^0\rangle$.

So, the double decay rate for $\Psi_0 \to (f,t_0; g,t_0+t)$ to the state f at t_0 and to the state g at t_0+t integrated for t_0 is given by [14]

$$\begin{aligned} I(f,g;t) &= \frac{e^{-\Gamma|t|}}{16\Gamma|pq|^2}\left|e^{-i\Delta Mt/2}A_L^f A_H^g - e^{+i\Delta Mt/2}A_H^f A_L^g\right|^2 \\ &= \frac{e^{-\Gamma|t|}}{16\Gamma|pq|^2}\left|\begin{matrix} \cos\left(\frac{\Delta M}{2}t\right)\left(A_L^f A_H^g - A_H^f A_L^g\right) \\ -i\sin\left(\frac{\Delta M}{2}t\right)\left(A_L^f A_H^g + A_H^f A_L^g\right)\end{matrix}\right|^2 \end{aligned} \qquad (18)$$

where $\Delta M, \Gamma$ have the usual relations [15] to the real and imaginary parts of $\mu_{H,L}$ and we work in the very good approximation $\Delta\Gamma = 0$ ($Im\mu_H = Im\mu_L$). For convenience we normalize (18) in terms of the averaged – for particle and antiparticle- decay rate $\langle\Gamma_f\rangle$. This way we define a reduced double decay rate [16]:

$$\hat{I}(f,g;t) \equiv \frac{\Gamma}{\langle\Gamma_f\rangle\langle\Gamma_g\rangle} I(f,g;t) = \\ = e^{-\Gamma|t|}\left(I_d^{fg}\cos^2\left(\frac{\Delta Mt}{2}\right) + I_m^{fg}\sin^2\left(\frac{\Delta Mt}{2}\right) + I_{od}^{fg}\sin(\Delta Mt)\right) \tag{19}$$

which has been proven [17] to verify an exact connection with the observables related to the evolution and transitions among B states.

$$\hat{I}(f,g;t) = \left|\langle B_{\nrightarrow g}^{\perp}|B_{\nrightarrow f}(t)\rangle\right|^2 \tag{20}$$

This reduced double decay rate is equal to the rate at which -our initial $B_{\nrightarrow f}$ meson state tagged by the first decay $B \to f$- evolves after t to the B-meson filtered at the final meson state $B_{\nrightarrow f}^{\perp}$ by the second decay $B \to g$.

In eq (19) we have introduce the "intensity parameters" $I_d^{fg}, I_m^{fg}$and I_{od}^{fg} for every pair of decay channel (f,g). The first one appears at $t = 0$ so clearly is a signal of the direct correlation between the decay amplitudes to the two channels (f,g), I_d^{fg}will not depend on the mixing parameters appearing in the evolution in time of the $|B_{\nrightarrow f}\rangle$. We use d for direct, m for mixing induced and od for odd under t.

3. Consistency conditions

The observables are the coefficients in eq (19): I_d^{fg}, I_m^{fg} and I_{od}^{fg}. These terms enjoy interesting properties, very useful for the experimental analysis. Eq (19) verifies formally the invariance of the reduced DDR under the simultaneous reversing of the order in the decay channels $(f,g) \to (g,f)$ and $t \to -t$

$$\hat{I}(f,g;t) = \hat{I}(g,f;-t) \tag{21}$$

It implies the following consistency conditions

$$I_d^{fg} = I_d^{gf};\ I_m^{fg} = I_m^{gf}; I_{od}^{fg} = -I_{od}^{gf} \tag{22}$$

A second set of consistency conditions, already explained in reference [18] is

$$I_d^{Sg} + I_d^{Lg} = 1;\ I_m^{Sg} + I_m^{Lg} = 1;\ I_{od}^{Sg} + I_{od}^{Lg} = 0 \tag{23}$$

Where we have used the simplified notation $J/\psi K_{S,L} = S, L$. The importance of equation (22) relies in the fact that for some of the intensity parameters is not necessary to distinguish if the f decay has occurred before or after the g decay. The non-homogeneity of equations (23) gives a controlled connection between the CP forbidden and CP-allowed time-dependent transitions for any of the four decay products g. A consequence of these connections is that one can measured all the three observables $I_{d,m,od}^{fg}$ for all (f,g) and (g,f) channels with $f = S, L$ and $g = (\rho_L^+\rho_L^-), (\rho_L^0\rho_L^0), (\pi^+\pi^-), (\pi^0\pi^0)$, *just measuring the ratios* $I_{d,m}^{Lg}/I_{od}^{Lg}, I_{d,m}^{Sg}/I_{od}^{Sg}$.

4. The observables

The observables just described, if we include both times ordering as separate channels, amount to a total of 16 channels. In full generality the magnitude where it enters our phase γ is the ratio of amplitudes, in eq (13),

$$\frac{\bar{A}_g}{A_g} \equiv \rho_g e^{-2i\phi_g} \tag{24}$$

In particular, we could have $\phi_g = \gamma$ and $\rho = 1$ in a channel dominated by a single amplitude like the $\pi\pi$ channel in the Isospin state $I = 2$ where only the tree level amplitude contributes. Therefore, our main target is the measurement of ϕ_g. In the next section we will describe the extraction of γ from ϕ_g, that is completely analogous to the Gronau and London isospin analysis [19] to extract α. Of course, looking at eq (13) we will need also

$$\frac{\bar{A}_L}{A_L} = -\frac{\bar{A}_S}{A_S} = 1 \qquad (25)$$

And once we are interested in measuring the observables in eq (19), we have to introduce the parameters relevant for the time evolution

$$\frac{q}{p} = e^{-2i\phi_M}\ ;\ \lambda_f = \frac{q\bar{A}_f}{pA_f} \qquad (26)$$

Where ϕ_M is the phase appearing in the mixing, in our case $\phi_M = \beta$, the well-known CP violating phase in $B \to J/\psi K_S$. Therefore, we will also use

$$\begin{aligned} \lambda_S &= -\lambda_L = -e^{-2i\phi_M} \\ \lambda_g &= \rho_g e^{-2i(\phi_g+\phi_M)} \end{aligned} \qquad (27)$$

For the observable present at $t = 0$ we get

$$I_d^{L,Sg} = \frac{\left|\frac{\bar{A}_g}{A_g} - \frac{\bar{A}_{L,S}}{A_{L,S}}\right|^2}{\left(1+|\lambda_{L,S}|^2\right)\left(1+|\lambda_g|^2\right)} = \frac{1}{2}\left[1 \mp \frac{2\rho_g\cos(2\phi_g)}{(1+\rho_g^2)}\right] \qquad (28)$$

It is CP forbidden for the channel L and CP allowed for the channel S, if there were not penguin pollution in the g decay $\rho_g = 1$ and we would have $I_d^{Lg} = \sin^2\gamma$ and $I_d^{Sg} = \cos^2\gamma$. It is clear why we have named I_d for direct CP, because mixing does not enter and is not needed.

For the other observables we get

$$\begin{aligned} I_m^{L,Sg} &= \frac{|(1-\lambda_g\lambda_{L,S})|^2}{2\left(1+|\lambda_g|^2\right)} = \frac{1}{2}\left[1 \mp \frac{2\rho_g\cos(4\phi_M+2\phi_g)}{(1+\rho_g^2)}\right] \\ I_{od}^{L,Sg} &= \frac{Im\left[(\lambda_g^* - \lambda_{L,S}^*)(1-\lambda_g\lambda_{L,S})\right]}{\left(1+|\lambda_g|^2\right)} = (\mp)\frac{(1-\rho_g^2)}{(1+\rho_g^2)}\sin(2\phi_M) \end{aligned} \qquad (29)$$

that also depend on the mixing phase. The quantities to be extracted from $I_{d,m,od}^{L,Sg}$, for each channel g are ϕ_g, ρ_g and ϕ_M. We repeat again that the observable containing information on γ is ϕ_g.

5. Isospin analysis

In general, we will have for each g channel a departure in ϕ_g from the universal γ value that we call

$$\epsilon_g = \gamma - \phi_g \qquad (30)$$

The neutral and charged B meson decays differ in the presence versus absence, respectively, of the penguin contribution to the amplitudes for each final state with $h = \pi, \rho_L$. The charged decay amplitudes $A_{+0} = A(B^+ \to h^+h^0)$ and $\bar{A}_{+0} = A(B^- \to h^-h^0)$ have a final isospin 2 state and, therefore, only the $\Delta I = 3/2$ tree level amplitude contributes with the weak phase γ: $\bar{A}_{+0}/A_{+0} = e^{-2i\gamma}$. It is convenient to define the quantities

$$a_g = \frac{A_g}{A_{+0}} \; ; \; \bar{a}_g = \frac{\bar{A}_g}{\bar{A}_{+0}} \qquad (31)$$

In such a way that the double ratio fixes the penguin pollution parameters ρ_g and ϵ_g

$$\frac{\bar{a}_g}{a_g} = \rho_g e^{2i\epsilon_g} \qquad (32)$$

The isospin triangular relations, with these complex ratios, are [20-21]

$$\frac{1}{\sqrt{2}} a_{+-} = 1 - a_{00} \; ; \; \frac{1}{\sqrt{2}} \bar{a}_{+-} = 1 - \bar{a}_{00} \qquad (33)$$

That allows to obtain the real parts of $a_{+-}, \bar{a}_{+-}, a_{00}, \bar{a}_{00}$ in terms of the corresponding moduli and consequently also the imaginary parts. In other words, we can get a_g and $\bar{a}_g$ from the branching ratios of the processes $B^\pm \to h^\pm h^0$; $B^0, \bar{B}^0 \to h^+h^-, h^0h^0$ fixing ϵ_g and ρ_g. The summary of our isospin analysis, with the PDG data, presented in reference [22] is

g	ρ_g	ϵ_g
$\rho_L^+\rho_L^-$	1.007 ± 0.076	$\mathbf{0.008 \pm 0.091}$
$\rho_L^0\rho_L^0$	0.972 ± 0.241	0.007 ± 0.345
$\pi^+\pi^-$	1.392 ± 0.062	$\pm(0.307 \pm 0.170)$
$\pi^0\pi^0$	1.306 ± 0.206	$\pm(0.427 \pm 0.172)$

Table I. Results of the real isospin analysis

Because the $\rho_L^+\rho_L^-$ channel is the one with the largest branching ratio, $\delta\epsilon_{\rho_L^+\rho_L^-} = 0.091 = 5.2°$ give us an estimate of the uncertainty, due to the present knowledge of the penguin pollution, in the determination of γ/ϕ_3. Note that we will have improvements from Belle II and LHCb.

6. The accuracy of the method

The intrinsic accuracy of the proposed method is controlled by the ability to extract ϕ_g. Under the assumption that Belle II [23-25] can collect 1000 $\rho_L^+\rho_L^-$ events in the categories $(L, \rho_L^+\rho_L^-)$, $(S, \rho_L^+\rho_L^-)$,$(\rho_L^+\rho_L^-, L)$, $(\rho_L^+\rho_L^-, S)$, 50 $\rho_L^0\rho_L^0$, 200 $\pi^+\pi^-$ and 50 $\pi^0\pi^0$we generate simulated data. For each *g*, we generate values of *t*, the events, distributed according to the four double-decay intensities. To incorporate the effect of experimental time resolution, each *t* is randomly displaced following a normal distribution with zero mean and $\sigma = 1ps$. Additional experimental effects such as efficiencies are not included. Generation proceeds until the chosen number of events. Events are binned. The procedure is repeated in order to obtain mean values and standard deviations in each bin: these constitute our simulated data, as we illustrate with 20 bins. There are no significant differences if one considers, for example, 15 or 10 bins.

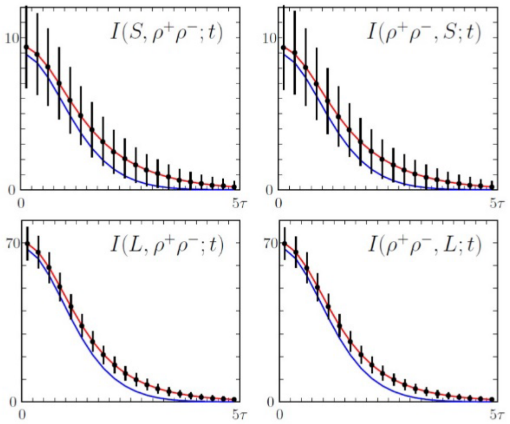

Figure 1. Simulated data, as explained in the text. Black dots with bars indicate mean values and associated uncertainties; the red curves are the extracted double-decay intensities, while the blue curves correspond to the term I_d^{fg} in each intensity.

The fit to these simulations gives $I_d^{S\rho_L^+\rho_L^-} = 0.1170 \pm 0.0138$, $I_m^{S\rho_L^+\rho_L^-} = 0.1658 \pm 0.0456$ and $I_{od}^{S\rho_L^+\rho_L^-} = 0.0000 \pm 0.0198$. Together with the other fits [14], it gives rise to

g	ϕ_g	ρ_g
$\rho_L^+\rho_L^-$	$1.222 \pm \mathbf{0.020}$	1.00 ± 0.06
$\rho_L^0\rho_L^0$	1.22 ± 0.09	1.00 ± 0.24
$\pi^+\pi^-$	1.57 ± 0.12	1.35 ± 0.12
$\pi^0\pi^0$	1.57 ± 0.18	1.35 ± 0.24
	$\phi_M = 0.384 \pm 0.031$	

Table II. Results of the fit to the simulation

We conclude that since $\gamma = \phi_g + \epsilon_g$, the error $\delta\phi_{\rho_L^+\rho_L^-} = \mathbf{0.020 = 1.1^\circ}$ gives us an idea of the intrinsic statistical limiting error we would expect in the determination of γ/ϕ_3 for the assumed number of events.

7. Non-CP channels

Looking at equation (28) we can keep the channels $f = S, L$ and used non CP channels where the ratio $\bar{A}_g/A_g$ depends on γ/ϕ_3. Some channels that contain γ in the ratio $\bar{A}_g/A_g$ are $g = \pi^+D^-$ and the flavour related $\rho^+D^-, \pi^+D^*(2010)^-, \rho^+D^*(2010)^-$, note that $g \neq \bar{g}$. In general, the diagrams contributing to these decays are

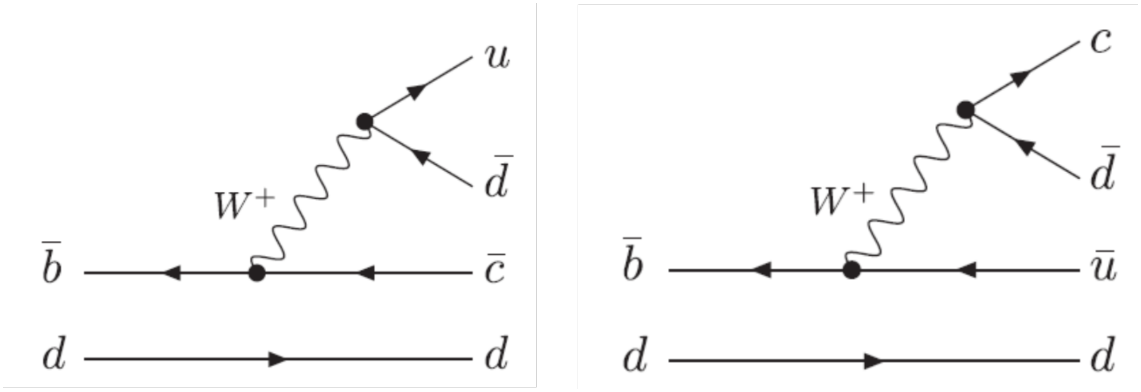

Figure 2. Diagrams contributing to $B_d^0 \to \pi^+ D^-$ (left) and $B_d^0 \to \pi^- D^+$ (right)

Both diagrams contribute to decays involving the channels $g = \pi^+ D^-$ and $\bar{g} = \pi^- D^+$ the one on the left contributes to $B_d^0 \to g$ and $\bar{B}_d^0 \to \bar{g}$ and the diagram on the right contributes to $B_d^0 \to \bar{g}$ and $\bar{B}_d^0 \to g$ and to the flavour analogue channels. The one on the left does not depend on γ, it is the diagram in the right that introduces the dependence on γ through the matrix element V_{ub} . Therefore, we have

$$\begin{aligned} A_g &= V_{ud} V_{cb}^* T_g \quad ; \quad \bar{A}_{\bar{g}} = V_{cb} V_{ud}^* T_g \\ \bar{A}_g &= V_{ub} V_{cd}^* T_{\bar{g}} \quad ; \quad A_{\bar{g}} = V_{cd} V_{ub}^* T_{\bar{g}} \end{aligned} \tag{34}$$

For the relevant ratios entering in our observables in equation (28) we have

$$\frac{\bar{A}_g}{A_g} = -\rho_g e^{-i(\gamma+\Delta_g)} \quad ; \quad \frac{\bar{A}_{\bar{g}}}{A_{\bar{g}}} = -\frac{1}{\rho_g} e^{-i(\gamma-\Delta_g)} \tag{35}$$

Where $\rho_g \sim \lambda^2$ is the square root of the ratio of branching ratios $B_r(B_d^0 \to \bar{g})/B_r(B_d^0 \to g)$ and Δ_g is the difference of the strong phases of T_g and $T_{\bar{g}}$.

A very important conclusions can be extracted from equations (35) when we plug it in our observable equation (28). By measuring the channels $(S, \pi^+ D^-), (L, \pi^+ D^-), (S, \pi^- D^+)$ and $(L, \pi^- D^+)$ one can extract $\gamma + \Delta_g$ and $\gamma - \Delta_g$ without isospin analysis. Therefore, a priori, *it is possible to extract in these channels γ without isospin analysis*. In favour is the larger branching ratios of these channels, but an important drawback is the large departure of ρ_g and $1/\rho_g$ from 1, that will compromise the sensibility of $I_d^{L,Sg}$ to γ.

In the following table we include the known data in order to get a rough idea of what we can expect from these potential channels:

g	ρ_g	$B_r(B^0 \to g)$	$B_r(B^0 \to \bar{g})$	$\delta\gamma$
$D^- \pi^+$	0.0178	2.5×10^{-3}	7.3×10^{-7}	8°
$D^{*-} \pi^+$	0.0181	2.7×10^{-3}	5.9×10^{-7}(cal)	7°
$D^- \rho^+$	0.0071	7.6×10^{-3}	3.8×10^{-7}(cal)	11°
$D^{*-} \rho^+$	0.0145	6.8×10^{-3}	14×10^{-7}(cal)	
$D^- a_1^+$		6.0×10^{-3}		
$D^{*-} a_1^+$		13.0×10^{-3}		

Table III. Non CP channels and $\delta\gamma$

In the first column of table III, we have the possible channels, in the third column we have the branching ratios as in the PDG. In the fourth column, (cal) means that these values has been calculated by BaBar [26] by imposing SU(3) flavour symmetry. With those numbers we have calculated the values for ρ_g in the second column. The estimated precision that can be achieved for the extraction of γ is in the last column under the name of $\delta\gamma$. This $\delta\gamma$ has been estimated by assuming that relative errors in $I_d^{L,Sg}$ scale as the inverse of the square root of the number of events.

8. Comments on the Upsilon (5S)

About 90% of all $B_s\bar{B}_s$ pairs produced at the $\Upsilon(5S)$ are $CP = -$ eigenstates, as Atwood and Soni points out [27] and therefore they are entangled states, in its center of mass system, of the form

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}(|B_s^0\rangle|\bar{B}_s^0\rangle - |\bar{B}_s^0\rangle|B_s^0\rangle) \qquad (36)$$

In this system there is an analogous channel to $B_d^0 \to \pi^+ D^-$: $B_s^0 \to D_s^- K^+$. This channel is much more interesting because the ratio $\bar{A}_g/A_g$ is proportional to $|V_{ub}V_{cs}^*/V_{us}V_{cb}^*|\sim 1$ and, as Fleischer and Malami estimates [28], it turns to be:

$$\rho_g = \left|\frac{\bar{A}_g}{A_g}\right| = 0.40 \pm 0.13$$

This situation is more similar, as interference is concerned, to the $\rho\rho$ channel but with the advantage of having a larger branching ratio. Of course, we must not forget that the initial state will be a statistical mixture, that we have to include $\Delta\Gamma$ and that probably for the f channel we have to use the $J/\psi\phi$ channel.

9.Conclusions

We have explained that $B^0 - \bar{B}^0$ entanglement at the $\Upsilon(4S)$ peak has two decay paths to measure interfering phases. With CP eigenstates as decay channels, one can choose CP allowed and CP forbidden decays. The possibility of measuring γ appears: the phases entering in the time evolution, coming from mixing are not needed, neither the strong final state interaction phases. The channels in the double decay rate $\Upsilon(4S) \to (f, g)$ with $f = J/\psi K_S, J/\psi K_S$ and $g = \pi\pi, \rho_L\rho_L$ have a tree level common relative phase γ. $\rho_L^+\rho_L^-$ is the benchmark channel. Combining the channels $(J/\psi K_S, g)$ and $(J/\psi K_L, g)$, general constraints allow the full measurement of all the observables. To extract γ, the proposal has to be completed with an isospin analysis of $B \to \pi\pi, \rho_L\rho_L$, where the mixing phase is not included, so being a different analysis than the one performed to get the phase α. The intrinsic accuracy we estimate, according to Belle II design and to the more optimistic expectations is of 1° [17-18]. The accuracy associated to isospin analysis is 5° according to the actual data (to be improved by LHCb and Belle II...).

Non CP eigenstates in one of the final sates can also be used, in particular we have considered $(S, \pi^+D^-), (L, \pi^+D^-), (S, \pi^-D^+)$ and (L, π^-D^+), where the isospin analysis is not needed, but one has to measure both channels (f, g) and $(f, \bar{g})$. Because one channel is double Cabibbo suppressed respect to the CP conjugate, the interference is suppressed and therefore the precision in the extraction of γ does not improve, even if the statistics increases. This situation could be improved a lot with the decay channels $B_s^0 \to D_s^\mp K^\pm$ and going to the $B_s - \bar{B}_s$ system at the $\Upsilon(5S)$ peak, but several issues have to be fixed previously, before arriving to a definite conclusion.

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References

[1] N. Cabibbo, Unitary Symmetry and Leptonic Decays, Phys.Rev.Lett. 10 (1963) 531-533. And M. Kobayashi and a T. Maskawa, CP Violation in the Renormalizable Theory of Weak Interaction, Prog.Theor.Phys. 49 (1973) 652-657

[2] L-L. Chau and W-Y Keung, Comments on the Parametrization of the Kobayashi-Maskawa Matrix, Phys.Rev.Lett. 53 (1984) 1802-1805

[3] C. Jarlskog, Commutator of the Quark Mass Matrices in the Standard Electroweak Model and a Measure of Maximal CP Non conservation, Phys.Rev.Lett. 55 (1985) 1039

[4] R. Aaij et al. (LHCb Collaboration), Simultaneous determination of CKM angle γ and charm mixing parameters, J. High Energy Phys. 12 (2021) 141.

[5] M. Gronau and D. London, How to determine all the angles of the unitarity triangle from $B_d^0 \to DK_S$ and $B_s^0 \to D\phi$, Phys. Lett. B 253, 483 (1991); M. Gronau and D. Wyler, On determining a weak phase from CP asymmetries in charged B decays, Phys. Lett. B 265, 172(1991)

[6] D.Atwood, I. Dunietz, and A.Soni, Improved methods for observing CP violation in B_ →KD and measuring the CKM phase gamma, Phys. Rev. D 63,036005 (2001).

[7] A. Giri,Y. Grossman,A. Soffer, and J. Zupan, Determining gamma using $B^{\pm}\to DK^{\pm}$with multibody D decays, Phys.Rev. D 68, 054018 (2003).

[8] A. Einstein, B. Podolsky, and N. Rosen, Can quantum mechanical description of physical reality be considered complete? Phys. Rev. 47, 777 (1935).

[9] L. Wolfenstein, CP violation in B0 anti-B0 mixing, Nucl.Phys. B246, 45 (1984).

[10] M. B. Gavela, A. Le Yaouanc, L. Oliver, O. Pene, J. C. Raynal, M. Jarfi, and O. Lazrak, A test of non-superweak CP violation, Phys. Lett. 162B, 197 (1985).

[11] A. F. Falk and A. A. Petrov, Measuring Gamma Cleanly with CP Tagged B(s) and B(d) Decays, Phys. Rev. Lett. 85,252 (2000).

[12] E.Alvarez and J. Bernabeu, Correlated neutral B meson decays into CP eigenstates, Phys. Lett. B579, 79 (2004).

[13] J. Bernabeu and C. Jarlskog, CP violation in decay rates of charged bottom mesons, Z. Phys. C 8,233 (1981).

[14] J. Bernabeu, F. J. Botella, M. Nebot and A. Segarra. $B^0 - \bar{B}^0$entanglement for an ideal experiment for the direct CP violation ϕ_3/γ phase, Phys.Rev.D 106, 054026 (2022).

[15] G. Branco, L. Lavoura and J. P. Silva, CP Violation, *Int.Ser.Monogr.Phys.* 103 (1999) 1-536

[16] J. Bernabeu, F. J. Botella, and M. Nebot, Genuine T, CP, CPT asymmetry parameters for the entangled B_d system, J. High Energy Phys.06 (2016) 100.

[17] J. Bernabeu, F. J. Botella, and M. Nebot, Novel T-Violation observable open to any pair of decay channels at meson factories, Phys. Lett. B 728, 95 (2014).

[18] F. J. Botella, $B^0 - \bar{B}^0$ entanglement for the direct CP violation $\phi_3 = \gamma$ phase, *PoS* DISCRETE2022 (2024) 039.

[19] M. Gronau and D. London, Isospin analysis of CP asymmetries in B decays. Phys.Rev.Lett. 65 (1990) 3381-3384.

[20] F. J. Botella and J. P. Silva, Reparameterization invariance of B decay amplitudes and implications for new physics searches in B decays, Phys. Rev. D 71,094008 (2005).

[21] S. Baek, F. J. Botella, D. London, and J. P. Silva, Can one detect new physics in I=0 and/or I=2 , ys B $\rightarrow\pi\pi$? Phys. Rev. D 72, 036004(2005).

[22] P. Zyla et al. (Particle Data Group), Review of particle physics, Prog.Theor. Exp. Phys. 2020, 083C01(2020).

[23] T. Abe et al. (Belle-II Collaboration), Belle II technical design report, arXiv:1011.0352.

[24] W.Altmannshofer et al.(Belle-II Collaboration), The Belle II physics book, Prog.Theor. Exp. Phys.2019, 123C01(2019); Erratum, Prog.Theor. Exp. Phys.2020, 029201(2020).

[25] F. Forti (Belle-II Collaboration), Snowmass Whitepaper: The Belle II detector upgrade program, in2022 Snowmass Summer Study (2022), arXiv:2203.11349.

[26] B. Aubert et al. (Babar Collaboration), Measurements of the branching fractions of the rare decays $B^0 \to D_s^{(*)+}\pi^-, B^0 \to D_s^{(*)+}\rho^-$ and $B^0 \to D_s^{(*)-}K^{(*)+}$, Phys. Rev. D 78, 032005 (2008).

[27] D. Atwood and A. Soni, Measuring B_s width difference at the $\Upsilon(5S)$ using quantum entanglement, Phys. Rev. D 82, 036003 (2010).

[28] R. Fleischer and E. Malami, Using $B_s^0 \to D_s^{\mp}K^{\pm}$ decays as a portal to new physics, Phys. Rev. D 106, 056004 (2022)