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Alleviating both H_0 and σ_8 tensions through Tsallis entropy

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We present how Tsallis cosmology can alleviate both H_0 and σ_8 tensions simultaneously. Such a modified cosmological scenario is obtained by the application of the gravity-thermodynamics conjecture, but using the non-additive Tsallis entropy, instead of the standard Bekenstein-Hawking one. Hence, one obtains modified Friedmann equations, with extra terms that depend on the new Tsallis exponent that quantifies the departure from standard entropy. We show that for particular choices we can obtain a phantom effective dark energy, which is known to be one of the sufficient mechanisms that can alleviate H_0 tension. Additionally, for the same parameter choice we obtain an increased friction term and an effective Newton's constant smaller than the usual one, and thus the σ_8 tension is also solved. These features act as a significant advantage of Tsallis modified cosmology.

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1. Introduction

The Standard Paradigm of Cosmology, namely ΛCDM concordance model, has been proven very successful in describing the universe evolution, at both background and perturbative levels. Nevertheless, it exhibits possible disadvantages, either theoretical or observational [1]. In the first class of potential issues one has the non-renormalizability of general relativity or the cosmological constant problem. In the second class one may find the dynamical behavior of dark energy, the realization of the inflationary phase, as well as various cosmological tensions.

Among cosmological tensions, one has the H_0 tension, namely the fact that the present value of the Hubble parameter is estimated by Planck collaboration to be $H_0 = (67.27 \pm 0.60)$ km/s/Mpc [2], whereas local measurements of the 2019 SH0ES collaboration (R19) lead to $H_0 = (74.03 \pm 1.42)$ km/s/Mpc. Additionally, one may have the σ_8 tension, which is related to the matter clustering, and the fact that the Cosmic Microwave Background (CMB) estimation [2] differs from the SDSS/BOSS direct measurement [3, 4].

If these tensions are not due to unknown systematics, they probably need a modification of standard lore in order to be alleviated. There are many directions one can follow in order achieve this. In particular, since $\theta_s = \frac{r_s}{D_A}$, where $r_s \propto \int_0^{t_{recom}} dt \frac{c_s(t)}{\rho(t)}$ is the sound horizon and $D_A \propto \frac{1}{H_0} \int_{t_{recom}}^{t_{today}} dt \frac{1}{\rho(t)}$ is the angular diameter distance, one could try to change either r_s or D_A or both. Solutions that affect r_s are referred to as "early-time" solutions, and solutions that alter D_A are called "late-time" solutions. Hence, in the literature one can find a large class of solutions, including modified gravity, early dark energy, extra relativistic degrees of freedom, bulk viscous models, clustering dark energy, holographic dark energy, interacting dark energy, running vacuum models, Horndeski theories, decaying dark matter, string-inspired models, etc [5–58] (for a review see [59]).

In the present work we are interested in presenting a novel alleviation of both H_0 and σ_8 tension, obtained in the framework of Tsallis cosmology [60]. Such a modified cosmological scenario arises from the application of the standard gravity-thermodynamics conjecture, namely the procedure to obtain the Friedmann equations from the first law of thermodynamics applied in the universe horizon [61–63], but using Tsallis non-additive entropy [64–66], instead of the usual Bekenstein-Hawking one. Tsallis cosmology has been shown to lead to interesting cosmological phenomenology [67–76]. Nevertheless, in the following we will show that in such a framework we can obtain phantom behavior for the effective dark-energy sector, which is one of the sufficient mechanisms that can alleviate the H_0 tension, as well as an increased friction term in the matter-perturbation evolution equation, which leads to smaller σ_8.

The plan of the work is the following: In Section 2 we briefly review Tsallis cosmology, both at the background and perturbative levels. Then in Section 3 we show how Tsallis cosmology can lead to the alleviation of both H_0 and σ_8 tensions simultaneously. Finally, in Section 4 we summarize the obtained results.

2. Modified cosmology through Tsallis entropy

In this section we briefly review Tsallis cosmology. Tsallis non-additive entropy [64–66] generalizes the standard thermodynamics to non-extensive one, and it possess the standard Boltzmann-

Gibbs statistics as a limit. In cosmology, this is characterized by an exponent δ, and hence the Tsallis entropy can be written in the form [77]

$$S_T = \frac{\tilde{\alpha}}{4G} A^{\delta}, \tag{1}$$

in units where $\hbar = k_B = c = 1$. In the above expression G is the gravitational constant, $A \propto L^2$ is the area of the system with characteristic length L, $\tilde{\alpha}$ is a positive constant with dimensions $[L^{2(1-\delta)}]$ and δ is the non-additivity parameter. As mentioned in the Introduction, in the case $\delta = 1$ and $\tilde{\alpha} = 1$, Tsallis entropy recovers the standard Bekenstein-Hawking additive entropy.

We consider a Friedmann-Robertson-Walker (FRW) metric of the form

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \tag{2}$$

with $a(t)$ the scale factor, and $k = 0, +1, -1$ the spatial curvature. We substitute Tsallis entropy (1) into the first law of thermodynamics $-dE = TdS$, and we perform all the steps of gravity-thermodynamics conjecture [61–63]. Specifically, we consider the boundary of the system to be the Universe apparent horizon $\tilde{r}_a = (H^2 + \frac{k}{a^2})^{-1}$, having temperature $T = 1/(2\pi r_h)$, and being filled by the universe matter fluid, with energy density ρ_m and pressure p_m [78–91]. Hence, this leads to (we consider only the more physically interesting case $\delta \neq 2$) [60]

$$-\frac{(4\pi)^{2-\delta}G}{\tilde{\alpha}}(\rho_m + p_m) = \delta \frac{\dot{H} - \frac{k}{a^2}}{\left(H^2 + \frac{k}{a^2}\right)^{\delta-1}}, \tag{3}$$

and this by integration to

$$\frac{2(4\pi)^{2-\delta}G}{3\tilde{\alpha}}\rho_m = \frac{\delta}{2-\delta}\left(H^2 + \frac{k}{a^2}\right)^{2-\delta} - \frac{\tilde{\Lambda}}{3\tilde{\alpha}}, \tag{4}$$

with dots denoting time-derivatives, and where $\tilde{\Lambda}$ is an integration constant, which can be considered as the cosmological constant. For the non-extensive scenario with Tsallis entropy, equations (3) and (4) are the two modified Friedmann equations. Focusing on flat geometry, i.e. $k = 0$, we can re-write them into the standard form

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_{DE}\right) \tag{5}$$

$$\dot{H} = -4\pi G\left(\rho_m + p_m + \rho_{DE} + p_{DE}\right), \tag{6}$$

where we have defined an effective dark energy density and pressure as [60]

$$\rho_{DE} = \frac{3}{8\pi G}\left\{\frac{\Lambda}{3} + H^2\left[1 - \alpha\frac{\delta}{2-\delta}H^{2(1-\delta)}\right]\right\}, \tag{7}$$

$$p_{DE} = -\frac{1}{8\pi G}\left\{\Lambda + 2\dot{H}\left[1 - \alpha\delta H^{2(1-\delta)}\right] + 3H^2\left[1 - \alpha\frac{\delta}{2-\delta}H^{2(1-\delta)}\right]\right\}, \tag{8}$$

as well as the new constants $\Lambda \equiv (4\pi)^{\delta-1}\tilde{\Lambda}$ and $\alpha \equiv (4\pi)^{\delta-1}\tilde{\alpha}$. In these lines, the equation-of-state parameter for the effective dark energy is

$$w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} = -1 - \frac{2\dot{H}\left[1 - \alpha\delta H^{2(1-\delta)}\right]}{\Lambda + 3H^2\left[1 - \frac{\alpha\delta}{2-\delta}H^{2(1-\delta)}\right]}. \tag{9}$$

We mention that for $\delta = 1$ and $\alpha = 1$ the above expressions recover the standard ones as expected.

Let us elaborate the aforementioned equations. For simplicity we consider dust matter ($p_m = 0$) and we introduce the density parameters through

$$\Omega_m = \frac{8\pi G}{3H^2}\rho_m \tag{10}$$

$$\Omega_{DE} = \frac{8\pi G}{3H^2}\rho_{DE}. \tag{11}$$

Doing so, the Hubble parameter can be written as

$$H = \frac{\sqrt{\Omega_{m0}}H_0}{\sqrt{a^3(1-\Omega_{DE})}}, \tag{12}$$

where "0" denotes the present value of the corresponding quantity. In the following it proves more convenient to introduce the redshift z, defined as $1 + z = 1/a$, having imposed the present scale factor to 1. Substituting (7) into (11) and taking into account (12) we acquire

$$\Omega_{DE}(z) = 1 - H_0^2\Omega_{m0}(1+z)^3\left\{\frac{(2-\delta)}{\alpha\delta}\left[H_0^2\Omega_{m0}(1+z)^3 + \frac{\Lambda}{3}\right]\right\}^{\frac{1}{\delta-2}}, \tag{13}$$

while from (9) we find [60]

$$w_{DE}(z) = -1 + \frac{\left\{3[1-\Omega_{DE}(z)] + (1+z)\Omega'_{DE}(z)\right\}\left\{1 - \alpha\delta\left[\frac{H_0^2\Omega_{m0}(1+z)^3}{1-\Omega_{DE}(z)}\right]^{1-\delta}\right\}}{[1-\Omega_{DE}(z)]\left\{\frac{\Lambda[1-\Omega_{DE}(z)]}{H_0^2\Omega_{m0}(1+z)^3} + 3\left\{1 - \frac{\alpha\delta}{2-\delta}\left[\frac{H_0^2\Omega_{m0}(1+z)^3}{1-\Omega_{DE}(z)}\right]^{1-\delta}\right\}\right\}}, \tag{14}$$

where primes denote differentiation with respect to z. Finally, note that applying (13) at present time ($z = 0$) we acquire a relation of the parameters, namely

$$\Lambda = \frac{3\alpha\delta}{2-\delta}H_0^{2(2-\delta)} - 3H_0^2\Omega_{m0}, \tag{15}$$

from which we deduce that our model has two extra free parameters, namely α and δ.

We close this section by examining Tsallis cosmology at the perturbative level. Introducing as usual the matter overdensity $\delta_m := \delta\rho_m/\rho_m$, and focusing without loss of generality on the case $\alpha = 1$, one can show that its evolution equation is given by [92]

$$\delta''_m + \frac{2(4-2\delta) - \left(9-6\delta+8\pi G\Lambda H^{2\delta-4}\right)}{(4-2\delta)(1+z)}\delta'_m + \frac{3^{\frac{1}{\delta-2}}\left[(1-2\delta)\rho_m^{\frac{1}{2-\delta}} - 9(1-\delta)\Lambda\rho_m^{\frac{\delta-1}{2-\delta}}\right]8\pi G}{2(2-\delta)^2H^2(1+z)^2}\delta_m = 0, \tag{16}$$

with Λ given by (15). Note that comparing to standard result, in the above expression we have a different friction term (the second term), as well as an effective Newton's constant (the last term). Clearly, in the case $\delta = 1$, one recovers the standard result, which in the case of matter domination ($\Omega_m \approx 1$) gives

$$\delta_m'' + \frac{1}{2(1+z)}\delta_m' - \frac{3}{2(1+z)^2}\delta_m = 0 \tag{17}$$

as expected. Lastly, after obtaining the solution for $\delta_m(z)$, we can calculate the physically interesting observational quantity

$$f\sigma_8(z) = f(z)\,\sigma(z), \tag{18}$$

with $f(z) := -\frac{d\ln\delta_m(z)}{d\ln z}$ and $\sigma(z) := \sigma_8\frac{\delta_m(z)}{\delta_m(0)}$ [59].

3. Alleviating H_0 and σ_8 tensions

Let us now investigate how the scenario of Tsallis cosmology can alleviate both H_0 and σ_8 tensions. As we observe, the Tsallis exponent δ affects both the background and perturbation evolution. As it was in [59] and in [93, 94], one of the efficient mechanisms that can alleviate the H_0 tension is to obtain an effective dark-energy equation-of-state parameter lying in the phantom regime, while one of the efficient mechanisms that can alleviate the σ_8 tension is to obtain an increased friction term or a smaller effective Newton's constant in the evolution equation of δ_m. Hence, our strategy will be to choose δ in order to fulfill the above requirements.

In ΛCDM cosmology the Hubble function is given by the relation

$$H_{\Lambda CDM}(z) \equiv H_0\sqrt{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}}. \tag{19}$$

On the other hand, in Tsallis cosmology the Hubble function is given by (12) with $\Omega_{DE}(z)$ provided by (13). Hence, we can choose model parameters which our $H(z)$ coincides with $H_{\Lambda\mathrm{CDM}}(z)$ of (19) at $z = z_{\mathrm{CMB}} \approx 1100$, namely $H(z \to z_{\mathrm{CMB}}) \approx H_{\Lambda\mathrm{CDM}}(z \to z_{\mathrm{CMB}})$, but give $H(z \to 0) > H_{\Lambda\mathrm{CDM}}(z \to 0)$. Finally, as usual, we desire to have the standard evolution of Ω_m and Ω_{DE}, with the sequence of matter and dark-energy epochs, and with $\Omega_{m0} \approx 0.31$, according to observations [2]. We mention here that the range of values $\{\delta, \Omega_{m0}\}$ that we are using is well within the observational bounds in these kinds of theories [60, 95].

In Fig. 1 we depict the normalized $H(z)/(1+z)^3$ as a function of the redshift parameter, for ΛCDM scenario, as well as for Tsallis cosmology for various values of the entropic exponent δ. As we observe, for small deviations of δ below the standard entropy value $\delta = 1$ we can have a coincidence to ΛCDM cosmology at high and intermediate redshifts, while at small redshifts the modified Tsallis scenario stabilizes in higher values of H_0. The dependence of the H_0 value on δ, leads to the estimation $H_0 \approx 74$ km/s/Mpc for $\delta = 0.993$. Hence, Tsallis cosmology, with δ values slightly less than the standard value, can indeed alleviate H_0 tension ($\delta > 1$ values do not lead to alleviation).

As an additional verification, we confront the derived evolution with Cosmic Chronometer (CC) data [97], namely datasets based on $H(z)$ measurements through the relative ages of massive passively evolving galaxies [96]. In Fig. 2 we present $H(z)$ for ΛCDM scenario and for Tsallis

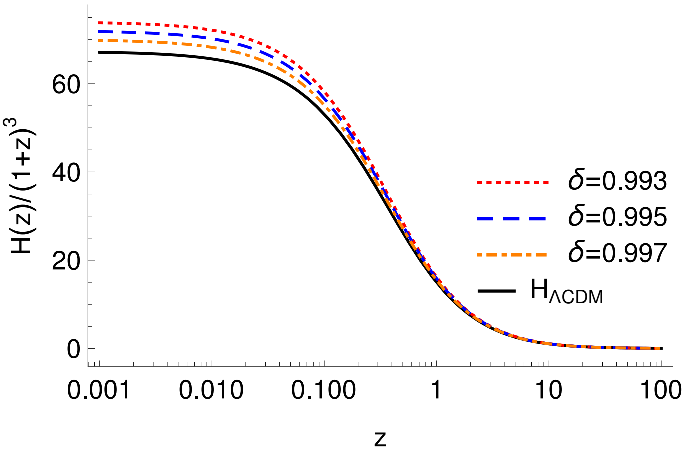

Figure 1: *The normalized $H(z)/(1+z)^3$ in units of $km/s/Mpc$ as a function of the redshift, for ΛCDM cosmology (black-solid) and for Tsallis cosmology for $\alpha = 1$ and for $\delta = 0.993$ (red - dotted), $\delta = 0.995$ (blue - dashed) and $\delta = 0.997$ (orange - dashed-dotted). We have imposed $\Omega_{m0} \approx 0.31$.*

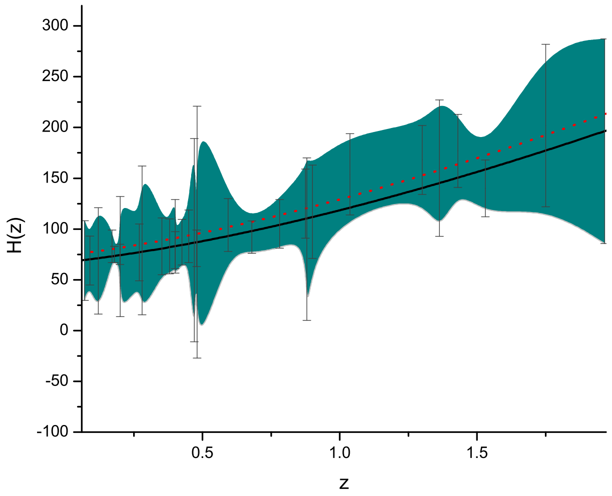

Figure 2: *The $H(z)$ evolution in units of km/s/Mpc as a function of the redshift, for ΛCDM scenario (black line) and for Tsallis cosmology with $\alpha = 1$ and $\delta = 0.993$ (red dotted line), on top of the CC data points at 2σ confidence level [96]. We have imposed $\Omega_{m0} \approx 0.31$.*

cosmology, on top of CC data from [96] at 2σ confidence level. As we see, the agreement is very good, $H(z)$ of Tsallis cosmology lies within the CC data, exhibiting a slightly higher accelerating behavior compared to that of the ΛCDM model at low redshifts, for the parameter sets $\{\Omega_{m0}, \delta\} = \{0.31, 0.993\}$.

Let us now examine what is the mechanism behind the H_0 tension alleviation. In Fig. 3 we present the evolution of the dark-energy equation-of-state parameter (14), for the value of the parameter δ that alleviates the tension. As we observe, it lies in the phantom regime, which as we mentioned above is one of the sufficient ways to alleviate the tension [93, 94]. Note that according to

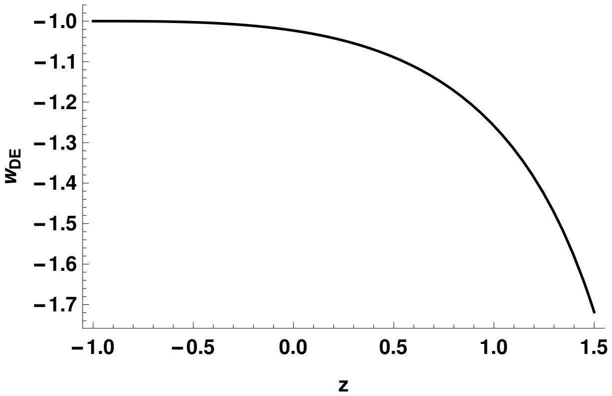

Figure 3: *The evolution of the dark-energy equation-of-state parameter w_{DE} as a function of the redshift, for Tsallis cosmology with $\alpha = 1$ and $\delta = 0.993$.*

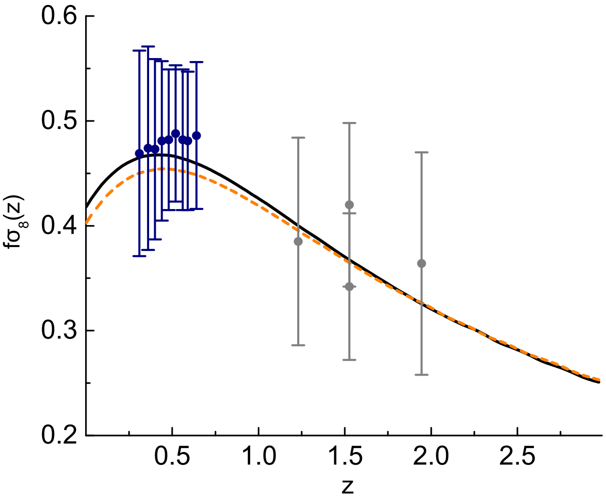

Figure 4: *Evolution of $f\sigma_8$ in ΛCDM scenario (black solid) and in Tsallis cosmology with $\alpha = 1$ and $\delta = 0.993$ (orange dashed). The blue data points are from Baryonic Acoustic Oscillations (BAO) observations in SDSS-III DR12 [98], while the gray data points at higher redshifts are from SDSS-IV DR14 [99–101].*

(14) in principle $w_{DE}(z)$ can be quintessence-like, phantom-like, or experience the phantom-divide crossing during the evolution, according to the value of δ, and this is the reason we we constrained our analysis to values $\delta \lesssim 1$, since these are needed to obtain the appropriate amount of phantom behavior (concerning other potential astrophysical implications of phantom behavior see [102]). Finally, note that we choose to extend Fig. 3 to negative redshifts, namely to the future, in order to show that in the scenario at hand as time passes the phantom behavior becomes less and less significant, and that the dark energy sector will asymptotically behave as a cosmological constant.

We now proceed to the examination of the σ_8 tension. As we mentioned above, the evolution equation of matter overdensity δ_m is given by (16). In Fig. 4, we present the evolution of $f\sigma_8$ for ΛCDM scenario, as well as for Tsallis cosmology, on top of observational data. As we observe, Tsallis cosmology can indeed reduce $f\sigma_8$ and alleviate σ_8 tension too, for the same parameter choice that can alleviate H_0 tension. This simultaneous alleviation of the tensions is not easy to be obtained in alternative cosmological scenarios, and it is the main result of the present work.

Finally, let us examine the mechanism behind the σ_8 tension alleviation. As we observe from (16), the scenario at hand has a different friction term as well as an effective Newton's constant. One can see that under the above parameter choice, we obtain an increased friction and an effective Newton's constant smaller than the usual one. And this indeed serves as one of the sufficient mechanisms to alleviate σ_8 tension [93, 94].

4. Conclusions

In this work we presented how Tsallis cosmology can alleviate both H_0 and σ_8 tensions simultaneously. Such a modified cosmological scenario is obtained by the application of the gravity-thermodynamics conjecture, but using the non-additive Tsallis entropy, instead of the standard Bekenstein-Hawking one. Hence, one obtains modified Friedmann equations, with extra terms that depend on the new Tsallis exponent δ that quantifies the departure from standard entropy.

In Tsallis cosmology one acquires an effective dark-energy sector with equation-of-state parameter that can be quintessence-like or phantom-like. Additionally, at the perturbative level one extracts the evolution equation for matter overdensity, with extra terms in the friction term as well as in the effective Newton's constant.

As we showed, for particular choice of the Tsallis parameter δ we can obtain a phantom effective dark energy, which is known to be one of the sufficient mechanisms that can alleviate H_0 tension. Interestingly enough, for the same parameter choice we obtain an increased friction term and an effective Newton's constant smaller than the usual one, and thus the σ_8 tension is also solved.

In summary, Tsallis cosmology has the capability to simultaneously mitigate both H_0 and σ_8 tensions. Note that in general this is not easily obtained in alternative cosmological scenarios [59]. Hence, this attribute stands as a considerable advantage of Tsallis modified cosmology.

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References

[1] L. Perivolaropoulos and F. Skara, *Challenges for ΛCDM: An update*, *New Astron. Rev.* **95** (2022) 101659 [`2105.05208`].

[2] Planck collaboration, *Planck 2018 results. VI. Cosmological parameters*, *Astron. Astrophys.* **641** (2020) A6 [`1807.06209`].

[3] P. Zarrouk et al., *The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample: measurement of the growth rate of structure from the anisotropic correlation function between redshift 0.8 and 2.2*, Mon. Not. Roy. Astron. Soc. **477** (2018) 1639 [`1801.03062`].

[4] BOSS collaboration, *The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample*, Mon. Not. Roy. Astron. Soc. **470** (2017) 2617 [`1607.03155`].

[5] E. Di Valentino, O. Mena, S. Pan, L. Visinelli, W. Yang, A. Melchiorri et al., *In the realm of the Hubble tension—a review of solutions*, Class. Quant. Grav. **38** (2021) 153001 [`2103.01183`].

[6] E. Di Valentino et al., *Snowmass2021 - Letter of interest cosmology intertwined II: The hubble constant tension*, Astropart. Phys. **131** (2021) 102605 [`2008.11284`].

[7] E. Di Valentino, A. Melchiorri and J. Silk, *Beyond six parameters: extending ΛCDM*, Phys. Rev. D **92** (2015) 121302 [`1507.06646`].

[8] B. Hu and M. Raveri, *Can modified gravity models reconcile the tension between the CMB anisotropy and lensing maps in Planck-like observations?*, Phys. Rev. D **91** (2015) 123515 [`1502.06599`].

[9] J.L. Bernal, L. Verde and A.G. Riess, *The trouble with H_0*, JCAP **10** (2016) 019 [`1607.05617`].

[10] S. Kumar and R.C. Nunes, *Probing the interaction between dark matter and dark energy in the presence of massive neutrinos*, Phys. Rev. D **94** (2016) 123511 [`1608.02454`].

[11] N. Khosravi, S. Baghram, N. Afshordi and N. Altamirano, *H_0 tension as a hint for a transition in gravitational theory*, Phys. Rev. D **99** (2019) 103526 [`1710.09366`].

[12] E. Di Valentino, A. Melchiorri and O. Mena, *Can interacting dark energy solve the H_0 tension?*, Phys. Rev. D **96** (2017) 043503 [`1704.08342`].

[13] E. Di Valentino, C. Bøehm, E. Hivon and F.R. Bouchet, *Reducing the H_0 and σ_8 tensions with Dark Matter-neutrino interactions*, Phys. Rev. D **97** (2018) 043513 [`1710.02559`].

[14] E. Di Valentino, A. Melchiorri, E.V. Linder and J. Silk, *Constraining Dark Energy Dynamics in Extended Parameter Space*, Phys. Rev. D **96** (2017) 023523 [`1704.00762`].

[15] J. Solà, A. Gómez-Valent and J. de Cruz Pérez, *The H_0 tension in light of vacuum dynamics in the Universe*, Phys. Lett. B **774** (2017) 317 [`1705.06723`].

[16] W. Yang, S. Pan, E. Di Valentino, R.C. Nunes, S. Vagnozzi and D.F. Mota, *Tale of stable interacting dark energy, observational signatures, and the H_0 tension*, JCAP **09** (2018) 019 [`1805.08252`].

[17] F. D'Eramo, R.Z. Ferreira, A. Notari and J.L. Bernal, *Hot Axions and the H_0 tension*, *JCAP* **11** (2018) 014 [1808.07430].

[18] V. Poulin, T.L. Smith, T. Karwal and M. Kamionkowski, *Early Dark Energy Can Resolve The Hubble Tension*, *Phys. Rev. Lett.* **122** (2019) 221301 [1811.04083].

[19] A. El-Zant, W. El Hanafy and S. Elgammal, *H_0 Tension and the Phantom Regime: A Case Study in Terms of an Infrared $f(T)$ Gravity*, *Astrophys. J.* **871** (2019) 210 [1809.09390].

[20] S. Basilakos, S. Nesseris, F.K. Anagnostopoulos and E.N. Saridakis, *Updated constraints on $f(T)$ models using direct and indirect measurements of the Hubble parameter*, *JCAP* **08** (2018) 008 [1803.09278].

[21] S.A. Adil, M.R. Gangopadhyay, M. Sami and M.K. Sharma, *Late-time acceleration due to a generic modification of gravity and the Hubble tension*, *Phys. Rev. D* **104** (2021) 103534 [2106.03093].

[22] R.C. Nunes, *Structure formation in $f(T)$ gravity and a solution for H_0 tension*, *JCAP* **05** (2018) 052 [1802.02281].

[23] W. Yang, S. Pan, E. Di Valentino and E.N. Saridakis, *Observational constraints on dynamical dark energy with pivoting redshift*, *Universe* **5** (2019) 219 [1811.06932].

[24] S. Pan, W. Yang, E. Di Valentino, E.N. Saridakis and S. Chakraborty, *Interacting scenarios with dynamical dark energy: Observational constraints and alleviation of the H_0 tension*, *Phys. Rev. D* **100** (2019) 103520 [1907.07540].

[25] S. Pan, W. Yang, C. Singha and E.N. Saridakis, *Observational constraints on sign-changeable interaction models and alleviation of the H_0 tension*, *Phys. Rev. D* **100** (2019) 083539 [1903.10969].

[26] S.-F. Yan, P. Zhang, J.-W. Chen, X.-Z. Zhang, Y.-F. Cai and E.N. Saridakis, *Interpreting cosmological tensions from the effective field theory of torsional gravity*, *Phys. Rev. D* **101** (2020) 121301 [1909.06388].

[27] R. D'Agostino and R.C. Nunes, *Measurements of H_0 in modified gravity theories: The role of lensed quasars in the late-time Universe*, *Phys. Rev. D* **101** (2020) 103505 [2002.06381].

[28] F.K. Anagnostopoulos, S. Basilakos and E.N. Saridakis, *Observational constraints on Myrzakulov gravity*, *Phys. Rev. D* **103** (2021) 104013 [2012.06524].

[29] K.L. Pandey, T. Karwal and S. Das, *Alleviating the H_0 and σ_8 anomalies with a decaying dark matter model*, *JCAP* **07** (2020) 026 [1902.10636].

[30] S. Adhikari and D. Huterer, *Super-CMB fluctuations and the Hubble tension*, *Phys. Dark Univ.* **28** (2020) 100539 [1905.02278].

[31] D. Benisty, *Cosmology of fermionic dark energy coupled to curvature*, *Nucl. Phys. B* **992** (2023) 116251 [1912.11124].

[32] S. Vagnozzi, *New physics in light of the H_0 tension: An alternative view*, *Phys. Rev. D* **102** (2020) 023518 [1907.07569].

[33] F.K. Anagnostopoulos, S. Basilakos and E.N. Saridakis, *Bayesian analysis of $f(T)$ gravity using $f\sigma_8$ data*, *Phys. Rev. D* **100** (2019) 083517 [1907.07533].

[34] M. Braglia, M. Ballardini, F. Finelli and K. Koyama, *Early modified gravity in light of the H_0 tension and LSS data*, *Phys. Rev. D* **103** (2021) 043528 [2011.12934].

[35] S. Pan, W. Yang and A. Paliathanasis, *Non-linear interacting cosmological models after Planck 2018 legacy release and the H_0 tension*, *Mon. Not. Roy. Astron. Soc.* **493** (2020) 3114 [2002.03408].

[36] S. Capozziello, M. Benetti and A.D.A.M. Spallicci, *Addressing the cosmological H_0 tension by the Heisenberg uncertainty*, *Found. Phys.* **50** (2020) 893 [2007.00462].

[37] E.N. Saridakis, S. Myrzakul, K. Myrzakulov and K. Yerzhanov, *Cosmological applications of $F(R,T)$ gravity with dynamical curvature and torsion*, *Phys. Rev. D* **102** (2020) 023525 [1912.03882].

[38] M. Petronikolou, S. Basilakos and E.N. Saridakis, *Alleviating H_0 tension in Horndeski gravity*, 2110.01338.

[39] M. Petronikolou and E.N. Saridakis, *Alleviating the H_0 Tension in Scalar–Tensor and Bi-Scalar–Tensor Theories*, *Universe* **9** (2023) 397 [2308.16044].

[40] C. Escamilla-Rivera and J. Levi Said, *Cosmological viable models in $f(T,B)$ theory as solutions to the H_0 tension*, *Class. Quant. Grav.* **37** (2020) 165002 [1909.10328].

[41] E. Di Valentino, A. Melchiorri, O. Mena and S. Vagnozzi, *Nonminimal dark sector physics and cosmological tensions*, *Phys. Rev. D* **101** (2020) 063502 [1910.09853].

[42] G. Benevento, W. Hu and M. Raveri, *Can Late Dark Energy Transitions Raise the Hubble constant?*, *Phys. Rev. D* **101** (2020) 103517 [2002.11707].

[43] A. Banerjee, H. Cai, L. Heisenberg, E.O. Colgáin, M.M. Sheikh-Jabbari and T. Yang, *Hubble sinks in the low-redshift swampland*, *Phys. Rev. D* **103** (2021) L081305 [2006.00244].

[44] E. Elizalde, M. Khurshudyan, S.D. Odintsov and R. Myrzakulov, *Analysis of the H_0 tension problem in the Universe with viscous dark fluid*, *Phys. Rev. D* **102** (2020) 123501 [2006.01879].

[45] A. De Felice, S. Mukohyama and M.C. Pookkillath, *Addressing H_0 tension by means of VCDM*, *Phys. Lett. B* **816** (2021) 136201 [2009.08718].

[46] B.S. Haridasu, M. Viel and N. Vittorio, *Sources of H_0-tension in dark energy scenarios*, *Phys. Rev. D* **103** (2021) 063539 [2012.10324].

[47] O. Seto and Y. Toda, *Comparing early dark energy and extra radiation solutions to the Hubble tension with BBN*, *Phys. Rev. D* **103** (2021) 123501 [`2101.03740`].

[48] T. Adi and E.D. Kovetz, *Can conformally coupled modified gravity solve the Hubble tension?*, *Phys. Rev. D* **103** (2021) 023530 [`2011.13853`].

[49] M. Ballardini, M. Braglia, F. Finelli, D. Paoletti, A.A. Starobinsky and C. Umiltà, *Scalar-tensor theories of gravity, neutrino physics, and the H_0 tension*, *JCAP* **10** (2020) 044 [`2004.14349`].

[50] F.X. Linares Cedeño and U. Nucamendi, *Revisiting cosmological diffusion models in Unimodular Gravity and the H_0 tension*, *Phys. Dark Univ.* **32** (2021) 100807 [`2009.10268`].

[51] S.D. Odintsov, D. Sáez-Chillón Gómez and G.S. Sharov, *Analyzing the H_0 tension in $F(R)$ gravity models*, *Nucl. Phys. B* **966** (2021) 115377 [`2011.03957`].

[52] G. Alestas and L. Perivolaropoulos, *Late-time approaches to the Hubble tension deforming H(z), worsen the growth tension*, *Mon. Not. Roy. Astron. Soc.* **504** (2021) 3956 [`2103.04045`].

[53] E. Elizalde, J. Gluza and M. Khurshudyan, *An approach to cold dark matter deviation and the H_0 tension problem by using machine learning*, `2104.01077`.

[54] S. Basilakos, D.V. Nanopoulos, T. Papanikolaou, E.N. Saridakis and C. Tzerefos, *Signatures of Superstring theory in NANOGrav*, `2307.08601`.

[55] C. Krishnan, R. Mohayaee, E.O. Colgáin, M.M. Sheikh-Jabbari and L. Yin, *Does Hubble tension signal a breakdown in FLRW cosmology?*, *Class. Quant. Grav.* **38** (2021) 184001 [`2105.09790`].

[56] T. Papanikolaou, A. Lymperis, S. Lola and E.N. Saridakis, *Primordial black holes and gravitational waves from non-canonical inflation*, *JCAP* **03** (2023) 003 [`2211.14900`].

[57] A. Theodoropoulos and L. Perivolaropoulos, *The Hubble Tension, the M Crisis of Late Time H(z) Deformation Models and the Reconstruction of Quintessence Lagrangians*, *Universe* **7** (2021) 300 [`2109.06256`].

[58] T. Papanikolaou, *Primordial black holes in loop quantum cosmology: the effect on the threshold*, *Class. Quant. Grav.* **40** (2023) 134001 [`2301.11439`].

[59] E. Abdalla et al., *Cosmology intertwined: A review of the particle physics, astrophysics, and cosmology associated with the cosmological tensions and anomalies*, *JHEAp* **34** (2022) 49 [`2203.06142`].

[60] A. Lymperis and E.N. Saridakis, *Modified cosmology through nonextensive horizon thermodynamics*, *Eur. Phys. J. C* **78** (2018) 993 [`1806.04614`].

[61] T. Jacobson, *Thermodynamics of space-time: The Einstein equation of state*, *Phys. Rev. Lett.* **75** (1995) 1260 [`gr-qc/9504004`].

[62] T. Padmanabhan, *Gravity and the thermodynamics of horizons*, *Phys. Rept.* **406** (2005) 49 [`gr-qc/0311036`].

[63] T. Padmanabhan, *Thermodynamical Aspects of Gravity: New insights*, *Rept. Prog. Phys.* **73** (2010) 046901 [`0911.5004`].

[64] C. Tsallis, *Possible Generalization of Boltzmann-Gibbs Statistics*, *J. Statist. Phys.* **52** (1988) 479.

[65] M.L. Lyra and C. Tsallis, *Nonextensivity and Multifractality in Low-Dimensional Dissipative Systems*, *Phys. Rev. Lett.* **80** (1998) 53 [`cond-mat/9709226`].

[66] G. Wilk and Z. Wlodarczyk, *On the interpretation of nonextensive parameter q in Tsallis statistics and Levy distributions*, *Phys. Rev. Lett.* **84** (2000) 2770 [`hep-ph/9908459`].

[67] A. Sheykhi, *Modified Friedmann Equations from Tsallis Entropy*, *Phys. Lett. B* **785** (2018) 118 [`1806.03996`].

[68] H.F. Lalus and G. Hikmawan, *Analytical solutions of modified Friedmann equation in Tsallis Cosmology for nonflat universe*, *International Journal of Innovation, Creativity and Change* **5** (2019) 638.

[69] S. Nojiri, S.D. Odintsov and E.N. Saridakis, *Modified cosmology from extended entropy with varying exponent*, *Eur. Phys. J. C* **79** (2019) 242 [`1903.03098`].

[70] C.-Q. Geng, Y.-T. Hsu, J.-R. Lu and L. Yin, *Modified Cosmology Models from Thermodynamical Approach*, *Eur. Phys. J. C* **80** (2020) 21 [`1911.06046`].

[71] A. Ghoshal and G. Lambiase, *Constraints on Tsallis Cosmology from Big Bang Nucleosynthesis and Dark Matter Freeze-out*, `2104.11296`.

[72] G.G. Luciano, *Tsallis statistics and generalized uncertainty principle*, *Eur. Phys. J. C* **81** (2021) 672.

[73] D.J. Zamora and C. Tsallis, *Thermodynamically consistent entropic late-time cosmological acceleration*, *Eur. Phys. J. C* **82** (2022) 689 [`2201.03385`].

[74] G.G. Luciano and J. Gine, *Baryogenesis in non-extensive Tsallis Cosmology*, *Phys. Lett. B* **833** (2022) 137352 [`2204.02723`].

[75] S. Nojiri, S.D. Odintsov and T. Paul, *Early and late universe holographic cosmology from a new generalized entropy*, *Phys. Lett. B* **831** (2022) 137189 [`2205.08876`].

[76] P. Jizba and G. Lambiase, *Tsallis cosmology and its applications in dark matter physics with focus on IceCube high-energy neutrino data*, *Eur. Phys. J. C* **82** (2022) 1123 [`2206.12910`].

[77] C. Tsallis and L.J.L. Cirto, *Black hole thermodynamical entropy*, *Eur. Phys. J. C* **73** (2013) 2487 [`1202.2154`].

[78] R.-G. Cai and S.P. Kim, *First law of thermodynamics and Friedmann equations of Friedmann-Robertson-Walker universe*, *JHEP* **02** (2005) 050 [`hep-th/0501055`].

[79] M. Akbar and R.-G. Cai, *Thermodynamic Behavior of Friedmann Equations at Apparent Horizon of FRW Universe*, *Phys. Rev. D* **75** (2007) 084003 [`hep-th/0609128`].

[80] G. Izquierdo and D. Pavon, *Dark energy and the generalized second law*, *Phys. Lett. B* **633** (2006) 420 [`astro-ph/0505601`].

[81] R.-G. Cai and L.-M. Cao, *Unified first law and thermodynamics of apparent horizon in FRW universe*, *Phys. Rev. D* **75** (2007) 064008 [`gr-qc/0611071`].

[82] M. Akbar and R.-G. Cai, *Friedmann equations of FRW universe in scalar-tensor gravity, f(R) gravity and first law of thermodynamics*, *Phys. Lett. B* **635** (2006) 7 [`hep-th/0602156`].

[83] A. Paranjape, S. Sarkar and T. Padmanabhan, *Thermodynamic route to field equations in Lancos-Lovelock gravity*, *Phys. Rev. D* **74** (2006) 104015 [`hep-th/0607240`].

[84] A. Sheykhi, B. Wang and R.-G. Cai, *Thermodynamical properties of apparent horizon in warped DGP braneworld*, *Nucl. Phys. B* **779** (2007) 1 [`hep-th/0701198`].

[85] M. Jamil, E.N. Saridakis and M.R. Setare, *Thermodynamics of dark energy interacting with dark matter and radiation*, *Phys. Rev. D* **81** (2010) 023007 [`0910.0822`].

[86] R.-G. Cai and N. Ohta, *Horizon Thermodynamics and Gravitational Field Equations in Horava-Lifshitz Gravity*, *Phys. Rev. D* **81** (2010) 084061 [`0910.2307`].

[87] M. Wang, J. Jing, C. Ding and S. Chen, *First law of thermodynamics in IR modified Hořava-Lifshitz gravity*, *Phys. Rev. D* **81** (2010) 083006 [`0912.4832`].

[88] Y. Gim, W. Kim and S.-H. Yi, *The first law of thermodynamics in Lifshitz black holes revisited*, *JHEP* **07** (2014) 002 [`1403.4704`].

[89] Z.-Y. Fan and H. Lu, *Thermodynamical First Laws of Black Holes in Quadratically-Extended Gravities*, *Phys. Rev. D* **91** (2015) 064009 [`1501.00006`].

[90] E.N. Saridakis, *Modified cosmology through spacetime thermodynamics and Barrow horizon entropy*, *JCAP* **07** (2020) 031 [`2006.01105`].

[91] A. Hernández-Almada, G. Leon, J. Magaña, M.A. García-Aspeitia, V. Motta, E.N. Saridakis et al., *Observational constraints and dynamical analysis of Kaniadakis horizon-entropy cosmology*, *Mon. Not. Roy. Astron. Soc.* **512** (2022) 5122 [`2112.04615`].

[92] A. Sheykhi and B. Farsi, *Growth of perturbations in Tsallis and Barrow cosmology*, *Eur. Phys. J. C* **82** (2022) 1111 [`2205.04138`].

[93] L. Heisenberg, H. Villarrubia-Rojo and J. Zosso, *Simultaneously solving the H0 and σ8 tensions with late dark energy*, Phys. Dark Univ. **39** (2023) 101163 [`2201.11623`].

[94] L. Heisenberg, H. Villarrubia-Rojo and J. Zosso, *Can late-time extensions solve the H0 and σ8 tensions?*, Phys. Rev. D **106** (2022) 043503 [`2202.01202`].

[95] M. Asghari and A. Sheykhi, *Observational constraints on Tsallis modified gravity*, Mon. Not. Roy. Astron. Soc. **508** (2021) 2855 [`2106.15551`].

[96] H. Yu, B. Ratra and F.-Y. Wang, *Hubble Parameter and Baryon Acoustic Oscillation Measurement Constraints on the Hubble Constant, the Deviation from the Spatially Flat ΛCDM Model, the Deceleration–Acceleration Transition Redshift, and Spatial Curvature*, Astrophys. J. **856** (2018) 3 [`1711.03437`].

[97] R. Jimenez and A. Loeb, *Constraining cosmological parameters based on relative galaxy ages*, Astrophys. J. **573** (2002) 37 [`astro-ph/0106145`].

[98] Y. Wang, G.-B. Zhao, C.-H. Chuang, M. Pellejero-Ibanez, C. Zhao, F.-S. Kitaura et al., *The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: a tomographic analysis of structure growth and expansion rate from anisotropic galaxy clustering*, Mon. Not. Roy. Astron. Soc. **481** (2018) 3160 [`1709.05173`].

[99] H. Gil-Marín et al., *The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample: structure growth rate measurement from the anisotropic quasar power spectrum in the redshift range* $0.8 < z < 2.2$, Mon. Not. Roy. Astron. Soc. **477** (2018) 1604 [`1801.02689`].

[100] J. Hou et al., *The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample: anisotropic clustering analysis in configuration-space*, Mon. Not. Roy. Astron. Soc. **480** (2018) 2521 [`1801.02656`].

[101] G.-B. Zhao et al., *The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample: a tomographic measurement of cosmic structure growth and expansion rate based on optimal redshift weights*, Mon. Not. Roy. Astron. Soc. **482** (2019) 3497 [`1801.03043`].

[102] Y.-F. Cai, E.N. Saridakis, M.R. Setare and J.-Q. Xia, *Quintom Cosmology: Theoretical implications and observations*, Phys. Rept. **493** (2010) 1 [`0909.2776`].