

Predictions for Composite Higgs models from gauge/gravity dualities

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Gauge/gravity dualities provide a very useful approach into solving strongly coupled systems. We apply this to Composite Higgs models and determine the mass hierarchies of the corresponding bound states. As a cross check we apply this to QCD and compare the results to existing lattice calculations for which we find good agreement. We then focus on a particular example whose phenomenology has recently been studied in the literature in a generic way and outline first phenomenological implications of our findings for the spectrum.

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1. Introduction

The extension of the the AdS/CFT correspondence [1–3] to less symmetric gauge/gravity dualities has proven to be a powerful tool in describing group by adding probe branes [4]. This allows to study the related meson operators [5, 6]. These methods were successfully used to obtain gravity duals of chiral symmetry breaking (χSB) in confining non-abelian gauge theories [7, 8]. It is natural to apply this approach to other strongly coupled theories. We focus here on composite Higgs models, see e.g. [9, 10] for reviews. These models are characterised by strongly coupled gauge theory and an underlying set of fermions dubbed hyperquarks in the following.

In composite Higgs models χSB in the fermion sector is caused by a by a strongly coupled gauge theory, similar to QCD. In this way at least four Nambu-Goldstone bosons are generated [11]. By weakly gauging part of the global chiral symmetries, four then pseudo-Nambu Goldstone bosons (pNGBs) can be placed in the fundamental representation of $SU(2)_L$ to become the complex Higgs field. The composite nature of the Higgs removes the huge level of fine tuning in the Standard Model (SM) hierarchy problem. This strong dynamics would occur at a scale of a few TeV, the expected mass scale for the bound states.

We use non-conformal gauge/gravity models that explicitly include the gauge theories' dynamics through the running of the anomalous dimension γ of the hyperquark mass [12, 13]. The models are inspired by top-down models involving probe D7-branes embedded into ten-dimensional supergravity. However, this is combined with a phenomenological approach and sensible guesses for the running of γ which are based on perturbation theory. With this we obtain prediction for part of the mesonic and baryonic spectrum of the theory. Here we will demonstrate that power of this methods by comparing our results with data for QCD bound states. Then we will focus on two models: (i) an $Sp(4)$ theory with 4 fundamental and 6 sextet Weyl fermions [14] for which also lattice studies exist [15, 16]; and an $Sp(4)$ theory with five sextet Weyl and 6 fundamental fermions [17]. Further examples can be found in ref. [13].

2. Gauge/gravity duality, basic idea and some technical aspects

We introduce here briefly the model based on gauge/gravity duality, which was first suggested in ref. [18]. In [13] it has been dubbed *Dynamic AdS/YM* (Anti-de Sitter/Yang-Mills) to emphasise that it can be used to holographically describe the chiral symmetry breaking dynamics of any gauge theory. These models include hyperquarks in several, potentially inequivalent, representations.

We summarize here key elements and refer to [13] for further details. In this modelling, the renormalization group (RG) scale emerges as an extra holographic direction. In a five dimensional AdS_5 space one has the metric

$$ds^2 = \frac{dr^2}{r^2} + r^2 dx_{(1,3)} . \quad (1)$$

The radial direction r is interpreted as the RG scale and slices at some fixed r describe the gauge theory on the corresponding 1 + 3 dimensional sub-space. Fields in the AdS bulk have solutions which describe gauge invariant observables, i.e. operators \mathcal{O} and sources \mathcal{J} .

In QCD with two light flavours the chiral flavour symmetry $SU(2)_L \times SU(2)_R$ is broken to $SU(2)_V$ by the formation of a vacuum expectation value for the operator $\bar{q}_L q_R + \text{h.c.}$ Holographically

the operator is described as a dimension one scalar field L in AdS₅ satisfying the Klein-Gordon equation [6]

$$\frac{\partial}{\partial r} \left(r^3 \frac{\partial L}{\partial r} \right) - r \Delta m^2 L = 0, \quad L = \frac{m}{r^\gamma} + \frac{\langle \bar{q}q \rangle}{r^{2-\gamma}}, \quad \gamma(\gamma - 2) = \Delta m^2. \quad (2)$$

The solutions describe a dimension one mass m and the dimension three quark condensate $\langle \bar{q}q \rangle$ if $\Delta^2 m = 0$. For non-zero Δm^2 these operators develop an anomalous dimension γ .

In AdS₅ the instability bound for a scalar is given by the Breitenlohner-Freedman (BF) bound [19] corresponding to $\Delta m^2 = -1$. The massless $L = 0$ solution becomes unstable if Δm^2 passes through this bound. This corresponds to $\gamma = 1$ or in other words if the dimension of $\bar{q}q$ falls to two then there is an instability due to the generation of a quark condensate and χSB takes place. Note, that this criteria matches that from gap equation analysis in ref. [20]. Therefore, one can make holographic models of χSB which are essentially Higgs-like theories with the scalar field L but whose potential Δm^2 changes with the RG scale r . The potential is determined by the running of the anomalous dimension γ which in turn is determined by the gauge dynamics. Chiral symmetry breaking occurs at the scale where $\gamma = 1$.

Holography allows one to determine the low energy mesonic theory as follows. For a given background vacuum, e.g. a solution for the field L , one allows small fluctuations of the form $\delta(r)e^{ik \cdot x}$ with $k^2 = -M^2$, which describe fluctuations of the $\bar{q}q$ operator, for example the f_0 state in QCD. Generically one obtains a Sturm-Louville system which only has normalizable solutions for particular values of M^2 . Other states such as spin-1 states like the ρ meson can be included via a gauge field dual to $\bar{q}\gamma^\mu q$ and so forth. In this way one obtains the meson spectrum. Substituting these solutions back into the action and integrating over the radial direction r yields a 1 + 3 dimensional theory of the mesons and their interaction couplings. In a similar way one can also include baryons and their couplings with the mesons [13] which eventually also include the Yukawa coupling of the top-quark [12, 13].

Last but not least, we note, that holography is a strong weak duality and so the gravitational dual should only describe the infrared (IR) region of an asymptotically free gauge theory with a large coupling. Models based on gauge/gravity duality show some remnant of the N=4 super Yang-Mills theory left in the description that enforces conformality in the UV. This matches the weak coupling physics of these models and one might be tempted to let the dual extend to the far UV. However, one should cut off the description, e.g. around a few GeV in QCD, where the coupling becomes weak. Here, one can simply impose a cut off in the bulk but then there is a matching problem: one should align the dual to meet at QCD the intermediate coupling. This means the inclusion of some higher dimension operators (HDO) at the UV boundary Λ_{UV} . This can be implemented using Witten's multi-trace prescription [21] as has been detailed in ref. [13] to which refer for further details. The basic idea behind this is, that one catches in this way the first corrections describing the stringy nature of excited states. In ref. [13] the following operators have been considered

$$\frac{g_S^2}{\Lambda_{UV}^2} |\bar{q}q|^2, \quad \frac{g_V^2}{\Lambda_{UV}^2} |\bar{q}\gamma^\mu q|^2, \quad \frac{g_A^2}{\Lambda_{UV}^2} |\bar{q}\gamma^\mu \gamma_5 q|^2, \quad \frac{g_B^2}{\Lambda_{UV}^5} |qqq|^2, \quad (3)$$

with g_i being dimensionless couplings.

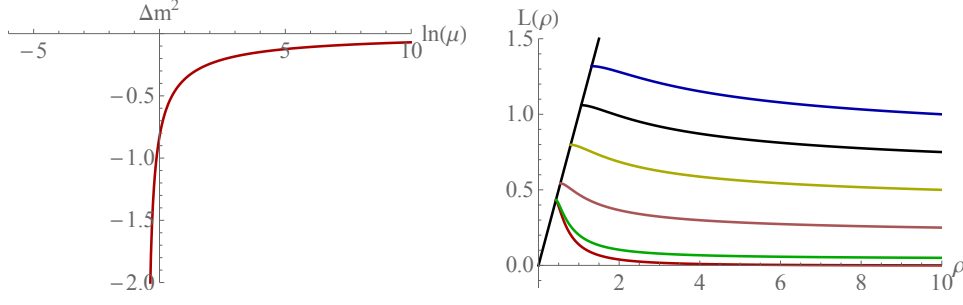


Figure 1: The $N_c = 3, N_f = 2$ QCD model: on the left we display the running of the AdS scalar mass Δm^2 against log RG scale (we use $\mu = \sqrt{\rho^2 + L^2}$ in the holographic model). On the right we show the the vacuum solution for $|X| = L(\rho)$ against ρ . The 45° line is where we apply the on mass shell infrared boundary condition. The $L(\rho)$ with a massless UV quark has $L_{IR} = 0.43$. The quark masses from top to bottom are 1, 0.75, 0.5, 0.25, 0.05, 0. Here units are set by $\alpha(\rho = 1) = 0.65$. See ref. [13] for further details.

We first demonstrate the power of the gauge/gravity duality by applying it to QCD as a test case. The corresponding model is described by the action [13, 18]

$$S = \int d^4x d\rho \text{Tr} \rho^3 \left[\frac{1}{\rho^2 + |X|^2} |DX|^2 + \frac{\Delta^2 m}{\rho^2} + \frac{1}{2g_5^2} (F_V^2 + F_A^2) \bar{\Psi} (\not{D}_{\text{AdS}} - m) \Psi \right] \quad (4)$$

where $X = Le^{2i\pi T}$ describes the quark condensate/ σ and pion fields, F_V the ρ meson and F_A the a meson. Ψ is a Dirac field corresponding to the nucleon. The factors with $r^2 = \rho^2 + |X|^2$ of X in the metric are deduced from top down (probe D7 brane) models [6] and are the simplest ansatz to communicate the background quark condensate to the fluctuation fields.

The starting point is the perturbative results for the running of γ . Expanding $\Delta m^2 = \gamma(\gamma - 2)$ at small γ gives

$$\Delta m^2 = -2\gamma = -\frac{3(N_c^2 - 1)}{2N_c\pi} \alpha \quad (5)$$

with α being the QCD coupling constant. Here we allow ourselves to extend the perturbative results as a function of renormalization RG scale $\mu = r = \sqrt{\rho^2 + L^2}$ to the non-perturbative regime. The resulting running of Δm in the Dynamic AdS/QCD model is shown in fig. 1 on the left – the BF bound is violated close to the scale $r = \mu = 1$.

In table 1 we display our results for the spectrum. Note, that we use here and below the following notation: we label the model as AdS/SU(3) to indicate the gauge group and $2F \ 2\bar{F}$ to show that there are 2 Weyl fermions in the fundamental and two in the anti-fundamental representation which is equivalent to 2 Dirac fermions in the fundamental representation. The second column gives the measured masses of the QCD bound states whereas the third one gives the results for AdS/SU(3) setting all quark masses to zero. Moreover, we have used the ρ -meson mass to set the scale. We see, that the ground states are reasonably well described but the pion decay constant is somewhat low which is partially caused by neglecting the UV quark mass(es). The σ (S) mass is high, but possibly should be compared to the $f_0(980)$ if the $f_0(500)$ is a pion bound state. The proton mass is clearly too high. The first excited states are clearly more off but this had to be expected as no

	QCD	AdS/SU(3) $2F\ 2\bar{F}$		AdS/SU(3) $2F\ 2\bar{F}$	with HDO
M_ρ	775	775*		775*	
M_A	1230	1183	- 4%	1230*	$g_A^2 = 5.76149$
M_S	500/990	973	+64%/-2%	597	+20%/-40%
M_B	938	1451	+43%	938*	$g_B^2 = 25.1558$
f_π	93	55.6	-50%	93*	$g_S^2 = 4.58981$
f_ρ	345	321	- 7%	345*	$g_V^2 = 4.64807$
f_A	433	368	-16%	444	+2.5%
$M_{\rho,n=1}$	1465	1678	+14%	1532	+4.5%
$M_{A,n=1}$	1655	1922	+19%	1789	+8%
$M_{S,n=1}$	990 /1200-1500	2009	+64%/+35%	1449	+46%/0%
$M_{B,n=1}$	1440	2406	+50%	1529	+6%

Table 1: Predictions for masses in MeV for $N_f = 2$ SU(3) gauge theory. The columns show the QCD value [22], the holographic prediction for the theory with massless quarks and the improved holographic QCD theory with HDOs. The ρ -meson mass has been used to set the scale and other quantities that are used to fix the HDO couplings are indicated by *s.

string like structures are taking into account. The situation clearly improves once HDOs are taken into account as shown in the fourth column. However, this comes at the cost of fewer predictions at this stage as we have fitted the coefficients of the HDOs to data because a calculation from first principles is quite difficult.

We note here for completeness, that strictly speaking these results do not fully take into account the flavour structure of the theory, e.g. they do not take into account for example mass splittings between different isospin multiplets nor the fact that the quarks have different UV masses. This can be taken into account by starting with a non-abelian Dirac-Born-Infeld action as has been shown in ref. [23]. Table 2 gives a corresponding results from ref. [23] assuming $m_u = m_d \neq m_s$ and one finds a reasonable agreement between data and our model.

3. Example Sp(4) gauge theories

We focus here on Sp(4) gauge theories which have been proposed in refs. [14, 17, 24, 25] as candidates for CH models. Moreover, phenomenological aspects of a particular example of such models, dubbed M5 in ref. [25], have been recently worked out in a series of papers, see [26–30]. We will briefly comment on the impact of our findings to phenomenological aspects in the subsequent section. We discuss first a realisation that allows for a comparison with lattice data. It contains 4 Weyl fermions in the fundamental representation F . Thus, the global group containing eventually the electroweak sector of the SM is SU(4) which gets broken to Sp(4) via the condensate. One can introduce top partners [14] into the Sp(4) model by including of six additional fermions in the sextet, two index anti-symmetric representation A_2 of the gauge group. In the nomenclature of ref. [25] this is model M8. The condensate breaks the SU(6) global group of this sector to SO(6)

	QCD	AdS/SU(3) 3 F 3 \bar{F}	
$\rho(770), \omega(782)$	775.26 ± 0.23	775*	
$K^*(892)$	891.67 ± 0.26	966	8%
$\phi(1020)$	1019.461 ± 0.016	1120	9%
$a_1(1260), f_1(1260)$	1230 ± 40	1103	11%
$K_1(1400)$	1403 ± 7	1432	2%
$f_1(1420)$	1426.3 ± 0.9	1847	26%
$a_0(980), f_0(980)$	980 ± 20	930	5%
$K_0^*(700)$	845 ± 17	987	16%
$f_0(1370)$	1370	1031	28%
$\pi^{0,\pm}$	139.57039 ± 0.00017	128	9%
$K^{0,\pm}$	497.611 ± 0.013	497	O(0.1) %

Table 2: Meson masses in MeV in the three flavour case compared with the experimental data [22]. We have fixed the masses for the vector bosons ρ and ω as indicated by the * and calculated the masses for the axial vectors, the scalars and the pNGBs. The UV quark masses used are $m_u = m_d = 3.1$ MeV and $m_s = 95.7$ MeV.

which contains SU(3) as subgroup. The full symmetry breaking pattern is given by

$$\text{SU}(4) \times \text{SU}(6) \times \text{U}(1) \rightarrow \underbrace{\text{Sp}(4)}_{\text{SU}(2)_L \times \text{U}(1)} \times \underbrace{\text{SO}(6)}_{\text{SU}(3) \times \text{U}(1)} \times \text{U}(1) \quad (6)$$

where the U(1) factors give eventually the hypercharge.

For the holographic model one needs the running of the gauge coupling as outlined in the previous section, see e.g. [13] for the corresponding formulae. In this model one has two condensates $\langle FF \rangle$ and $\langle A_2 A_2 \rangle$ and the relevant one-loop anomalous dimensions are

$$\gamma_F = \frac{15}{8\pi} \alpha \quad \text{and} \quad \gamma_{A_2} = \frac{6}{2\pi} \alpha. \quad (7)$$

We note, that one would expect the A_2 fermions to condense ahead of the fundamental fields since the corresponding critical value for α is smaller. Once the A_2 s condense, SU(6) is broken to SO(6). At this point the A_2 s become massive but it is unclear how quickly they decouple from the running of α . This is still an open question and we refer to ref. [13] for a first discussion.

We give in table 3 the spectrum for various approximations which are needed on the one hand to estimate on the on hand the uncertainty due to the decoupling of the A_2 fermions and on the other hand for comparison with lattice data. The first column gives our results without approximation for the various bound states: scalar S , axial spin-1 A , vector spin-1 V and baryons B . Mesonic bound states which are either A_2 or F bound states carry a corresponding index. There difference in the masses arises as the different vacuum solutions of their respective condensate enter their equations of motions [13]. The baryons are FA_2F bound states. It is still an open question how to precisely model such states within gauge/gravity duality. We compute their masses using either the vacuum of the F condensate or the one of the A_2 condensate as indicated by the corresponding subscript.

	AdS/Sp(4) no decouple	AdS/Sp(4) A2 decouple	AdS/Sp(4) quench	lattice [15] quench	lattice [16] unquench	AdS/Sp(4) + NJL
M_{VA_2}	1*	1*	1*	1.000(32)		1*
M_{VF}	0.61	0.814	0.962	0.83(19)	0.83(27)	1.03
M_{AA_2}	1.35	1.35	1.28	1.75 (13)		1.35
M_{AF}	0.938	1.19	1.36	1.32(18)	1.34(14)	1.70
M_{SA_2}	0.375	0.375	1.14	1.65(15)		0.375
M_{SF}	0.325	0.902	1.25	1.52 (11)	1.40(19)	0.375
M_{BA_2}	1.85	1.85	1.86			1.85
M_{BF}	1.13	1.53	1.79			1.88

Table 3: AdS/Sp(4) $4F$ $6A_2$. Ground state spectra for various gauge/gravity models and comparison to lattice results. The subscript A_2 and F indicate the quantity in each of the two different representation sectors. Note here for the unquenched lattice results, which do not include the A_2 fields, we have normalized the F vector meson mass to that of the quenched computation.

For the results in the first/second column we do not/do decouple the A_2 fermions at the scale where the A_2 condense. This clearly does not affect the masses of the A_2 mesons which is the reason for us to fix the overall scale by the mass of the vector spin-1 state VA_2 . The decoupling of the A_2 implies an increase in the running of α which in turn leads to the observed increase of the masses with F index in the second column. Independent of this uncertainty we see that the A_2 mesons are heavier than the F mesons. Note, that in particular the mass of the scalar SV depends strongly on the decoupling behaviour of the A_2 fermions. In case of the baryons, the variation gives a first measure of the uncertainty on the underlying mass calculation. Last but not least we have added a NJL term to enforce an equal scale of condensation and the corresponding results are given in the last column of this table.

Lattice studies of this model have been made in ref. [15] in the quenched approximation. Within gauge/gravity duality the quenching corresponds to neglecting the hyperquark contributions to the running of the gauge coupling. This implies a steeper running of the coupling resulting in a more compressed spectrum as can be seen in the third column of table 3. In [16] the group followed up that work of [15] by unquenching the fundamental hyperquark sector using Wilson fermions. We show the results of both lattice studies in column 4 and 5 of this table for direct comparison to the holographic results. The quenched results from both, the lattice and the holographic model, show considerable correlation. This provides confidence that trends as the fields are unquenched may be trustworthy. Consequently we would expect that if the A_2 fermions were included as unquenched fields, the F sector would decrease in mass by about 20-40%. Moreover, we also expect the scalar meson masses to be considerably lower than predicted by the quenched lattice computation.

A huge advantage of calculations in the gauge/gravity framework is, that one can easily incorporate changes in the number of the underlying fermions which is much more difficult in case of lattice calculations. We take as an example the M5 model of ref. [25] which contains 5 A_2 and

	M_{VA_2}	M_{VF}	M_{AA_2}	M_{AF}	M_{SA_2}	M_{SF}	M_{BA_2}	M_{BF}
$4F\ 6A_2$	1*	0.618	1.4	0.862	0.376	0.348	1.85	1.15
$5A_2\ 6F$	1*	0.61	1.35	0.938	0.38	0.33	1.85	1.13

Table 4: For the masses in Sp(4) theories with two different matter representations that can trigger chiral symmetry breaking: $4F\ 6A_2$ (M8 model) and $5A_2\ 6F$ (M5 model).

6 F corresponding to the following breaking pattern of the global group

$$SU(5) \times SU(6) \times U(1) \rightarrow \underbrace{SO(5)}_{SU(2)_L \times U(1)} \times \underbrace{Sp(6)}_{SU(3) \times U(1)} \times U(1) \quad (8)$$

where again the U(1) factors give eventually the hypercharge. The resulting masses are given in table 4. As one would naively expect, the results are quite similar as we have changed the fermion content only slightly. However, note that the embedding of the SM in the corresponding global groups differ in both models and, thus, consequently the phenomenological implications. We take as an example the spin-1 vectors states: in case of $4F\ 6A_2$ ($5A_2\ 6F$) the ones carrying electroweak quantum numbers will be significantly lighter (heavier) compared to the ones carrying SU(3) quantum numbers.

4. Phenomenological aspects

We would like to emphasize that the implications of our results for phenomenological investigations discussed below have to be taken with care. The results presented here so far take are based originally on an abelian Born-Dirac-Infeld action and, thus, they do not take into account possible multiplet structures within a given type of bound states. Taking for example the M5 model, there is actually not one spin-1/2 state but several of them [26] which transform as follows under the SM-gauge group $SU(3) \times SU(2)_L \times U(1)_Y$: $(3, 2, \pm 7/6)$, $(3, 2, \pm 1/6)$, $(3, 1, \pm 2/3)$, $(8, 2, \pm 1/2)$, $(8, 1, 0)$, $(1, 2, \pm 1/2)$ and $(1, 1, 0)$. The colour triplets are the usual top-partners. For a proper calculation of the corresponding masses one has to consider a non-abelian Born-Dirac-Infeld action as starting which complicates the calculation quite a bit. However, we have recently started this for some simple cases, $SU(3) \times SU(3)/SU(3)$ [23] and $SU(4)/Sp(4)$ [31]. While these are not applicable for the M5 model directly, one can nevertheless infer some requirements needed for a successful description.

In [26] it has been shown that the M5 model contains an accidental ‘baryon number’ implying that either one of the baryons or the coloured pNGB transforming as $(3, 1, 2/3)$, called π_3 in [26], has to be stable. Obviously this should be the $(1, 1, 0)$ baryon, called \tilde{B} in [26], which could then serve as a DM candidate. It might surprise at first glance that a baryon should be lighter than a pNGB. Such a behaviour can in principle occur if the underlying hyperquarks have mass terms which break the global groups explicitly as has been shown in [13, 31]. In such a scenario π_3 would decay into $\tilde{B}t$ like the right-handed scalar top in supersymmetric models if there is sufficient phase space. In case of smaller mass differences, three-body decays via an off-shell top-quark would

become important, again similar to supersymmetric models [32–34]. Actually, this scenario could easily be confused with supersymmetric models at first glance as several signatures are very similar.

Assuming, that the results in table 4 still give the same rough patterns, the coloured baryons would be significantly heavier than the coloured spin-1 resonances represented by VF and AF , which actually represents different $SU(3)$ states including V_8^0 and $A_3^{-2/3}$ [30]. Here the subscripts give the $SU(3)$ representation and the superscripts the electric charge. These would yield completely new decay channels of the baryons such as

$$\tilde{g} \rightarrow A_6^{-2/3} t \rightarrow \pi_8 \pi_8 \pi_3^* t \rightarrow 4t + 2\bar{t} + \tilde{B} \quad (9)$$

$$T_R \rightarrow V_8^0 t \rightarrow \pi_3 \pi_3^* t \rightarrow 2t + \bar{t} + 2\tilde{B}. \quad (10)$$

which to our knowledge have not yet been considered in the literature up to now. We denoted here the $(8, 1, 0)$ and $(3, 1, 2/3)$ baryons by \tilde{g} and T_R , respectively. Moreover, π_8 are strongly interacting pNGBs decaying dominantly into $t\bar{t}$. Note also, that \tilde{B} would lead to missing transverse momentum in the signal. The model M5 contains also an extended pNGB sector in the electroweak sector containing for example a doubly charged scalar S^{++} which has similar quantum numbers as the one from type II seesaw models [35, 36] and thus the same production cross section at the LHC [28]. However, it has a quite different dominant decay mode [28], namely

$$S^{++} \rightarrow t\bar{b}W^+. \quad (11)$$

The resulting increased hadronic activity implies that bounds on its mass are quite weak [28]. This scalar pNGB can also be produced in the decays of top-partners. For example, it leads to an additional decay channel for the $X_{5/3}$ baryon from the $(3, 2, 7/6)$ multiplet:

$$X_{5/3} \rightarrow W^+ t \quad (\text{standard channel}) \quad (12)$$

$$X_{5/3} \rightarrow S^{++} b \rightarrow t\bar{b}W^+ \quad (\text{new channel}). \quad (13)$$

The importance of the new channel depends of course what is the mass of the S^{++} compared to the baryons. We are investigating currently how important this and related decay modes are in scenarios in which the required mass splitting of the baryons can be achieved by giving different masses to the underlying fermions. We note for completeness, that exotic signatures of top-partners haven been discussed in [27, 37–39].

5. Conclusions and outlook

We have shown in this contribution that gauge/gravity duality methods are a powerful tool for obtaining sensible estimates for masses of the bound states, both, in QCD as well as in Composite Higgs models. We have seen that reasonable agreement with data in case of QCD and with lattice data in case of a specific Composite Higgs model are obtained. The advantage of this framework is, that it allows for a relatively fast computations of the spectrum, more precisely ratio of masses. Most of the results have been obtained neglecting the fact the bound states form multiplets of the unbroken global sub-group which can have somewhat different masses. We have started recently to take this into account, see ref. [31], for the case of $SU(4)/Sp(4)$. This will be extended in different directions in the future: (i) we will take into account fermionic bound states. (ii) We will systematically explore the models proposed in [25]. (iii) We will study the impact of our results on the phenomenology of these models in a coherent global picture.

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