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Dark Energy from topology change at the foam level

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Microscopic entities with distinct topology and Euler characteristics, such as instantons and wormholes, at the spacetime foam level, within the framework of Euclidean quantum gravity, initiate changes in spacetime topology. These alterations could theoretically influence the field equations derived from the semiclassical variation of gravitational actions. While in the case of the Einstein-Hilbert action the presence of microscopic wormholes yields no significant outcomes, the inclusion of the Gauss-Bonnet term into the gravitational action brings about an effective topological variation procedure. This process generates an effective cosmological term that is influenced by both the Gauss-Bonnet coupling and the density of wormholes. Given that wormhole density can vary with time in a dynamic spacetime, this mechanism introduces a topologically derived effective dark energy component.

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1. Introduction

Recent observations suggest that the Universe has recently entered a phase of accelerated expansion [1–6]. The simplest theoretical response involves the introduction of a positive cosmological constant, Λ, which gives rise to the "cosmological constant problem" as quantum field theoretical predictions vastly exceed observed values [7, 8]. This discrepancy, alongside tensions within the ΛCDM model such as the H_0 [9] and σ_8 tensions [10] (for a review see [11]), prompts exploration into alternative explanations. These include considering a dynamic cosmological constant or dark energy concept within general relativity [12, 13], or modifying the underlying gravitational theory to develop richer cosmological behaviors [14–18]. Additionally, more radical approaches like holographic dark energy are explored [19–21].

Modified gravity not only aims to address these cosmological issues but also offers better quantum behavior due to its potential to eliminate divergences [22] that general relativity cannot [23]. When higher order terms are included in the gravitational Langrangian they tend to eliminate the divergences [24], therefore higher-order gravity theories have been extensively developed [25–28]. From all the higher order Lagargian terms, the Gauss-Bonnet (GB) term is special as it represents the Euler density in 4D, making it a topological invariant as per the Chern-Gauss-Bonnet Theorem [29] and crucial for maintaining heterotic string theory's local supersymmetry [30]. In M-theory, it's vital for renormalization by addressing beta-function divergences at high energies [31].

In Euclidean quantum gravity, entities like instantons and wormholes exhibit different topology from the background at the microscopic level [32], while wormholes are also studied in the astrophysical scale [33–38]. These changes, potentially leading to singularities, challenge classical and quantum field theories [39, 40] but are considered necessary for a consistent quantum gravity framework [41]. Research, including work on Euclidean quantum gravity [42–45] and connections to Ricci flow [46, 47] and string theory [48–52], supports the viability and significance of topology changes. Among others, topology changes may in principle affect the field equations that arise through the semiclassical variation procedure of gravitational actions. Although in the case of Einstein-Hilbert action this procedure reproduces the standard field equations, one could investigate whether variation of the Gauss-Bonnet action on a topologically altered spacetime due to the formation of microscopic wormholes could lead to a non-trivial result. Interestingly enough, such an extended analysis does induce extra terms in the field equations, which can be interpreted as an effective dark energy sector of topological origin.

2. Topology change in Euclidean quantum gravity

In Euclidean quantum gravity (EQG) context the time dimension is Wick rotated so that the complex path integral to converge and yield saddle point solutions, namely instantons, with different topology from the background [53, 54] which mediate topology change [32]. These solutions can represent the creation of a pair of black holes or Euclidean wormholes under a strong field as in Schwinger process [32, 55]. In the sum over history approach, Sorkin [41] has developed a calculus for topology change based on Morse theory, where the transition between two manifolds of distinct topology is being performed by a Morse function provided a cobordism exists between the manifolds [44, 56].

The topological structure of a manifold M is characterized by the Euler characteristic $\chi(M)$ which is defined as the alternating sum of the Betti numbers of the manifold M [57]

$$\chi(M) = \sum_p (-1)^p B_p, \tag{1}$$

where the Betti numbers B_p of a manifold are defined as the dimension of the p^{th} de Rahm cohomology group [57]

$$B_p = dim H^p(M). \tag{2}$$

In the above expression the p^{th} de Rahm cohomology group $H^p(M)$ is the set of all closed p-forms $Z^p(M)$ modulo the set of all exact p-forms $B^p(M)$

$$H^p(M) = Z^p(M)/B^p(M), \tag{3}$$

where a closed form satisfies $d\omega = 0$ (where ω is a p-form and d denotes the exterior derivative), and an exact form satisfies $\omega = dn$ (where n is a p-form).

The Poincare lemma states that a closed form defined on a domain $V \subseteq M$ is also exact, if the domain V is contractible to a point. In the light of Poincare lemma, de Rham cohomology can be seen as a restriction on the global exactness of closed forms [57]. In summary, Betti numbers measure the global inexactness of closed forms as obstructions to contractibility to a point, originated from holes and discontinuities of the domain [58].

In Table 1 the value of Euler characteristics for different spacetime manifolds is presented.

Spacetime	Euler characteristic χ
Minkowski	0
Extreme Black Holes	0
Self-dual Taub-Newman-Unti-Tamburino	1
Schwarchild and Kerr Black Holes	2
Nariai $S_2 \times S_2$	4
Euclidean Wormhole $S_1 \times S_3$	0

Table 1: Euler characteristics as it has been calculated in [54, 59–62]. By the product property for product manifolds $\chi(M_1 \times M_2) = \chi(M_1) \cdot \chi(M_2)$, one can easily verify that for a Nariai instanton $\chi_{Na} = 2 \cdot 2 = 4$ and similarly for a Euclidean wormhole $\chi_{EW} = 0 \cdot 3 = 0$.

In order to investigate the topology change, one can decompose a 4D manifold M into a connected sum (symbolized by #) of two 4D manifolds M_1 and M_2, by gluing them together at the boundaries left by the removal of a four-ball. For connected sums, the Euler characteristic is given by [57]

$$\chi\,(M_1 \# M_2) = \chi(M_1) + \chi(M_2) - 2. \tag{4}$$

Then, following Gibbons [32, 63], the formation of a Euclidean wormhole with topology $(S_1 \times S_3)$, namely

$$M \longrightarrow M\#(S_1 \times S_3), \tag{5}$$

decreases χ by 2, thus $\delta\chi = -2$, while the formation of a Nariai instanton with topology $(S_2 \times S_2)$, namely

$$M \longrightarrow M\#(S_2 \times S_2), \tag{6}$$

increases χ by 2 thus $\delta\chi = 2$.

Therefore the formation of gravitational instantons or wormholes change the Euler characteristic of the 4D spacetime manifold, where equation (1) becomes [58]

$$\chi = b_0 - b_1 + b_2 + b_3, \tag{7}$$

with b_0 the number of connected components, b_1 the number of one-dimensional holes, b_2 the number of two-dimensional holes, and b_3 the number of three-dimensional holes. Hence, when the Euler characteristic changes, there is a change in the Betti numbers of spacetime [64], which implies that there is an area of the manifold, a wormhole, which is not contractible to a point.

3. The effective topological variation procedure

Inspired by Wheeler's conceptualization of spacetime foam [65], where quantum fluctuations of the metric are considered to cause fluctuations of the topology of the spacetime manifold, which was later developed in the context of Euclidean quantum gravity by Hawking, Gibbons, Sorkin and others [53, 66, 67], we are interested in investigating the behavior of the variation of higher-order gravitational actions under the assumption that the variation of the quantum field fluctuations δh causes a variation in the topology of the spacetime manifold $\delta\chi$.

We consider that the Euclidean quantum gravity process, which yields instanton solutions of different spacetime topology, can be encapsulated into an effective topology change operation (eff_{TC}), as illustrated in Fig. 1. Specifically, for manifolds with metric g_i and Euler characteristics χ_i, it can be encapsulated into the variation of the gravitational quantum field δh, namely

$$\begin{aligned} M(g_1, \chi_1) &\xrightarrow{eff_{TC}} M'(g_2.\chi_2), \\ eff_{TC} &: \delta h \longrightarrow \delta\chi. \end{aligned} \tag{8}$$

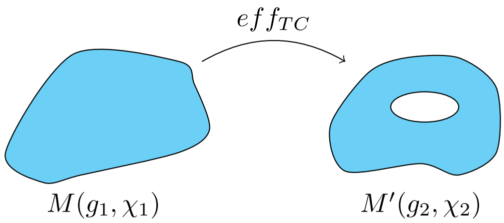

Figure 1: *A three-dimensional illustration of the effective topology change from a manifold of Euler characteristic χ_1 to a manifold of Euler characteristic χ_2.*

A common technique in many approaches to quantum gravity is to split linearly the full metric g into a background metric $\tilde{g}$ and the quantum fluctuation field h around it [68]

$$g^{\mu\nu} = \tilde{g}^{\mu\nu} + h^{\mu\nu}. \tag{9}$$

According to Wheeler's argument [69, 70] and similar more recent ones [71], the quantum fluctuations are scale-dependent as $\delta h \sim \frac{l_P}{l}$, and they become large near the Planck scale, remaining always smaller than one as long as we consider the Planck scale as cut-off. However, from the viewpoint of asymptotic safety, one could argue that interactions could become weak at the Planck scale by an appropriate renormalization-group flow [71]. If one follows the first consideration, fluctuations may induce topology change, nevertheless higher-order terms in the expansion could be non negligible. If one follows the second consideration, then classical expansion techniques can be employed but ambiguities arise on the ability of fluctuations to induce topology change. Since a solid theory of quantum gravity remains far from being complete, in this work we assume that a compromise exists between the two extremes, suggesting that quantum fluctuations can produce topology change at small scales, while being small enough in order for perturbation theory to hold. Consequently, this scaling approach facilitates a reduction in the significance of higher-order terms, thereby allowing the phenomena to be predominantly described by the first-order term.

As mentioned in [72], the quantum fluctuation field $h = g - \tilde{g}$ of the linear split is not a metric and lacks a geometrical meaning, therefore among the other types of split, it is the most suited for describing the fluctuation field that causes topology change. Consequently, we consider $\tilde{g}_{\mu\nu}$ to be the dynamical background metric and we handle $h_{\mu\nu}$ as an effective "matter" field. By demanding background independence, one can introduce split symmetry [72], given by all the transformations of the background metric and fluctuation fields that preserve the full metric, namely

$$g(\tilde{g}, h) \to g(\tilde{g} + \delta\tilde{g}, h + \delta h) = g(\tilde{g}, h). \tag{10}$$

In the quantization scheme of [73], background independence is guaranteed by the class of metrics that are self-consistent. Self-consistent metrics $\tilde{g}^{SC}$ are those that allow the effective field equations obtained from the effective action $\Gamma[h_{\mu\nu}, \tilde{g}]$ to admit the solution $h_{\mu\nu} = 0$. Thus, if one incorporates background independence into the extremization condition of the effective action, then one obtains the tadpole condition [73]

$$\frac{\delta}{\delta h_{\mu\nu}} \Gamma[h, \tilde{g}] \bigg|_{h=0,\ \tilde{g}=\tilde{g}^{SC}} = 0. \tag{11}$$

4. Effective cosmological constant of topological origin

We now have all the machinery to perform the variation of gravitational actions in cases where there are topology changes in the underlying spacetime manifold.

4.1 Einstein-Hilbert action

Let us start by presenting the semiclassical approach in the case of Einstein-Hilbert action, as it has been demonstrated firstly by 't Hooft in [74]. By performing a Wick rotation the action will be Euclideanized, i.e.

$$S_{EH} = -\frac{1}{2\kappa^2} \int d^4x \sqrt{g} R, \tag{12}$$

with κ^2 the gravitational constant. We expand the Einstein-Hilbert action around the background field according to the metric split (9), in orders of the "quantum field", namely

$$S_{EH} = S_0 + S_1 + S_2 + \sum_{n=3}^{\infty} S_n. \tag{13}$$

In order to calculate each term we expand the inverse metric as

$$g^{\mu\nu} = \tilde{g}^{\mu\nu} - h^{\mu\nu} + h^{\mu}_{\lambda} h^{\lambda\nu} + O(h^3), \tag{14}$$

and then by employing the property $\log\det A = \mathrm{tr}\log A$ and performing logarithmic and exponential expansions, we express the determinant of the metric in terms of powers of h as

$$\sqrt{g} = \sqrt{\tilde{g}}\left[1 + \frac{1}{2}\tilde{g}_{\mu\nu}h^{\mu\nu} - \frac{1}{4}h^{\mu\nu}h_{\mu\nu} + \frac{1}{8}(h^{\mu}_{\mu})^2 + O(h^3)\right], \tag{15}$$

where the uppering and lowering of the quantum field indices are performed using the background metric, namely $h = h^{\mu}_{\mu} = \tilde{g}^{\mu\nu}h_{\mu\nu}$. Since the Ricci scalar is $R = g^{\mu\nu}R_{\mu\nu}$, by employing (14) and (15) the Ricci tensor and Ricci scalar can be expanded in the same manner. After some algebra and neglecting the total derivatives, the first three terms of the Einstein-Hilbert expansion are expressed as

$$\begin{aligned}
S_0 &= -\frac{1}{2\kappa^2}\int d^4x\sqrt{\tilde{g}}\tilde{R} \\
S_1 &= \frac{1}{2\kappa^2}\int d^4x\sqrt{\tilde{g}}\left(\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R}\right)h^{\mu\nu} \\
S_2 &= -\frac{1}{2\kappa^2}\int d^4x\sqrt{\tilde{g}}\left\{\frac{1}{4}h^{\mu\nu}\nabla^2 h_{\mu\nu} - \frac{1}{8}h\nabla^2 h \right. \\
&\quad + \frac{1}{2}\left(\nabla^{\nu}h_{\nu\mu} - \frac{1}{2}\nabla_{\mu}h\right)^2 + \frac{1}{2}h^{\mu\lambda}h^{\nu\sigma}\tilde{R}_{\mu\lambda\nu\sigma} \\
&\quad \left. + \frac{1}{2}\left(h^{\mu\lambda}h^{\nu}_{\lambda} - hh^{\mu\nu}\right)\tilde{R}_{\mu\nu} + \frac{1}{8}\left(h^2 - 2h^{\mu\nu}h_{\mu\nu}\right)\tilde{R}\right\}.
\end{aligned} \tag{16}$$

In summary, the effective action up to one-loop approximation will be

$$\Gamma = S_{EH} + \Gamma_{1L} + O(2-loop), \tag{17}$$

where the quadratic terms of the quantum field h are absorbed in the one-loop part

$$\Gamma_{1L} = \Gamma_{GF} + \Gamma_{Fgh}, \tag{18}$$

with Γ_{GF} and Γ_{Fgh} corresponding to the effective action for the gauge fixing and ghost terms respectively [73]. One can then vary the Einstein-Hilbert action due to quantum fluctuations of the field $h^{\mu\nu} \to h^{\mu\nu} + \delta h^{\mu\nu}$, i.e. calculate $\delta_h S_{EH}$. Imposing the tadpole condition (11) for the effective action (17) and taking into account (16), one finally retrieves the Einstein equations for the classical background as [73, 75]

$$\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} = \kappa^2 T_{\mu\nu}, \tag{19}$$

where the stress tensor originates from the one-loop part of the effective action, containing matter as correction terms in the right-hand-side, i.e.

$$T^{\mu\nu} = -\frac{2}{\sqrt{g}}\frac{\delta}{\delta h^{\mu\nu}}\Gamma_{1L}\Big|_{h=0}. \tag{20}$$

Actually, this was expected, since the Einstein-Hilbert action term is the Euler density in two-dimensions, and thus its variation due to variations of the quantum field h in 4D will be the standard one [76].

4.2 Gauss-Bonnet action

Let us now perform the above procedure in the case of the Gauss-Bonnet action. The Gauss-Bonnet (GB) curvature polynomial $\mathcal{G}$ is defined as

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}, \tag{21}$$

and it is known that in four dimensions such a term is a topological invariant. In order to see this in the context of the present manuscript, we recall that the Chern-Gauss-Bonnet theorem [29] states that for the case of a compact orientable manifold M with boundary ∂M of dimension $D = 4$, the Euler characteristic is

$$\chi(M) = \frac{1}{32\pi^2}\int_M d^4x\sqrt{g}\,\mathcal{G} + \int_{\partial M} Q, \tag{22}$$

with Q an appropriate correction form integrated on the boundary ∂M [77]. The essence of the theorem is that despite any local deformation of the manifold, its total curvature, as expressed by the integral of the GB curvature polynomial, depends only on the topology of the manifold. Consequently, for a manifold of fixed topology, χ is considered a topological invariant under smooth variations of the metric [78].

We can now perform the steps of the previous subsections in the case of the Gauss-Bonnet action. Its Euclideanized form is

$$S_{GB} = -\frac{\alpha}{2\kappa^2}\int d^4x\sqrt{g}\left(R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}\right), \tag{23}$$

where α is the coupling parameter. The effective action under the one-loop approximation will be

$$\Gamma = S_{GB} + \Gamma_{1L} + O_{(2-Loop)}. \tag{24}$$

Varying the GB action with respect to quantum fluctuations of the field $h^{\mu\nu} \to h^{\mu\nu} + \delta h^{\mu\nu}$ according to the topological variational procedure and applying the 4D Chern-Gauss-Bonnet theorem (22) without a boundary, we obtain

$$\begin{aligned}
&\delta_h S_{GB} \\
&= \delta_h\left[-\frac{\alpha}{2\kappa^2}\int_M d^4x\sqrt{g}\left(R^2 - 2R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right] \\
&= -32\pi^2\frac{\alpha}{2\kappa^2}\frac{\delta\chi}{\delta h^{\mu\nu}}\delta h^{\mu\nu}.
\end{aligned} \tag{25}$$

Implementing the substitution $\delta\chi \to \partial\chi$ and applying the chain rule, we find

$$\begin{aligned}\delta_h S_{GB} &= -16\pi^2 \frac{\alpha}{2\kappa^2}\frac{\partial\chi}{\partial V}\frac{\delta V}{\delta h^{\mu\nu}}\delta h^{\mu\nu} \\ &= -16\pi^2 \frac{\alpha}{2\kappa^2}\frac{\partial\chi}{\partial V}\delta_h\left(\int_M d^4x\sqrt{g}\right) \\ &= -16\pi^2 \frac{\alpha}{2\kappa^2}\frac{\partial\chi}{\partial V}\int_M d^4x \frac{\delta\sqrt{g}}{\delta h^{\mu\nu}}\delta h^{\mu\nu},\end{aligned} \tag{26}$$

with V the manifold volume. If one implements the expansion of the metric determinant (15), then the functional derivative into the integral becomes $\frac{\delta\sqrt{g}}{\delta h^{\mu\nu}} = \frac{1}{2}\sqrt{\tilde{g}}\tilde{g}_{\mu\nu} + O(h)$, so after inserting it into (26) we finally acquire

$$\delta_h S_{GB} = -16\pi^2 \frac{\alpha}{2\kappa^2}\frac{\partial\chi}{\partial V}\int_M d^4x\sqrt{-\tilde{g}}\tilde{g}_{\mu\nu}\delta h^{\mu\nu}. \tag{27}$$

Finally, we make the reasonable approximation that for an infinitesimal integration volume the topology change per volume $\frac{\partial\chi}{\partial V}$ remains constant and thus it can enter inside the integral, in which case the topological variation of the Gauss-Bonnet term is expressed as

$$\frac{1}{\sqrt{\tilde{g}}}\frac{\delta S_{GB}}{\delta h^{\mu\nu}} = -16\pi^2\frac{\alpha}{2\kappa^2}\frac{\partial\chi}{\partial V}\tilde{g}_{\mu\nu} + O(h). \tag{28}$$

Interestingly enough, the variation of the Gauss-Bonnet term on a manifold that has topology changes due to the formation of wormholes is not zero.

4.3 Einstein-Gauss-Bonnet action

Let us now consider the full case of general relativity plus a Gauss-Bonnet correction, namely

$$S_{tot} = S_{EH} + S_{GB}. \tag{29}$$

As we analyzed above, although the Einstein-Hilbert term gives the standard classical field equations, the Gauss-Bonnet term leads to a nontrivial semiclassical result. In particular, the effective action under the one-loop approximation will be

$$\Gamma = S_{EH} + S_{GB} + \Gamma_{1L} + O_{(2-Loop)}. \tag{30}$$

Calculating the fuctional derivative of the effective action by employing Eq. (16) and Eq. (28), we finally obtain

$$\begin{aligned}\frac{1}{\sqrt{\tilde{g}}}\frac{\delta\Gamma}{\delta h^{\mu\nu}} &= \frac{1}{\sqrt{\tilde{g}}}\frac{\delta S_{EH}}{\delta h^{\mu\nu}} + \frac{1}{\sqrt{\tilde{g}}}\frac{\delta S_{GB}}{\delta h^{\mu\nu}} + \frac{1}{\sqrt{\tilde{g}}}\frac{\delta\Gamma_{1L}}{\delta h^{\mu\nu}} \\ &= \frac{1}{2\kappa^2}\left\{\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} - 16\pi^2\alpha\frac{\partial\chi}{\partial V}\tilde{g}_{\mu\nu} + \frac{1}{\sqrt{\tilde{g}}}\frac{\delta\Gamma_{1L}}{\delta h^{\mu\nu}} + O(h)\right\}.\end{aligned} \tag{31}$$

Hence, imposing the tadpole condition (11) that removes the terms $O(h)$, we find the semiclassical field equations

$$\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} + \tilde{g}_{\mu\nu}\Lambda_{eff} = \kappa^2 T_{\mu\nu}, \tag{32}$$

where the stress tensor is given by (20), and where we have defined

$$\Lambda_{eff} \equiv -16\pi^2\alpha\frac{\partial\chi}{\partial V}. \tag{33}$$

As we observe, we have obtained an effective cosmological constant term of topological origin, induced by the Gauss-Bonnet correction term due to the topology change that microscopic wormholes brought about. This is the main result of the present work.

5. Dark energy from microscopic Gauss-Bonnet wormholes

The effective cosmological constant $\frac{\partial\chi}{\partial V}$, can be interpreted as the density of the nontrivial microscopic objects per four-volume $\rho_{obj} = \frac{N_{obj}}{V}$, since these objects induce the topology change (for instance $\delta\chi = -2$ corresponds to the formation of a Euclidean wormhole, while $\delta\chi = 2$ to the formation of a Nariai instanton). Hence, according to (33), the effective cosmological constant equals the density ρ_w of N_w topology changing wormholes per four-volume, i.e.

$$\Lambda_{eff} = -16\pi^2\alpha\rho_w = -16\pi^2\alpha\frac{N_w}{V}, \tag{34}$$

namely it depends on the GB coupling parameter α and on the wormhole density.

Concerning the value of GB coupling, there is a consensus that since the GB term appears in the low-energy limit of an effective action [79], α is related to the inverse of the string tension [80–82] $\alpha \sim (1/\sqrt{a'})$ or equivalently to the square of the string scale $\alpha \sim l_s^2$ [76, 83]. Since the string scale l_s cannot be far from the Planck scale in four dimensions $l_s \sim l_p$ [84, 85], one first estimation could be $\alpha = l_p^2$. In such a case, if we identify the effective cosmological constant Λ_{eff} of (34) with the observed cosmological constant $\Lambda_{obs} = 10^{-52}m^{-2}$, we need a microscopic wormhole density of $\rho_w = 10^{16}$ wormholes per cubic meter per second, which is quite reasonable according to Hawking and Schulz estimations for the spacetime foam [40, 53, 64]. On the other hand, since the upper bound for the wormhole density is one wormhole per Planck volume, namely $\rho_{Mw} = \frac{1}{l_p^4} \sim 10^{140}$, according to (34) the upper bound of Λ_{eff} is $\Lambda_M \sim 10^{72}m^{-2}$, or approximately 10^{124} larger than Λ_{obs}.

Note that in different frameworks there have also been approaches where the cosmological constant is driven by spacetime wormholes, but they typically have $\Lambda \to 0$ at late times. For instance, in [53] Hawking considers space time foam as a gas of instantons of different topology and in the Euclidean quantum gravity one-loop approximation he obtains a negative cosmological constant $\Lambda_s \sim \alpha\frac{\chi}{V}$, which although having been extracted in a totally different framework, it resembles our result (33). However, Hawking's calculations are based on the trace anomaly expressed by the invariant GB term, and for that reason χ appears constant (refinements of Hawkings spacetime foam model were presented in [64, 86]). In [87], Coleman proposed a mechanism where wormholes and topological fluctuations of space time induce a distribution of the values of Nature's constants, which smear Λ distribution to peak at zero. In [88, 89] it was claimed that the behavior of the fundamental coupling constants in Coleman's scenario was controlled by the trace anomaly and a similar proposition was the Giddings-Strominger wormhole solution where a wormhole is coupled to an instanton [90]. Additionally, in [55] a semiclassical model of spacetime foam was proposed, in which Casimir-like quantum fluctuations give rise to an arbitrary number of wormholes, as pairs of black hole and antiblack hole, which drive the induced cosmological constant to zero as they grow.

Nevertheless, in our approach microscopic wormholes lead to an effective dark energy sector not directly, but due to the topology change they induce on the manifold, which in turn affects the variation of the GB term. That is why it can have an arbitrary dynamical behavior. Additionally, apart from its effects at late-time cosmology, such a dynamical effective sector could play a role in the early universe too, potentially driving inflation. Finally, note that since Nariai instantons correspond to a negative component while Euclidean wormholes to a positive one, one could have richer cosmological behavior as well.

6. Conclusions

It is known that the appearance of microscopic objects, such as instantons and wormholes, at the spacetime-foam level in Euclidean quantum gravity approaches, leads to spacetime topology changes, which in principle may affect the field equations that arise through the variational procedure of gravitational actions. Although in the case of Einstein-Hilbert action the presence of microscopic wormholes does not lead to any nontrivial result, when the Gauss-Bonnet term is added in the action the above procedure induces an effective cosmological term that depends on the Gauss-Bonnet coupling and the wormhole density. Since the later in a dynamical spacetime is in general time-dependent, one results with an effective dark energy sector of topological origin.

In particular, the appearance of objects with distinct topology and thus with different Euler characteristics, leads to a change of the topological character of the spacetime manifold. This process can be encapsulated into an effective approach in which the variation of the quantum fluctuations induces a variation in the Euler characteristic, constituting the effective topological variation procedure. By employing the semiclassical one-loop approach on the linear split of the metric and, additionally, incorporating the background independence through the tadpole condition, we showed that the variation of the Gauss-Bonnet term in the Lagrangian gives rise to a nontrivial term in the field equations. The obtained effective cosmological constant can coincide with the observed value $10^{-52}m^{-2}$ for densities of the order of 10^{16} microscopic wormholes per cubic meter per second, which is quite reasonable according to estimations.

It would be interesting to consider scenarios of time-dependent wormhole density and investigate the behavior of the resulting dynamical dark energy sector, including the confrontation with observational data from supernovae type I (SNIa), baryonic acoustic oscillations (BAO), and cosmic microwave background (CMB) observations, as well as with direct Hubble constant measurements through cosmic chronometers (CC). Additionally, one could examine the matter perturbation evolution in such a dynamical scenario. Moreover, one could apply the same considerations at early times and examine the possibility of a successful inflation realization. At the more theoretical level, one could investigate the effective topological variation procedure going beyond the linear expansion level, as well as examine the effect of a topologically dynamical GB term in the trace anomaly behavior, in heterotic strings renormalizability, and in M-theory's β-function. All these studies extend beyond the scope of this manuscript and will be performed in future projects.

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