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Landscape, Swampland, and Extra Dimensions

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By combining swampland conjectures with observational data, it was recently suggested that the cosmological hierarchy problem (i.e. the smallness of the dark energy in Planck units) could be understood as an asymptotic limit in field space, corresponding to a decompactification of one extra (dark) dimension of a size in the micron range. In these Proceedings we examine the fundamental setting of this framework and discuss general aspects of the effective low energy theory inherited from properties of the overarching string theory. We then explore some novel phenomenology encompassing the dark dimension by looking at potential dark matter candidates, decoding neutrino masses, and digging into new cosmological phenomena.

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1. Introduction

The challenge for a fundamental theory of nature is to describe both particle physics and cosmology. Accelerator experiments and cosmological observations provide complementary information to constrain the same theory. We have long known that only about 4% of the content of the universe is ordinary baryonic matter; the remainder is dark matter ($\sim 22\%$) and dark energy ($\sim 74\%$). The ΛCDM model, in which the expansion of the universe today is dominated by the cosmological constant Λ and cold dark matter (CDM), is the simplest model that provides a reasonably good account of all astronomical and cosmological observations [1].

The cosmological evolution is described by Einstein's field equation,

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, \mathcal{R} + g_{\mu\nu}\, \Lambda = \frac{8\pi G}{c^4}\, T_{\mu\nu}\,, \tag{1}$$

where $\mathcal{R}_{\mu\nu}$ and $\mathcal{R}$ are respectively the Ricci tensor and scalar, $g_{\mu\nu}$ is the metric tensor, $T_{\mu\nu}$ is the energy momentum tensor, Greek subscripts run from 0 to 3, and G is Newton's gravitational constant. The cosmological constant encapsulates two length scales: the size of the observable Universe $[\Lambda] = L^{-2}$ and of the dark energy $[\Lambda/G \times c^3/\hbar] = L^{-4}$. The observed value of the cosmological constant $\Lambda_{\rm obs} \simeq 0.74 \times 3H_0^2/c^2 \simeq 1.4 \times (10^{26}\ {\rm m})^{-2}$ gives a characteristic length of dark energy $\simeq 85\ \mu$m, where we have adopted the recent measurement of the Hubble constant $H_0 \simeq 73$ km/s/Mpc by the SH0ES team [2, 3].

The $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ Standard Model (SM) of strong and electroweak interactions encapsulates our current best understanding of physics at the smallest distances ($< 10^{-21}$ m) and highest energies (center-of-mass energies $\sqrt{s} \sim 14$ TeV). Even though the SM continues to survive all experimental tests at accelerators [1], it is widely considered to be an effective theory. Besides missing gravity, the SM does not include a mechanism for giving neutrinos their masses, and does not incorporate dark matter or dark energy. Moreover, if we compare the strength of gravity to that of the SM interactions we find that

$$GM_H^2/(\hbar c) = (M_H/M_p)^2 \approx 10^{-34} \ll 1\,, \tag{2}$$

where M_H is the Higgs mass and M_p is the Planck mass.

Leaving aside for the moment the SM downsides, a question to ask ourselves is: *why is the gravitational interaction between SM particles so much weaker than the other SM interactions?* On the flip side, *why is the Planck mass so huge relative to the SM or dark energy scales?*

A way to connect the hierarchies between particle physics and cosmology is via the size of extra dimensions which are necessary ingredients for consistency of string theory [4–6]. Indeed, if their size is large compared to the fundamental (string) length, the strength of gravitational interactions becomes strong at distances larger than the actual four-dimensional (4D) Planck length [7, 8]. As a result, the string scale is detached from the Planck mass consistently with all experimental bounds if the observable universe is localized in the large compact space [8].

In these Proceedings we summarize the state-of-the-art in this subject area, and discuss future research directions. We begin by reviewing the emergence of a new paradigm of quantum gravity.

2. The Landscape and the Swampland

The Swampland Program seeks to amplify our understanding of the fundamental constraints accompanying a consistent theory of quantum gravity (QG) [9]. The basic thought is that an effective field theory (EFT) might seem consistent as a stand-alone theory in the IR (anomaly free), but coupling the theory to gravity in the UV may uproot its consistency. Actually, given the scarcity of consistent EFTs coupled to QG, it is easier to rule inconsistent theories out. The goal is therefore to circumscribe the subset of 4D EFTs that can be UV-completed to a QG theory and are said to belong in the landscape from the complementary subset of theories that do not admit such a completion and are relegated to the swampland. This is done by enumerating criteria classifying the properties that an EFT must satisfy in order to enable a consistent completion into QG. As the energy increases and we get closer to the QG scale the swampland criteria provide stronger constraints on the boundary that separates the swampland from the landscape. This implies that the space of UV-consistent EFTs encircles a conical-shape structure that is cartooned in the left panel of Fig. 1.

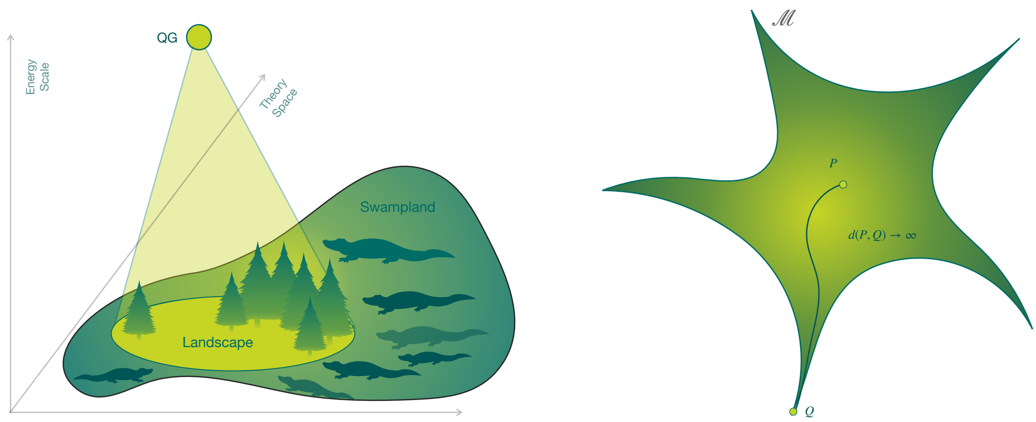

Figure 1: *Left.* The space of EFTs that are a low energy limit of quantum gravity forms a cone in theory space, because the swampland constraints become stronger at high energies. *Right.* Moduli space $\mathscr{M}$ with an infinite distance limit between points P and Q. From Ref. [10].

The swampland criteria have evolved to some set of conjectures, which can be used as new guiding principles to construct compelling UV-completions of the SM. There are many swampland conjectures in the literature; actually, too many to be listed here and so readers are referred to comprehensive reviews [11–14]. Conjectures linked to towers of states, however, deserve some attention because they are closely related to the topics we will explore in this proceedings.

Consider a gravitational EFT with a moduli space $\mathscr{M}$ (i.e. a space parameterized by the massless scalar fields in the theory) and whose metric is given by the kinetic terms of the scalar fields.[1] The **distance conjecture** (DC) states [15] that infinite distance limits $\Delta\phi \to \infty$, with reference to geodesic field distance $\Delta\phi \equiv d(P,Q)$ illustrated in Fig. 1, are accompanied by an infinite tower of exponentially light states

$$m(Q) \sim m(P)\, e^{-\lambda\Delta\phi}\,, \tag{3}$$

[1] In the presence of a mechanism of moduli stabilization, the moduli space is discretized corresponding to a finite landscape of vacua which is assumed to be large enough for the following arguments to hold.

where distance and masses are measured in Planck units, and λ is an order one positive constant.

The archetypal example to gain intuition about the DC is a theory compactified on a circle. Taking R to be the modulus controlling the radius (or radion), it is straightforward to see that the Kaluza-Klein (KK) modes have masses that scale as $m_n^2 = n^2/R^2$, with $n \in \mathbb{Z}$. Now, after dimensional reduction of the gravitational piece of the action and the corresponding field redefinition to go into the Einstein-frame, the kinetic term of the Lagrangian for the radion field takes the form $\mathscr{L}_{\rm kin} \supset (\partial R)^2/R^2$. The distance in field space between points $R_{\rm i}$ and $R_{\rm f}$ is measured by the field space metric, given by $1/R^2$, and is found to be

$$d(R_{\rm i}, R_{\rm f}) = \int_{R_{\rm i}}^{R_{\rm f}} \sqrt{\frac{1}{R^2}}\, dR = |\ln(R_{\rm f}/R_{\rm i})| \,. \tag{4}$$

As predicted by the DC, Taking the decompactification limit $R \to \infty$ we see that the KK tower becomes light, $m_n \sim e^{-d(R_{\rm i}, R_{\rm f}\to\infty)}$. In the opposite limit at infinite distance $R \to 0$ the KK tower does not satisfy the DC. However, in this limit there is another infinite tower that is becoming light; namely, the winding modes (wrapping strings), which present the same behaviour as the KK modes in the decompactification limit. All in all, we arrive at a staggering conclusion: *to satisfy the DC in KK compactifications we need extended objects that can wrap some compact directions and become light in the limit in which they shrink to zero size*.

Before moving on, we bring up a refinement of the DC, which is known as the **emergent string conjecture** (ESC) and states that any infinite distance limit is either a decompactification limit or a limit in which there is a weakly coupled string becoming tensionless [16]. Recently it was argued [17, 18] that bottom-up arguments from black hole properties provide general evidence for the ESC.

The DC and the ESC can be generalized to other field space configurations beyond the moduli space. For example, we can define a notion of distance between different metric and flux configurations of anti-de Sitter spacetimes [19–21], such that the flat space limit $\Lambda \to 0$ is located at infinite distance in this metric configuration space. The related distance in the space of AdS vacua is then given as

$$d(\Lambda_{\rm i}, \Lambda_{\rm f}) = |\ln(\Lambda_{\rm f}/\Lambda_{\rm i})| \,, \tag{5}$$

and the **anti-de Sitter distance conjecture** (AdS-DC) states [19] that any AdS vacuum has an infinite tower of states that becomes light in the flat space limit $\Lambda \to 0$, satisfying

$$m \sim |\Lambda|^{\alpha} \,, \tag{6}$$

with α a positive constant of order one. Besides, under the hypothesis that this scaling behavior holds in de Sitter (dS) space, an unbounded number of massless modes also pop up in the limit $\Lambda \to 0$ [19]. A point worth noting at this juncture is that the AdS-DC modifies the EFT expectation $\Lambda \sim \Lambda_0 + m^{1/\alpha}$, in which Λ_0 stands for the contribution of the heavy modes. The vanishing of Λ_0 is argued to be intrinsically related to the modular invariance of string theory (see also the discussion in [22]). Furthermore one can formulate a conformal field theory (CFT) distance conjecture about the spectrum of certain (holographically dual) conformal and world sheet field theories [23–25], and one can also generalize it to topology change and non-geometric spaces at infinite distance [26].

In a way, similar to the AdS-DC, a distance conjecture for black hole solutions in the EFT was put forward [27]. Concretely the **black hole entropy distance conjecture** (BHEDC) states that the limit of infinite black entropy $S_{\rm BH} \to \infty$ is at infinite distance in the space of BH metrics and the associated BH entropy distance can be expressed as

$$d(S_{\rm BH,i}, S_{\rm BH,f}) = \left|\ln(S_{\rm BH,f}/S_{\rm BH,i})\right| . \tag{7}$$

Following the infinite distance conjecture, there must be a corresponding mass scale with a tower of “states", whose masses in Planck units are given as

$$m \sim (S_{\rm BH})^{-\beta} , \tag{8}$$

where β is a positive constant. Further discussion and more evidence for the BHEDC for extremal and non-extremal black holes was subsequently given in [28–31]. In addition, the BHEDC can be also applied for so called minimal black holes. Their minimal horizon size $L_{\rm min}$ serves as a working definition for the species scale $\Lambda_{\rm sp}$ [32–36] and the corresponding species length $l_{\rm sp}$ in the EFT (see also the discussion in section 3.1 and section 8.):[2]

$$l_{\rm sp} \equiv \Lambda_{\rm sp}^{-1} \equiv L_{\rm min}. \tag{9}$$

It follows that the species scale is related to the entropy of the minimal black hole in the following way:

$$\Lambda_{\rm sp} = \frac{M_p}{(S_{\rm BH,min})^{\frac{1}{d-2}}} . \tag{10}$$

Employing the BHEDC to minimal black holes one can then infer that the limit of vanishing species scale $\Lambda_{\rm sp} \to 0$ is at infinite distance in the space of EFTs, with a distance given as

$$d(\Lambda_{\rm sp,i}, \Lambda_{\rm sp,f}) = \left|\ln(\Lambda_{\rm sp,f}/\Lambda_{\rm sp,i})\right|, \tag{11}$$

and with a tower of massless states, in which masses scale as

$$m \sim (\Lambda_{\rm sp})^{\gamma} , \tag{12}$$

where γ is a positive constant. Combing this with the AdS-DC it follows that the cosmological constant Λ and the species scale, i.e. the UV cut-off of the EFT, are parametrically related at the boundary of the moduli space as

$$|\Lambda| \simeq (\Lambda_{\rm sp})^{\gamma/\alpha} . \tag{13}$$

So the limit of a small cosmological constant goes along with the limit of a small species scale and vice versa. Furthermore minimal black holes and the species scale can also be used to introduce the notion of species thermodynamics [41, 49], which connects the thermodynamic properties of the KK and string species to an entropy and temperature functional over the moduli space of vacua.

The DC, the AdS-DC, and the BHEDC can be further generalized for more general manifolds using geometric flow equations [50–56]. Specifically, the following **Ricci flow distance conjecture**

[2]Alternatively, $\Lambda_{\rm sp} = M_p/\sqrt{F_1}$ can be identified with the scale at which R^2 corrections to the Einstein action become important, with $F_1 \simeq N$ being the one-loop topological string free energy [37]. In [38] it was shown that this definition of the species agree with the one from minimal black holes for a specific type of charged black holes. Further recent discussion about the species scale can be found in [39–49] and recently further arguments for the agreement of these two definitions of the species scale were provided in [18].

was formulated [50]: For a d-dimensional Riemannian manifold, the distance in the field space of the background metrics along the Ricci-flow is determined by the scalar curvature $\mathcal{R}(g)$:

$$d(\mathcal{R}_i, \mathcal{R}_f) = |\ln(\mathcal{R}_f/\mathcal{R}_i)| \,. \tag{14}$$

Here $\mathcal{R}_i$ and $\mathcal{R}_f$ are the corresponding initial and final values of the scalar curvature. Furthermore it is stated that at $\mathcal{R} = 0$ there is an infinite tower of additional massless states in quantum gravity.

One more related distance conjecture, the **large D distance conjecture** [57, 58], states that the limit of large space-time dimension, $d \to \infty$, is also at infinite distance in the space of quantum gravity vacua.

Another interesting limit arises with the gravitino mass going to zero. This is because the gravitino mass is generally related to the scale of spontaneous supersymmetry (SUSY) breaking in non-supersymmetric vacua (like the one we happen to live in) [59, 60]. The **gravitino conjecture** states [61, 62] that in a supersymmetric theory with a non-vanishing gravitino mass $m_{3/2}$, a tower of states becomes light in the limit $m_{3/2} \to 0$ according to

$$m \sim \left(\frac{m_{3/2}}{\bar{M}_p}\right)^{\delta} \bar{M}_p \tag{15}$$

where δ is an order-one positive parameter and $\bar{M}_p = M_p/\sqrt{8\pi}$ is the reduced Planck mass.

In closing, we review one last conjecture, which is not linked to a tower of states, but will be relevant for our discussion. The **weak gravity conjecture** (WGC) states [63, 64] that for given a gauge theory, weakly coupled to Einstein gravity, there exists a charged state with

$$\frac{Q}{m} \geq \left.\frac{Q_{\rm BH}}{M_{\rm BH}}\right|_{\rm extremal} = \mathcal{O}(1) \tag{16}$$

in Planck units, where $Q_{\rm BH}$ and $M_{\rm BH}$ are the charge and mass of an extremal black hole, $Q = qg$, q is the quantized charge of the state, and g is the gauge coupling.

The WGC can be seen as a kinematic requirement that allows extremal black holes to decay. As can be inferred from the *weak cosmic censorship* [65], charged black holes must satisfy an extremality bound in order to avoid the presence of naked singularities. Intuition about the extremality bound (16) arises spontaneously from the the Reissner-Nordström (RN) metric that describes the simplest extremal black hole, which has its mass $M_{\rm BH}$ equal to its charge $Q_{\rm BH}$ in appropriate units. Note that if $Q_{\rm BH} = M_{\rm BH}$ the single option for the black hole to decay is if there exists a particle whose charge to mass ratio is at least one. If $M_{\rm BH} > Q_{\rm BH}$ the black hole has inner (Cauchy) and outer (event) horizons, whereas if $M_{\rm BH} < Q_{\rm BH}$, the RN metric describes a naked singularity.

3. Foundations of the Dark Dimension

3.1 In which region of the landscape do we live?

As demonstrated in [4], applying the AdS-DC to dS (or quasi dS) space [19] provides one possible pathway to answer this question. Moving forward, we will walk along this pathway.

Many asymptotic limits are expected to have towers of KK modes. According to the dS-DC, the vacuum energy scales as $\Lambda \sim m^{1/\alpha}$, with m the mass scale of the leading tower [19]. Since the

KK tower contains massive spin-2 bosons, there is a strong constraint from fundamental physics, *unitarity*, which is expressed in the form of the Higuchi bound and imposes an absolute upper limit $\alpha = 1/2$ [66]. Besides, α has a lower limit set by contributions of the Casimir energy; in four dimensions $\alpha \geq 1/4$. Another property of asymptotic limits is that both m and Λ are very small in Planck units. Therefore, since the observed amount of dark energy in our world is also very small, $\Lambda \sim 10^{-122} M_p^4$, it is tempting to speculate whether we could be living near an asymptotic limit. We will then assume that we live within sight of the space boundary in some infinite distance limit. This assumption automatically leads to the prediction of a tower of light fields at the energy scale

$$m_{\rm KK} \lesssim \Lambda^{1/4} \sim 2.31 \text{ meV} . \tag{17}$$

Since we have not seen yet experimental evidence of such a tower, it must couple very weakly (if at all) to SM fields.

The ESC connects infinite distance limits with the decompactification of n extra dimensions. Now, consistency of large-distance black hole physics in the presence of a tower of N light fields imposes a bound on the gravitational cutoff of the EFT, and the fundamental length is no longer $l_p = M_p^{-1}$, but rather the species length [32, 33]

$$l_{\rm sp} = \sqrt{N}\, l_p . \tag{18}$$

The origin of (18) can be traced back using different arguments, we follow here the reasoning given in [67] based on quantum information storage [68]. Consider a pixel of size L containing N species storing information. The minimal energy required to localize N wave functions is of order N/L. This energy can be associated to a Schwarzschild radius $r_s = N/(LM_p^2)$, which must be smaller than the pixel size if we want to avoid the system to collapse into a black hole. Now, $r_s \leq L$, implies there is a minimum size $l_{\rm sp} \equiv \Lambda_{\rm sp}^{-1} \equiv L_{\rm min} = \sqrt{N}\, M_p^{-1}$ associated to the scale where gravity becomes strongly coupled and the EFT necessarily breaks down. Since $l_{\rm sp}$ constitutes the smallest black hole size described by the EFT (involving only the Einstein term), $\Lambda_{\rm sp}$ codifies the "number of light degrees of freedom" (i.e., the number of KK excitations lighter than the cut-off), given by $N \sim R_\perp^n l_{\rm sp}^{-n}$, up to energies of order

$$M_* \sim \Lambda_{\rm sp} = m_{\rm KK}^{n/(n+2)} M_p^{2/(2+n)} , \tag{19}$$

where n is the number of decompactifying dimensions of radius $R_\perp \sim m_{\rm KK}^{-1}$ and where the species scale $\Lambda_{\rm sp}$ corresponds to the Planck scale M_* of the higher dimensional theory.

Decompactification limits are tightly constrained by observations. Indeed, astrophysical bounds from the requirement that neutron stars are not excessively heated by KK decays lead to a very restrictive limit on the mass scale of the KK tower, which depends on the number of dimensions that are decompactifying: $R_\perp \leq 44\ \mu$m for $n = 1$, and $R_\perp \leq 10^{-4}\ \mu$m for $n = 2$ [69]. The limit on $R_\perp$ becomes more restrictive with rising n. For $n = 1$, deviations from Newton's law impose a more restrictive constraint, $R_\perp \leq 30\ \mu$m [70]. Remember that the starting point here is the hypothesis that associates the cosmological constant to a tower of states whose mass scale satisfies

$$m_{\rm KK} \simeq \Lambda^{1/4}/\lambda , \tag{20}$$

where λ is an order one parameter (more on this below). Now, we have seen that $\Lambda^{-1/4} \sim 85\mu$m and so from (20) we conclude that if $n = 2$ the hypothesis is excluded by experiment, but if $n = 1$ the hypothesis can be made compatible with the experiment adjusting the proportionality factor, which is estimated to be within the range $10^{-4} \lesssim \lambda \lesssim 10^{-2}$. Substituting $n = 1$ into (19) we have $M_* \sim m_{\rm KK}^{1/3} M_p^{2/3}$, and so $10^9 \lesssim M_*/\text{GeV} \lesssim 10^{10}$.[3] We can combine this with the relation between $m_{\rm KK}$ and $\Lambda_{\rm sp}$ to derive the following relation

$$M_* \sim \Lambda_{\rm sp} = \Lambda^{1/12}\, \lambda^{-1/3}\, M_p^{2/3}\,. \tag{21}$$

In summary, when astrophysical bounds and gravitational tests of Newton's law are combined with the requirement that the size of the extra dimension is related to the cosmological constant, we arrive at the conclusion encapsulated in (20); namely, that there is one extra dimension of radius $R_\perp$ in the micron range, and that the lower bound for $\alpha = 1/4$ is basically saturated [4]. Because of its connection to the observed dark energy, this dimension has been nicknamed *the dark dimension*.

In closing we note that explicit string calculations of the vacuum energy (see e.g. [73–76]) show that the lower limit on α is saturated. Actually, a theoretical amendment on the connection between the cosmological and KK mass scales confirms $\alpha = 1/4$ [22].[4]

3.2 The scale of SUSY breaking

It is of course interesting to explore whether there is a relation between the SUSY breaking scale and the measured value of the dark energy density Λ. Such a relation can be derived by combining two quantum gravity consistency swampland constraints, which tie Λ and the gravitino mass $m_{3/2}$, to the mass scale of a light KK tower and, therefore, to the UV cut-off of the EFT [59, 61, 62]. One can then use the constraint on $m_{3/2}$ to infer the implications of the dark dimension scenario for the scale of supersymmetry breaking. In general, one can distinguish two situations. In the first case, the gravitino mass and the cosmological constant are related to the same tower of states. This is arguably the simplest scenario, in which the natural scale for SUSY signatures is of order $\Lambda^{1/8} \sim$ TeV, and therefore is within reach of LHC and/or of the next generation of hadron colliders [79]. In the second case, $m_{3/2}$ and Λ are related to different towers. This scenario requires a decoupling of the gravitino mass from the cosmological constant and is thus more difficult to realize in concrete models.

Possible string theory and effective supergravity realizations of the dark dimension scenario with broken supersymmetry are discussed in [79].

3.3 The dark dimension as a space with two boundaries

It was recently conjectured that the dark dimension can be viewed as a line interval with end-of-the-world 9-branes (EW9-brane) attached at each end [80]. This construction derives from the 10D $E_8 \times E_8$ heterotic string theory in the strong coupling limit that has an orbifold $S^1/\mathbb{Z}_2$ eleventh

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[3] Auger data of highest energy cosmic rays favor $M_* \sim 10^{10}$ GeV [71, 72].

[4] Unsubstantiated criticisms raised in [77, 78] have been addressed in [22]. We reiterate herein that quantum gravity and string theory are different from field theory and lead to a finite result relating Λ to the KK scale via (20). Two arguments support this statement: *(i)* the AdS-dC stressing that all infinities cancel out (for AdS, this conjecture holds in many examples) and *(ii)* the explicit one-loop string computation, which is finite due to modular invariance and confirms $\alpha = 1/4$.

dimension (the dark dimension), which is not visible in perturbation theory. The gravitational field propagates in the bulk $\mathbb{R}^{10} \times S^1/\mathbb{Z}_2$, whereas the $E_8 \times E_8$ fields propagate only at the $\mathbb{Z}_2$ fixed points on the two EW9-branes.

The 10D coupling of the E_8 is a fixed number in 11D supergravity (M-theory) units. When six out of ten space-time dimensions are compactified on a Calabi-Yau (CY6) manifold, the tree-level couplings of the effective 4D theory are simply given by the volume V of CY6 (always in M_{11} units) [81, 82]. The SM gauge couplings (g) are observed to be $O(1)$, and this forces V on one of the branes to be $O(1)$ in M_{11} units.

For smooth compactifications, the volume of CY6 at given position of the coordinate x^{11},

$$v(x^{11}) = \int_{\mathrm{CY6}} \sqrt{g}\, d^6x\,, \tag{22}$$

becomes an approximate linear function of the extra coordinate, and decreases from one E_8 to the other E_8 [81]. This implies that v has different values at the two fixed points $x^{11} = 0$ and $x^{11} = \pi\rho$. Herein we identify $V = v(0)$ and $V' = v(\pi\rho)$. Note that when the theory in one E_8 is perturbative the theory on the other E_8 becomes non-perturbative when the radius ρ of the dark dimension is large. This forces an upper bound on the size of the 11th dimension to avoid V' to become negative. If the anomaly coefficient is $O(1)$, then $\rho \sim 1\ \mu$m is not allowed. This is a general phenomenon in orientifold compactifications in which couplings of localized interactions acquire a linear dependence in the extra dimension if one extra dimension (transverse to the brane) is large [83]. The absence of such divergences requires local tadpole cancellation (between branes and orientifolds).

However, there are some particular non-geometric compactifications where the correction to the other E_8 coupling vanishes and there is no constraint on ρ [84]. In such a particular case, we have the following connections between the 11D Planck mass, M_{11} (defined in terms of the coefficient of the Einstein Lagrangian in 11D supergravity, as $M_{11} = \kappa^{-2/9}$), the radius of the dark dimension ρ, and the compactification radius $R = V^{1/6}$,

$$\kappa = (2\alpha)^{3/4} R^{9/2}\,, \tag{23}$$

and

$$\rho = (\alpha/2)^{3/2}\, M_p^2\, R^3 \tag{24}$$

where V is the CY6 volume on the SM boundary and $\alpha \equiv g^2/(4\pi)$. For $\rho \sim 1\ \mu$m and phenomenological value $\alpha \sim 1/25$, we obtain $M_{\mathrm{KK}} \sim R^{-1} \sim 7 \times 10^8$GeV and $M_{11} \sim 10^9$GeV.

In summary, when six dimensions $(x^5, \cdots, x^{10})$ are compactified on a CY manifold, the eleven-dimensional bulk of the world becomes 5D while the 9-branes at its boundaries become 3-branes. The entire SM lives on one of those 3-branes and is oblivious to the bulk of the 5D world or its other boundary. The threshold structure of the $\rho \gg R \gtrsim l_{\mathrm{sp}} \sim M_{11}^{-1}$ regime of the M-theory can be summarized as follows:

- Gravity has a threshold at a rather low energy scale $m_{\mathrm{KK}} \sim 1/\rho \sim$ eV above which it becomes 5D. However, this threshold does not affect any gauge, Yukawa or scalar forces of the SM, which remains 4D at distances shorter than ρ.
- The next threshold arises at the KK scale $M_{\mathrm{KK}} \sim 1/R \sim 7 \times 10^8$ GeV, where six more dimensions open up for both gravity and gauge interactions.

- Almost immediately above this scale (around $\Lambda_{\rm sp}$), the effective field theory description breaks down and the fully quantized M-theory (whatever that is) takes over.

4. Dark Matter Candidates

The dark dimension provides a colosseum for dark matter contenders. In this section we review the general properties of the various dark matter candidates.

4.1 Primordial black holes

It has long been speculated that black holes could be produced from the collapse of large amplitude fluctuations in the early universe [85–88]. For an order of magnitude estimate of the black hole mass $M_{\rm BH}$, we first note that the cosmological energy density scales with time t as $\rho \sim 1/(Gt^2)$ and the density needed for a region of mass $M_{\rm BH}$ to collapse within its Schwarzschild radius is $\rho \sim c^6/(G^3 M_{\rm BH}^2)$, so that primordial black holes (PBHs) would initially have around the cosmological horizon mass [89]

$$M_{\rm BH} \sim \frac{c^3 t}{G} \sim 10^{15} \left(\frac{t}{10^{-23}\ {\rm s}}\right)\ {\rm g}\,. \tag{25}$$

This means that a black hole would have the reduced Planck mass ($\bar{M}_p \sim 10^{-5}$ g) if they formed at the Planck time (10^{-43} s), 1 $M_\odot$ if they formed at the QCD epoch (10^{-5} s), and $10^5 M_\odot$ if they formed at $t \sim 1$ s, comparable to the mass of the holes thought to reside in galactic nuclei. This back-of-the-envelope calculation suggests that PBHs could span an enormous mass range. Despite the fact that the mass spectrum of these PBHs is yet to be shaped, on cosmological scales they would behave like a typical cold dark matter particle.

However, an all-dark-matter interpretation in terms of PBHs is severely constrained by observations [89–92]. To be specific, the extragalactic γ-ray background [93], the cosmic microwave background (CMB) [94], the 511 keV γ-ray line [95–98], EDGES 21-cm signal [99], and the MeV Galactic diffuse emission [100–102] constrain evaporation of black holes with masses $\lesssim 10^{17}$ g, whereas the non-observation of microlensing events by MACHO [103], EROS [104], Kepler [105], Icarus [106], OGLE [107] and Subaru-HSC [108] set an upper limit on the black hole abundance for masses $M_{\rm BH} \gtrsim 10^{21}$ g.

Before proceeding, we pause and call attention to a captivating coincidence:

$$\texttt{size of the dark dimension} \sim \texttt{wavelength of visible light}\,, \tag{26}$$

which implies that the Schwarzschild radius of 5D black holes is well below the wavelength of light. For point-like lenses, this is precisely the critical length where geometric optics breaks down and the effects of wave optics suppress the magnification, obstructing the sensitivity to 5D PBH microlensing signals [108]. So 5D PBHs escape these microlensing constraints; at the same time, as pointed out in [109], they are: bigger, colder, and longer-lived than a usual 4D black hole of the same mass.

Throughout we rely on the probe brane approximation, which ensures that the only effect of the brane field is to bind the black hole to the brane [110]. This is an adequate approximation provided $M_{\rm BH}$ is well above the brane tension, which is presumably of the order of but smaller than $M_\star$. We also assume that the black hole can be treated as a flat d dimensional object. This assumption is valid for extra dimensions that are larger than the 5D Schwarzschild radius, which is given by [111–113]

$$r_s(M_{\rm BH}) \sim \frac{1}{M_*}\left[\frac{2}{3\,\pi}\,\frac{M_{\rm BH}}{M_*}\right]^{1/2}\,. \tag{27}$$

Schwarzschild black holes radiate all particle species lighter than or comparable to their Hawking temperature, which in four dimensions is related to the mass of the black hole by

$$T_H = \frac{\bar{M}_p^2}{8\pi M_{\rm BH}} \sim \left(\frac{M_{\rm BH}}{10^{16}\ {\rm g}}\right)^{-1}\ {\rm MeV}\,, \tag{28}$$

whereas for 5D black holes the Hawking temperature mass relation is found to be [114]

$$T_H \sim \frac{1}{r_s} \sim \left(\frac{M_{\rm BH}}{10^{12}\ {\rm g}}\right)^{-1/2}\ {\rm MeV}\,. \tag{29}$$

The numerical estimate of (29) applies to the dark dimension scenario with $M_* \sim 10^{10}$ GeV.[5] It is evident that 5D black holes are colder than 4D black holes of the same mass.

Armed with the Hawking temperature, we can now calculate the entropy of the 5D black hole [115]

$$S_{\rm BH} = \frac{4}{3}\pi\ M_{\rm BH}\ r_s\,. \tag{30}$$

In the rest frame of the Schwarzschild black hole, both the average number [116, 117] and the probability distribution of the number [118–120] of outgoing particles in each mode obey a thermal spectrum. However, in the neighborhood of the horizon the black hole produces an effective potential barrier that backscatters part of the emitted radiation, modifying the thermal spectrum. The so-called "greybody factor", which controls the black hole absorption cross section, depends upon the spin of the emitted particles s, their energy Q, and $M_{\rm BH}$ [121–125]. The prevailing energies of the emitted particles are $\sim T_{\rm H} \sim 1/r_s$, resulting in s-wave dominance of the final state. This implies that the black hole evaporates with equal probability to a particle on the brane and in the compact space [126, 127]. Thereby, the process of evaporation is driven by the large number of SM brane modes.

The Hawking radiation causes a 4D black hole to lose mass at the following rate [98]

$$\begin{aligned}\left.\frac{dM_{\rm BH}}{dt}\right|_{\rm evap} &= -\frac{\bar{M}_p^2}{30720\ \pi\ M_{\rm BH}^2}\ \sum_i c_i(T_H)\ \tilde{f}\ \Gamma_s \\ &\sim -7.5\times10^{-8}\left(\frac{M_{\rm BH}}{10^{16}\ {\rm g}}\right)^{-2}\ \sum_i c_i(T_H)\ \tilde{f}\ \Gamma_s\ {\rm g/s}\,,\end{aligned} \tag{31}$$

[5]We have taken the highest possible value of M_* to remain conservative in the estimated bound on the fraction of dark matter composed of primordial black holes $f_{\rm PBH}$.

whereas a 5D black hole has an evaporation rate of [114]

$$\left.\frac{dM_{\rm BH}}{dt}\right|_{\rm evap} \sim -9\,\pi^{5/4}\zeta(4)T_H^2 \sum_i c_i(T_H)\,\tilde{f}\,\Gamma_s$$
$$\sim -1.3\times 10^{-12}\left(\frac{M_{\rm BH}}{10^{16}\ {\rm g}}\right)^{-1} \sum_i c_i(T_H)\,\tilde{f}\,\Gamma_s\ {\rm g/s}, \tag{32}$$

where $c_i(T_H)$ counts the number of internal degrees of freedom of particle species i of mass m_i satisfying $m_i \ll T_H$, $\tilde{f} = 1$ ($\tilde{f} = 7/8$) for bosons (fermions), and where $\Gamma_{s=1/2} \approx 2/3$ and $\Gamma_{s=1} \approx 1/4$ are the (spin-weighted) dimensionless greybody factors normalized to the black hole surface area [128]. Now, comparing (31) and (32) it is easily seen that 5D black holes live longer than 4D black holes of the same mass.

Integrating (32) we can parametrize the 5D black hole lifetime as a function of its mass and temperature,

$$\tau_s \sim 13.8\left(\frac{M_{\rm BH}}{10^{12}\ {\rm g}}\right)^2\left(\frac{6}{\sum_i c_i(T_s)\,\tilde{f}\,\Gamma_s}\right)\ {\rm Gyr}, \tag{33}$$

where we have used (29) to estimate that $T_H \sim 1$ MeV and therefore $c_i(T_H)$ receives a contribution of 6 from neutrinos, 4 for electrons, and 2 from photons, yielding $\sum_i c_i(T_H)\,\tilde{f}\,\Gamma_s = 6$. Armed with (33) we can estimate the bound on the 5D PBH abundance by a simple rescaling procedure of the $d = 4$ bounds on the fraction of dark matter composed of primordial black holes $f_{\rm PBH}$. The key point for such a rescaling is that for a given photon energy, or equivalently a given Hawking temperature, we expect a comparable limit on $f_{\rm PBH}$ for both $d = 4$ and $d = 5$. For example, from (28) and (29) we see that the constraint of $f_{\rm PBH} \lesssim 5\times 10^{-5}$ for 4D black holes with $M_{\rm BH} \sim 10^{16}$ g [102], should be roughly the same for the abundance of 5D black holes with $M_{\rm BH} \sim 10^{12}$ g. Now, since in $d = 4$ for $M_{\rm BH} \sim 4\times 10^{17}$ g we have $f_{\rm PBH} \sim 1$ [102], this implies the same abundance for 5D black holes of $M_{\rm BH} \sim 10^{15}$ g. By duplicating this procedure for heavier black holes we conclude that for a species scale of $O(10^{10}$ GeV), an all-dark-matter interpretation in terms of 5D black holes must be feasible for masses in the range [114]

$$10^{15} \lesssim M_{\rm BH}/{\rm g} \lesssim 10^{21}\,. \tag{34}$$

This range is extended compared to that in the 4D theory by more than two orders of magnitude in the low mass region.

Extremal black holes trace the boundary between black-hole configurations and horizonless naked singularities. As put forward by the WGC, they are characterized by the minimally allowed mass (radius) for a given amount of black-hole charge $Q_{\rm BH}$ (or angular momentum). Near-extremal black holes are characterized by a finite (non-zero) mass gap of the first excited state above the extremal (zero-temperature) black-hole configuration [129]. The temperature of such near-extremal black holes is found to be

$$T_{ne} \sim \frac{\beta^{1/2}T_H}{S_{\rm BH}^{1/2}}\,, \tag{35}$$

where β is a factor of order-one that controls the differences between $M_{\rm BH}$ and $Q_{\rm BH}$; for details see [29, 49]. If there were 5D primordial near-extremal black holes in nature, then an all-dark-matter interpretation would be possible in the mass range [130]

$$10^5\sqrt{\beta} \lesssim M_{\rm BH}/{\rm g} \lesssim 10^{21}\,. \tag{36}$$

Up until now we have assumed that the 5D black holes stay attached to the brane during the evaporation process. Hereafter we relax this assumption and allow them wander off into the bulk. Without knowing more details of the bulk and brane theory it is not worth considering to calculate the probability of such wandering in detail. However, we can assume that the black holes are out of the brane-world and study the evaporation effects of these bulk PBHs. Furthermore, it is always possible that the PBHs are produced in the bulk to start with. This situation will be more appealing within the model discussed in Sec. 6, in which we theorize that the dark dimension may have undergone a uniform rapid expansion, together with the three-dimensional non-compact space, by regular exponential inflation driven by an (approximate) higher dimensional cosmological constant. If this were the case, then primordial fluctuations during inflation of the compact space could lead to the production of black holes in the bulk. In what follows, we then assume that PBHs are localized or propagate in the bulk.

Bulk black holes live longer than those attached to the brane. This is because KK modes are excitations in the full transverse space and so their overlap with small (higher dimensional) black holes is suppressed by the geometric factor $(r_s/R_\perp)$ relative to the brane fields. This geometric suppression precisely compensates for the enormous number of modes and the total KK contribution is only of same order as that from a single brane field [126]. Actually, greybody factors suppress graviton emission when compared to fermions and gauge bosons, and hence bulk black holes which do not have access to the brane degrees of freedom are expected to live longer. In addition, since there is no emission on the brane the bounds due to photon evaporation can be avoided. This implies that PBHs localized in the bulk can provide an all-dark-matter interpretation if

$$10^{11} \lesssim M_{\rm BH}/{\rm g} < 10^{21}\,, \tag{37}$$

where we have remained conservative, and following [131] we assumed that the ratio of the emitted flux into a single brane field over a single bulk field is roughly a factor of two [114].

4.2 KK gravitons

It was observed in [132] that the universal coupling of the SM fields to the massive spin-2 KK excitations of the graviton in the dark dimension provides an alternative dark matter candidate. Within this model the cosmic evolution of the hidden sector is primarily dominated by "dark-to-dark" decays, yielding a specific realization of the dynamical dark matter framework [133]. Consider a tower of equally spaced dark gravitons, indexed by an integer l, and with mass $m_l = l\; m_{\rm KK}$. The partial decay width of KK graviton l to SM fields is found to be,

$$\Gamma^l_{\rm SM} = \frac{\tilde{\lambda}^2\, m^3_{\rm KK}\, l^3}{80\pi \bar{M}_p^2}\,, \tag{38}$$

where $\tilde{\lambda}$ takes into account all the available decay channels and is a function of time [134].

In the absence of isometries in the dark dimension, which is the common expectation, the KK momentum of the dark tower is not conserved. This means that a dark graviton of KK quantum n

can decay to two other ones, with quantum numbers n_1 and n_2. If the KK quantum violation can go up to δn, the number of available channels is roughly $l\,\delta n$. In addition, because the decay is almost at threshold, the phase space factor is roughly the velocity of decay products, $v_{\rm r.m.s.} \sim \sqrt{m_{\rm KK}\,\delta n/m_l}$. Putting all this together we obtain the total decay width,

$$\begin{aligned} \Gamma^l_{\rm tot} &\sim \sum_{l'<l} \sum_{0<l''<l-l'} \Gamma^l_{l'l''} \sim \beta^2 \frac{m_l^3}{\bar{M}_p^2} \times \frac{m_l}{m_{\rm KK}}\,\delta n \times \sqrt{\frac{m_{\rm KK}\delta_n}{m_l}} \\ &\sim \beta^2\,\delta n^{3/2} \frac{m_l^{7/2}}{\bar{M}_p^2 m_{\rm KK}^{1/2}}, \end{aligned} \tag{39}$$

where β parametrizes our ignorance of decays in the dark dimension [132].

To estimate the time evolution of the dark matter mass assume that for times larger than $1/\Gamma^l_{\rm tot}$ dark matter which is heavier than the corresponding m_l has already decayed, and so it follows that

$$m_l \sim \left(\frac{\bar{M}_p^4\, m_{\rm KK}}{\beta^4\,\delta n^3}\right)^{1/7} t^{-2/7}, \tag{40}$$

where t indicates the time elapsed since the big bang [132].

Consistency with CMB anisotropies requires $\Gamma^l_{\gamma\gamma} < 5 \times 10^{-25}\ {\rm s}^{-1}$ between the last scattering surface and reionization [135]. Taking $\tilde{\lambda} = 1$ (to set out the decay into photons) and using (38) it follows that the CMB requirement is satisfied for $l \lesssim 10^8$ at the time $t_{\rm MR} \sim 6 \times 10^4$ yr of matter-radiation equality. In other words, by setting $\tilde{\lambda} \sim 1$ and $m_l(t_{\rm MR}) \lesssim 1$ MeV, the evolution of m_l with cosmic time given in (40) is such that at the last scattering surface the dominant KK state in the dynamical dark matter ensemble has the correct decay width to accommodate the CMB constraints [136].

Now, we have seen that dark matter decay gives the daughter particles a velocity kick. Self-gravitating dark-matter halos that have a virial velocity smaller than this velocity kick may be disrupted by these particle decays. Consistency with existing data requires roughly $\delta n \sim 1$, and $\beta \sim 635$ [137, 138]. For selected fiducial parameters, the cosmic evolution of the incredible bulk predicts via (40) a dominant particle mass of ~ 900 keV at CMB, of ~ 500 keV in the Dark Ages, of ~ 150 keV at Cosmic Dawn, and of ~ 50 keV in the local universe. This is in sharp contrast to typical dark matter decay scenarios with one unstable particle (such as sterile neutrinos [139]). Simultaneous observations of signals at Cosmic Dawn and in the local universe could constitute the smoking gun of the incredible bulk [5].

For many purposes, a black hole can be replaced by a bound state of gravitons [140]. As a matter of fact, a correspondence between 5D PBHs and massive KK gravitons as dark matter candidates has been conjectured in [141].

4.3 A fuzzy radion

The radion stabilizing the dark dimension could be yet another dark matter contender [142]. This is because in principle the radion could be ultralight, and if this were the case it would serve as a fuzzy dark matter candidate. A simple cosmological production mechanism brings into play unstable KK graviton towers which are fueled by the decay of the inflaton. As in the previous model, the cosmic evolution of the dark sector is mostly driven by "dark-to-dark" decay processes that regulate the decay of KK gravitons within the dark tower, conveying another realization of the dynamical dark matter framework [133]. In the spirit of [143], within this model it is assumed that the intra-KK decays in the bulk carry a spontaneous breakdown of the translational invariance in the compact space, such that the 5D momenta are not conserved (but now $\delta n \gg 1$). Armed with these two reasonable assumptions it is straightforward to see that the energy the inflaton deposited in the KK tower should have collapsed all into the radion well before BBN.

5. Neutrino Masses and Mixing

The dark dimension scenario provides a profitable arena to realize an old idea for explaining the smallness of neutrino masses by introducing the right-handed neutrinos as 5D bulk states with Yukawa couplings to the left-handed lepton and Higgs doublets that are localized states on the SM brane stack [144–146]. The neutrino masses are then suppressed due to the wave function of the bulk states.

More indicatively, we introduce three 5D Dirac fermions Ψ_α, which are singlets under the SM gauge symmetries and interact in our brane with the three active left-handed neutrinos in a way that conserves lepton number. The $S^1/\mathbb{Z}_2$ symmetry contains x^{11} to $-x^{11}$ which acts as chirality (γ_5) on spinors. In the Weyl basis each Dirac field can be decomposed into two two-component spinors $\Psi_\alpha \equiv (\psi_{\alpha L}, \psi_{\alpha R})^T$.

The generation of neutrino masses originates in 5D bulk-brane interactions of the form

$$\mathscr{L} \supset h_{ij}\, \overline{L}_i\, \tilde{H}\, \psi_{jR}(x^{11}=0)\,, \tag{41}$$

where $\tilde{H} = -i\sigma_2 H^*$, L_i denotes the lepton doublets (localized on the SM brane), ψ_{jR} stands for the 3 bulk (right-handed) R-neutrinos evaluated at the position of the SM brane, $x^{11} = 0$ in the dark-dimension coordinate x^{11}, and h_{ij} are coupling constants. This gives a coupling with the L-neutrinos of the form $\langle H\rangle\, \overline{\nu}_{L_i}\ \psi_{jR}(x^{11} = 0)$, where $\langle H\rangle = 175$ GeV is the Higgs vacuum expectation value. Expanding ψ_{jR} into modes canonically normalized leads for each of them to a Yukawa 3×3 matrix suppressed by the square root of the volume of the bulk $\sqrt{\pi R_\perp M_s}$, i.e.,

$$Y_{ij} = \frac{h_{ij}}{\sqrt{\pi R_\perp M_s}} \sim h_{ij}\frac{M_s}{M_p}\,, \tag{42}$$

where $M_s \lesssim M_*$ is the string scale, and where in the second rendition we have dropped factors of π's and of the string coupling.

Now, neutrino oscillation data can be well-fitted in terms of two nonzero differences $\Delta m^2_{ij} = m_i^2 - m_j^2$ between the squares of the masses of the three mass eigenstates; namely, $\Delta m^2_{21} = (7.53 \pm 0.18)\times 10^{-5}$ eV^2 and $\Delta m^2_{32} = (2.453 \pm 0.033)\times 10^{-3}$ eV^2 or $\Delta m^2_{32} = -(2.536\pm 0.034)\times 10^{-3}$ eV^2 [1]. It is easily seen that to obtain the correct order of magnitude of neutrino masses the coupling h_{ij} should be of order 10^{-4} to 10^{-5} for $10^9 \lesssim M_s/\text{GeV} \lesssim 10^{10}$.

Note that KK modes of the 5D R-neutrino fields behave as an infinite tower of sterile neutrinos, with masses proportional to $m_{\rm KK}$. However, only the lower mass states of the tower mix with the active SM neutrinos in a pertinent fashion. The non-observation of neutrino disappearance from oscillations into sterile neutrinos at long- and short-baseline experiments places a 90% CL upper limit $R_\perp < 0.4\ \mu$m for the normal neutrino ordering, and $R_\perp < 0.2\ \mu$m for the inverted neutrino ordering [147, 148].[6] This set of parameters corresponds to $\lambda \lesssim 10^{-3}$ and so $m_{\rm KK} \gtrsim 2.5$ eV [5].

Before proceeding, it is important to stress that the upper bounds on $R_\perp$ discussed in the previous paragraph are sensitive to assumptions of the 5$^{\rm th}$ dimension geometry. Moreover, in the presence of bulk masses [149, 150], the mixing of the first KK modes to active neutrinos can be suppressed, and therefore the aforementioned bounds on $R_\perp$ can be avoided [151, 152]. It is also worth mentioning that such bulk masses have the potential to increase the relative importance of the higher KK modes, yielding distinct oscillation signatures via neutrino disappearance/appearance effects.

Non-minimal extensions of the dark dimension, in which $m_{3/2}$ and Λ have different KK towers, allow a high-scale SUSY breaking and can therefore host a rather heavy gravitino together with a modulino with a mass of about 50 eV [153]. For a particular example, we note that the modulino could be the fermionic partner of the radion.[7] These models with high-scale SUSY breaking are fully predictive through neutrino-modulino oscillations [154] which can be confronted with data to be collected by experiments at the Forward Physics Facility [155, 156].

A seemingly different, but in fact closely related subject is the the *sharpened* version of the WGC forbidding the presence of non-SUSY AdS vacua supported by fluxes in a consistent quantum gravity theory [157]. This is because (unless the gravitino is very light, with mass in the meV range) *neutrinos have to be Dirac with right-handed states propagating in the bulk so that the KK neutrino towers compensate for the graviton tower to maintain stable dS vacua* [152].

6. Mesoscopic Extra Dimension from 5D Inflation

It is unnatural to entertain that the size of the dark dimension would remain fixed during the evolution of the Universe right at the species scale, since the Higuchi bound implies a very low inflation scale. One possible mechanism to accommodate this hierarchy is to inflate the size of the dark dimension. The required inflationary phase can be described by a 5D dS (or approximate) solution of Einstein equations, with cosmological constant and a 5D Planck scale $M_* \sim 10^9$ GeV [5]. All dimensions (compact and non-compact) expand exponentially in terms of the 5D proper time. It is straightforward to see that this set-up requires about 42 e-folds to expand the 5th dimension from the fundamental length $O(M_*^{-1})$ to the micron size $O(R_\perp)$. At the end of 5D inflation, or at any given moment, one can interpret the solution in terms of 4D fields using 4D Planck units from the relation $M_p^2 = M_*^3 R$, which amounts going to the 4D Einstein frame. This implies that if $M_*^{-1} \leq R \leq R_\perp$ expands N e-folds, then the 3D space would expand $3N/2$ e-folds as a result of a uniform 5D inflation. Altogether, the 3D space has expanded by about 60 e-folds to solve the horizon problem, while connecting this particular solution to the generation of large size extra dimension.

[6]We arrived at these upper bounds by looking at the low mass limit of the lightest neutrino state in Fig. 6 of [148] and rounding the numbers to one significant figure.

[7]In the standard moduli stabilization by fluxes, all complex structure moduli and the dilaton are stabilized in a supersymmetric way while Kähler class moduli need an input from SUSY breaking. The radion is Kähler class and exists in a model independent fashion within the dark dimension scenario.

Besides solving the horizon problem, 4D slow-roll inflation predicts an approximate scale-invariant Harrison-Zel'dovich power spectrum of primordial density perturbations [158, 159] consistent with CMB observations [160]. This is due to the fact that the 2-point function of a massless minimally coupled scalar field in dS space behaves logarithmically at distances larger than the cosmological horizon, a property which is though valid for any spacetime dimensionality [161]. When some dimensions are however compact, this behaviour is expected to hold for distances smaller than the compactification length, while deviating from scale invariance at larger distances, potentially conflicting with observations at large angles. Remarkably, consistency of 5D inflation with CMB observations is maintained if the size of the dark dimension is larger than about a micron, implying a change of behaviour in the power spectrum at angles larger than 10 degrees, corresponding to multiple moments $l \lesssim 30$, where experimental errors are getting large [6]. Actually, the scale invariance of the power spectrum is obtained upon summation over the contribution of the inflaton KK-modes' fluctuations that correspond to a tower of scalars from the 4D point of view. Spectral indices dependence on slow-roll parameters and tensor perturbations have been computed in [162]. The tensor-to-scalar ratio is found to be $r = 24\epsilon_V$, and so the 95% CL upper limit $r < 0.032$ (derived using a combination of BICEP/Keck 2018 and *Planck* data) [164, 165] places an experimental constraint on the potential slow-roll parameter: $\epsilon_V < 0.0013$.

Another interesting feature of 5D inflation is that the radion can be stabilized in a local (metastable) dS vacuum [6], using the contributions of bulk field gradients [166] or of the Casimir energy, assuming a mass for the bulk R-handed neutrinos of the same order of magnitude [167]. Consider 5D Einstein-de Sitter gravity compactified on a circle S^1 endowed with $S^1/\mathbb{Z}_2$ symmetry, and assume that the SM is localized on a D-brane transverse to the compact dimension, whereas gravity spills into the compact space. The effective 4D potential of the radion field R is found to be

$$V(R) = \frac{2\pi\,\Lambda_5\, r^2}{R} + \left(\frac{r}{R}\right)^2 T_4 + V_C(R), \tag{43}$$

where Λ_5 is the 5D cosmological constant, $r \equiv \langle R\rangle$ is the vacuum expectation value of the radion, T_4 is the total 3-brane tension, and V_C stands for the quantum corrections to the vacuum energy due to Casimir forces. These corrections are expected to become important in the deep infrared region, because the Casimir contribution to the potential falls off exponentially at large R compared to the particle wavelength. Indeed, as R decreases different particle thresholds open up,

$$V_C(R) = \sum_i \frac{\pi r^2}{32\pi^7 R^6}\,(N_F - N_B)\,\Theta(R_i - R), \tag{44}$$

where $m_i = R_i^{-1}$ are the masses of the 5D fields, Θ is a step function, and $N_F - N_B$ stands for the difference between the number of light fermionic and bosonic degrees of freedom. At the classical level, i.e. considering only the first two terms in (43), it is straightforward to see that the potential develops a maximum at

$$R_{\rm max} = -T_4/(\pi\Lambda_5), \tag{45}$$

requiring a negative tension T_4. Note that if the fermionic degrees of freedom overwhelm the bosonic contribution, they would give rise to possible minima, as long as $R_i < R_{\rm max}$. This could be the case if $N_F = 12$ takes for the three 5D Dirac neutrino fields and $N_B = 5$ accounts for the 5D graviton. In Fig. 2 we show an illustrative example.

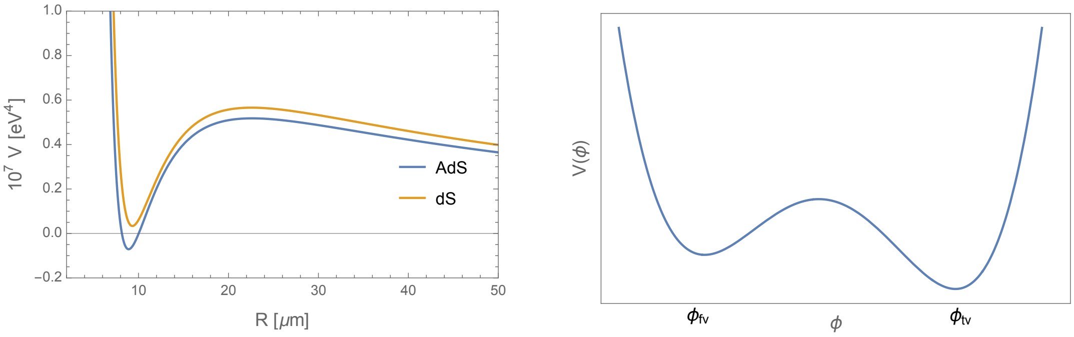

Figure 2: *Left.* The potential $V(R)$ for $(\Lambda_5)^{1/5} = 22.6$ meV and $|T_4|^{1/4} = 24.2$ meV, considering $N_F - N_B = 6$ (AdS) and $N_F - N_B = 7$ (dS). From Ref. [163]. *Right.* Schematic form of the real scalar potential $V(\phi)$.

7. Living on the Edge: Cosmology within sight of the space boundary

7.1 Cosmic Discrepancies

Over the last few years, low- and high-redshift observations set off tensions in the measurement of the present-day expansion rate H_0 and in the determination of the amplitude of the matter clustering in the late Universe (parameterized by S_8). More concretely, the values $H_0 = 67.4 \pm 0.5$ km/s/Mpc and $S_8 = 0.834 \pm 0.016$ inferred from *Planck*'s CMB data assuming ΛCDM [160] are in $\sim 5\sigma$ tension with $H_0 = 73.04 \pm 1.04$ km/s/Mpc from the SH0ES distance ladder measurement (using Cepheid-calibrated type-Ia supernovae) [2, 3] and in $\sim 3\sigma$ tension with $S_8 = 0.766^{+0.020}_{-0.014}$ from the cosmic shear data of the Kilo-Degree Survey (KiDS-1000) [168], respectively. These statistically significant discrepancies have become a new cornerstone of theoretical physics, and many beyond SM setups are rising to the challenge [169–172].

Λ_sCDM [173–176] is one of the many beyond SM physics models that have been proposed to simultaneously resolve the H_0 and S_8 tensions; see Appendix for details.[8] The model relies on an empirical conjecture which postulates that Λ may have switched sign (from negative to positive) at critical redshift $z_c \sim 2$;

$$\Lambda \quad \rightarrow \quad \Lambda_s \equiv \Lambda_0 \, \mathrm{sgn}[z_c - z], \tag{46}$$

with $\Lambda_0 > 0$, and where $\mathrm{sgn}[x] = -1, 0, 1$ for $x < 0$, $x = 0$ and $x > 0$, respectively. Apart from resolving the three major cosmological tensions, Λ_sCDM achieves quite a good fit to Lyman-α data provided $z_c \lesssim 2.3$ [173], and it is in agreement with the otherwise puzzling JWST observations [178, 179].

Despite the remarkable success of Λ_sCDM to accommodate the experimental data, the model is theoretically unsatisfactory because it postulates that the Universe experienced a rapid transition from an AdS vacuum to a dS vacuum, and this hods out against the AdS-DC conjecture, which posits that flat space limit is at infinite distance in the space of metric configurations and therefore these two vacua are an infinite distance appart in metric space [19]. However, it is important to stress that this no-go theorem is valid at zero temperature, where the number of light particles is (in

[8]An alternative model that accommodates the data has been presented in [177]. This model, however, calls for a violation of the weak energy condition.

general) constant. At finite temperature, particles can decay and hence the number of light particles can change. In this way the minima of the potential can be lifted [163].

7.2 AdS → dS transition driven by Casimir forces of bulk fields

A possible explanation for the required AdS → dS crossover transition in the vacuum energy can be obtained using the Casimir forces of fields inhabiting the dark dimension [163]. We assume that the 5D spectrum contains a light real scalar field ϕ, in addition to the graviton and the three neutrino generations. We further assume that the real scalar has a potential with two local minima with very small difference in vacuum energy and bigger curvature (mass) of the lower one, see Fig. 2. At z_c the false vacuum "tunnels" to its true vacuum state. After the quantum tunneling ϕ becomes more massive and its contribution to the Casimir energy becomes exponentially suppressed. The idea here is that for $z \gtrsim z_c$, we have $N_B = 6$, whereas for $z \lesssim z_c$, we have $N_B = 5$. Taking $N_F = 12$ to account for the three Dirac neutrino fields we can use (43) in combination with (44) to obtain an expression for the effective 4D radion potential. In Fig. 2 we show an illustrative example of the AdS → dS transition produced by $N_F - N_B = 6$ for $z \gtrsim z_c$, and $N_F - N_B = 5$ for $z \lesssim z_c$. It is important to note that the 5D vacuum transition creates a δV contribution to Λ_5, where $\delta V = V(\phi_{\rm fv}) - V(\phi_{\rm tv})$ corresponding to the vacuum energies of the upper (false vacuum) and the lower (true vacuum) minima. We have taken $\delta V \lll \Lambda_5$ so that it does not perturb the analysis producing the curves shown if Fig. 2.

Now, the AdS → dS transition shown in Fig. 2 slightly deviates from the model analyzed in [176], because the fields characterizing the deep infrared region of the dark sector contribute to the effective number of relativistic neutrino-like species $N_{\rm eff}$ [180]. Using conservation of entropy, fully thermalized relics with g_* degrees of freedom contribute

$$\Delta N_{\rm eff} = g_* \left(\frac{43}{4g_s}\right)^{4/3} \begin{cases} 4/7 & \text{for bosons} \\ 1/2 & \text{for fermions} \end{cases}, \tag{47}$$

where g_s denotes the effective degrees of freedom for the entropy of the other thermalized relativistic species that are present when they decouple [181]. The 5D graviton has 5 helicities, but the spin-1 helicities do not have zero modes, because we assume the compactification has $S^1/\mathbb{Z}_2$ symmetry and so the ± 1 helicities are projected out. The spin-0 is the radion and the spin-2 helicities form the massless (zero mode) graviton. This means that for the 5D graviton, $g_* = 3$. The scalar field ϕ contributes with $g_* = 1$. The (bulk) left-handed neutrinos are odd, but the right-handed neutrinos are even and so each counts as a Weyl neutrino, for a total $g_* = 2 \times 3$. Assuming that the dark sector decouples from the SM sector before the electroweak phase transition we have $g_s = 106.75$. This gives $\Delta N_{\rm eff} = 0.25$.[9] A numerical study shows that the addition of extra relativistic degrees of freedom does not spoil the resolution of the H_0 and S_8 tensions [186].

We end with an observation: the argument to understand the transition is essentially the same than the one in finite temperature models, because the number of light degrees of freedom changes due to a different transition of the 5D scalar field. In plain English, the model avoids

[9]It was recently noted that if the QCD axion is localized on the SM brane, a combination of theoretical and observational constraints forces it to have decay constant in a narrow range $10^9 \lesssim f/\text{GeV} \lesssim 10^{10}$ [182]. This corresponds to a mass for the QCD axion of $1 \lesssim m_a/\text{meV} \lesssim 10$. Although the axion would not affect the Casimir corrections to the potential, it would contributes to $\Delta N_{\rm eff}$ [183–185].

finite temperature requirements and relies on an ordinary vacuum decay in five dimensions. This obviously implies that the AdS vacuum is not a true vacuum. The vacuum in the radius modulus is determined by the contribution to the Casimir potential of the number of light degrees of freedom. This number changes discontinuously due to an ordinary vacuum decay of a 5D scalar field which satisfies the AdS-DC conjecture. This change drives the AdS to dS transition in the radius modulus, which is therefore discontinuous as in first order transitions.

8. The Black Hole Transition Conjecture

In Sec. 4.1, we have argued that if $r_s < R_\perp$, then black holes are 5D, where r_s is the Schwarzschild radius and $R_\perp$ the radius of the dark dimension. This implies that if the horizon size of a 4D black hole that is evaporating shrinks below the micron scale, then the black hole must undergo a 4D → 5D transition. As noted in [114], the black hole transition is instantly visible by analyzing the black hole entropy of a d-dimensional black hole,

$$S_{\rm BH} = \frac{4\pi \, M_{\rm BH} \, r_s}{d-2} \sim \left(\frac{M_{\rm BH}}{M_d}\right)^{(d-2)/(d-3)}, \tag{48}$$

where $M_{\rm BH}$ is the black hole mass and M_d is the d-dimensional Planck scale; note that for $d = 4$ we have $M_4 = M_p$. In Fig. 3 we show a comparison of the 4D and 5D scaling behavior of the black hole entropy as given by (48). By adding species in a higher dimensional theory, it follows from (48) that the scaling behavior of the entropy changes, and for the black hole it is more convenient to be in the 5D configuration because for given black hole mass its entropy is larger than the one in the 4D configuration. As can be seen in Fig. 3, for $R_\perp \sim 1\ \mu$m, the transition takes place at $M_{\rm BH} \sim 10^{21}$ g. Note that the 4D and 5D entropies as given by (48) are equal at the 5D-4D transition point where $M_{\rm BH} \sim M_p^2 R_\perp$. Moreover, the entropy crosses the horizontal axis where the black hole masses are the same as the 4D or 5D Planck masses. Then, the associated lengths are the 4D or 5D Planck lengths, where the two entropies are equal to one.

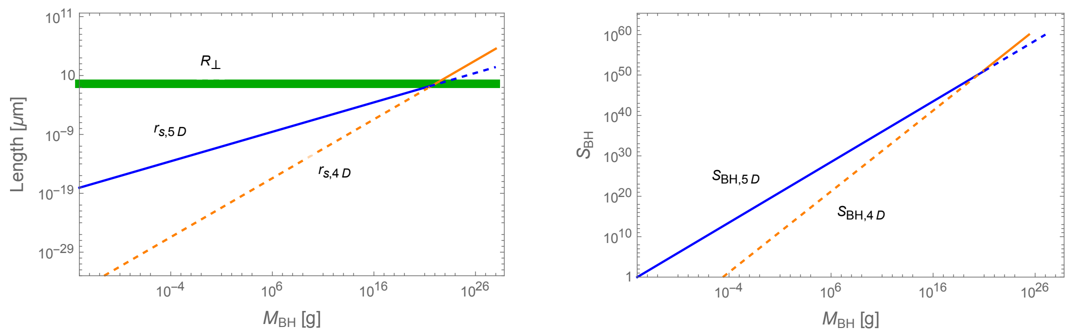

Figure 3: Scaling of the Schwarzschild radius (left) and black hole entropy (right) in $d = 4$ and $d = 5$ dimensions. From Ref. [114].

The transition between 4D and 5D black holes corresponds to Gregory-Laflamme phase transition [187] and is also visible using the free energy in terms of the temperature T_H [188]. This can be seen in the left plot in Fig. 3, where the black hole mass is approximately the same as the free energy, and the black hole radius corresponds to the inverse temperature. So the 5D configuration for given black hole temperature has smaller free energy than the 4D configuration.

More generally, this has lead in [188] to the **black hole transition conjecture** which states that any consistent EFT description of d-dimensional QG must exhibit three scales across its moduli space $\mathscr{M}_{\text{QG}}$: *(i)* the d-dimensional Planck scale M_d, which controls the strength of the Einstein term; *(ii)* the species scale Λ_{sp}, where the higher order gravitational corrections become important; and *(iii)* the black hole scale Λ_{BH}, where at this temperature, the black hole predicted by EFT undergoes a phase transition to a more stable solution. Furthermore, $\Lambda_{\text{BH}} \lesssim \Lambda_{\text{sp}} \lesssim M_d$ everywhere in $\mathscr{M}_{\text{QG}}$ and Λ_{BH} approaches the mass scale of the lightest tower at large distances in field space. So for a KK tower one asymptotically has that $\Lambda_{\text{BH}} = m_{\text{KK}}$. For the line interval with EW9-branes attached at each end [80], the three scales are $m_{\text{KK}} \ll M_{\text{KK}} \lesssim M_{11}$ [84].[10]

Let us also add a remark about the definition of the species scale by the minimal black hole in view of the phase transition from the d-dimensional, e.g. 4-dimensional, black hole to the higher-dimensional, e.g. 5-dimensional, black hole solution. Recall that the species length $l_{\text{sp}} = 1/\Lambda_{\text{sp}}$ corresponds to the minimal possible Schwarzschild radius of the BH solution in the EFT. First, from the higher dimensional perspective, the species length corresponds to the higher-dimensional Planck length, and this is the scale, where the entropy of the higher dimensional black hole becomes one. However, also the lower dimensional perspective makes sense for the definition of the species length, i.e. the species scale formula can be seen entirely from a d-dimensional perspective. Namely l_{sp} corresponds to the Schwarzschild radius, where the entropy of the corresponding minimal d-dimensional black hole is equal to N, the number of species. Using (10) one then obtains $l_{\text{sp}} = N^{\frac{1}{d-2}} M_d^{-1}$. For the species being the KK modes of an n-dimensional compact space one then obtains

$$l_{\text{sp}} = M_d^{\frac{2-d}{d+n-2}} m_{\text{KK}}^{-\frac{n}{d+n-2}} \,. \tag{49}$$

Replacing m_{KK} by Λ_{BH}, one obtains a definition of the species length entirely in terms of n and Λ_{BH}:

$$l_{\text{sp}} = M_d^{\frac{2-d}{d+n-2}} \Lambda_{\text{BH}}^{-\frac{n}{d+n-2}} \,. \tag{50}$$

It is not difficult to show that this d-dimensional definition of the species scale agrees with the higher dimensional one, using the specific form of the higher dimensional entropy formula and requiring the entropy in higher dimensions to be one.

Finally, we discuss the correspondence (conjectured in [141]) between the graviton-boundstate interpretation of 5D black holes and massive KK gravitons as dark matter candidates. We first consider a 4D black hole with entropy $S_{\text{BH,4d}} = N_{\text{tot}}$, with $N_{\text{tot}} = N_{g,4} N_{\text{sp}}$, where $N_{g,4}$ is the number of 4D gravitons in the black hole boundstate and N_{sp} is the number of additional species in the black hole boundstate [49]. These can be the KK gravitons, i.e. $N_{\text{sp}} = N_{\text{KK}}$. The radius of the 4D black hole is $r_{s,4} = N_{\text{tot}}^{1/2} l_p$, where $l_p = 1/M_p$. For the minimal black hole, whose radius sets the species scale, $N_{g,4} = 1$, leading to $S_{\text{BH},4d,\text{min}} = N_{\text{KK}}$. The corresponding minimal radius is $r_{s,4,\text{min}} = N_{\text{KK}}^{1/2} l_p$. This is indeed the species length, being just the 5D Planck length.

[10]Using universality of black hole thermodynamics and properties of gravitational scattering amplitudes it has been shown in [18] that some intrinsic features of the density of one-particle states above the minimum black hole mass are an indicator for the existence of large extra dimensions, and cannot be reproduced by any lower-dimensional field theory with finitely many fields satisfying the WEC.

Now, we consider a 5D black hole after the transition. There are no KK gravitons anymore, since they are part of normal 5D gravitions. The black hole is now made entirely of $N_{g,5}$ 5D gravitons and its entropy becomes $S_{\mathrm{BH},5d} = N_{g,5}$. The radius of the 5D black hole is $r_{s,5} = N_{g,5}^{1/3} l_5$, where $l_5 = 1/M_5$ is the 5D Planck length. For the minimal black hole, whose radius again sets the species scale, $N_{g,5} = 1$, leading to $S_{\mathrm{BH},5d,\mathrm{min}} = 1$. The corresponding minimal radius is $r_{s,5,\mathrm{min}} = l_5$. This is again the species length l_{sp}, so we get in a consistent way that $r_{s,5,\mathrm{min}} = r_{s,4,\mathrm{min}}$.

In summary, the 4D/5D phase transition for the KK gravitons can be understood assuming that above the KK scale these states are part of the 5D graviton and therefore the effective description of space time becomes 5D. So in this case we indeed would have a 5D (or d-dimensional) black hole graviton boundstate description, just like in the original black hole N-portrait picture of [140]. This implies that before the phase transition, the correspondence is that the 4D black hole is a bound state of the N_{KK} particles, whereas after the phase transition, there is the correspondence between the 5D black hole and the bound state of the 5D gravitons.

9. Concluding Thoughts

We have seen that the dark dimension scenario provides one possible explanation of the cosmological hierarchy problem and carries with it a rich phenomenology:

- It provides a profitable arena to accommodate a very light gravitino.
- It encompasses a framework for primordial black holes, KK gravitons, and a fuzzy radion to emerge as viable candidates to comprise some or all of the dark matter.
- It also encompasses an interesting framework for studying cosmology and astroparticle physics.
- It provides a natural set up for R-neutrinos propagating in the bulk to accommodate neutrino masses in the range $10^{-4} < m_\nu/\mathrm{eV} < 10^{-1}$, despite the lack of any fundamental scale higher than M_*. The suppressed neutrino masses are not the result of a see-saw mechanism, but rather because the bulk modes have couplings suppressed by the volume of the dark dimension (akin of the weakness of gravity at long distances).

We have also seen that uniform 5D inflation can relate the causal size of the observable universe to the present weakness of gravitational interactions by blowing up an extra compact dimension from the microscopic fundamental length of gravity to a large size in the micron range, as required by the dark dimension scenario. Moreover, uniform 5D inflation can lead to an approximate scale invariant power spectrum of primordial density perturbations. The predicted small-angle ($< 10°$) CMB power spectrum is compatible with observations. Such an angle corresponds to a distance ~ 2.3 Mpc and multipole moment $\ell \simeq 30$. For smaller ℓ multipoles (larger angles), one obtains more power spectrum than standard 4D inflation, corresponding to a nearly vanishing spectral index, that the present data cannot distinguish due to large errors. One caveat here is that the power spectrum is cosmic variance limited [189]. However, even though cosmic variance prevents identification at low ℓ, the transition region could provide a signal for experiments in the near future. To determine ℓ multipoles in the transition region, an exhaustive transfer-function analysis (numerically solving the linearized Einstein-Boltzmann equations) would be required. The tensor-to-scalar ratio is also consistent with observations. An estimate of the magnitude of isocurvature perturbations based on entropy perturbations indicates that they are suppressed [162]. A dedicated investigation along these lines is obviously important to be done.

On a separate track, it was noted in [190] that the cosmic scale factor a describing the evolution of Λ_sCDM is continuous and non-zero at $t = t_c$, but its first derivative $\dot{a}$ is discontinuous, and its second derivative $\ddot{a}$ diverges. In the spirit of [191], it would be interesting to investigate the evolution of (43) during the phase transition induced by the Casimir forces. This would allow a complete description of the background and perturbation evolution at all redshifts.

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Appendix

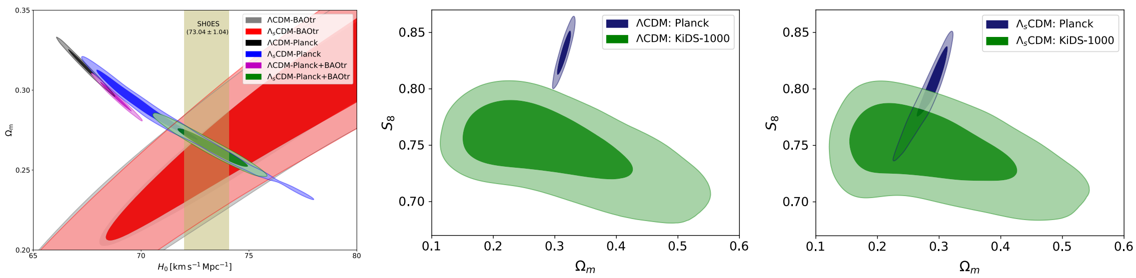

Figure 4: 2D contours at 68% and 95% CL in the H_0-Ω_m plane (left) for the ΛCDM and Λ_sCDM models, and in the Ω_m-S_8 plane for the ΛCDM (middle) and Λ_sCDM (right) models. In the left pannel the model predictions have been extracted fitting data from the *Planck* satellite and transversal baryon acoustic oscillations (BAOtr). In the middle and right panels the model predictions have its origins in *Planck* and KiDS-1000 data. Note that the Planck and BAOtr contours intersect exactly at the vertical band of the SH0ES measurement. From Ref. [176].

In this Appendix we briefly discuss the Λ_sCDM solution of the cosmic discrepancies, paying special attention to the role played by data from baryon acoustic oscillations (BAO). The most recent analysis presented in [176] is based on: the *Planck* CMB data [192], the Pantheon+ supernovae type Ia sample [193], the data release of KiDS-1000 [194], and the (angular) transversal 2D BAO data on the shell [195], which are less model dependent than the 3D BAO data used in previous studies of Λ_sCDM. The outcome, which is displayed in Fig. 4, shows that the Λ_sCDM model can simultaneously resolve both the H_0 and S_8 tensions. It is important to stress that the BAO 3D data sample assumes ΛCDM to determine the distance to the spherical shell, and hence could potentially introduce a bias when analyzing beyond ΛCDM models [196].

As shown in [197], 3D BAO data leave no room for low-z solutions to the H_0 tension if the absolute magnitude of supernova M is constant. This is generally phrased as a no-go theorem which states that if M is constant, to address the H_0 tension one needs to consider some sort of new physics at $z > 1000$. If one uses 2D BAO (instead of 3D BAO) data, though, it is possible to solve the Hubble tension without requiring new physics before recombination. However, in this case the effective dark energy density needs to be negative at $z \gtrsim 2$ in order to produce the correct angular diameter distance to the last scattering surface.

References

[1] R. L. Workman *et al.* [Particle Data Group], *Review of Particle Physics*, PTEP **2022**, 083C01 (2022) doi:10.1093/ptep/ptac097

[2] A. G. Riess, W. Yuan, L. M. Macri, D. Scolnic, D. Brout, S. Casertano, D. O. Jones, Y. Murakami, L. Breuval and T. G. Brink, *et al. A comprehensive measurement of the local value of the Hubble constant with 1 km s^{-1} Mpc^{-1} uncertainty from the Hubble Space Telescope and the SH0ES Team*, Astrophys. J. Lett. **934** (2022) no.1, L7 doi:10.3847/2041-8213/ac5c5b [arXiv:2112.04510 [astro-ph.CO]].

[3] Y. S. Murakami, A. G. Riess, B. E. Stahl, W. D. Kenworthy, D. M. A. Pluck, A. Macoretta, D. Brout, D. O. Jones, D. M. Scolnic and A. V. Filippenko, *Leveraging SN Ia spectroscopic similarity to improve the measurement of H_0*, JCAP **11**, 046 (2023) doi:10.1088/1475-7516/2023/11/046 [arXiv:2306.00070 [astro-ph.CO]].

[4] M. Montero, C. Vafa and I. Valenzuela, *The dark dimension and the Swampland*, JHEP **02**, 022 (2023) doi:10.1007/JHEP02(2023)022 [arXiv:2205.12293 [hep-th]].

[5] L. A. Anchordoqui, I. Antoniadis and D. Lüst, *Aspects of the dark dimension in cosmology*, Phys. Rev. D **107**, no.8, 083530 (2023) doi:10.1103/PhysRevD.107.083530 [arXiv:2212.08527 [hep-ph]].

[6] L. A. Anchordoqui and I. Antoniadis, *Large extra dimensions from higher-dimensional inflation*, Phys. Rev. D (in press) [arXiv:2310.20282 [hep-ph]].

[7] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *The Hierarchy problem and new dimensions at a millimeter*, Phys. Lett. B **429** (1998), 263-272 doi:10.1016/S0370-2693(98)00466-3 [arXiv:hep-ph/9803315 [hep-ph]].

[8] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, Phys. Lett. B **436** (1998), 257-263 doi:10.1016/S0370-2693(98)00860-0 [arXiv:hep-ph/9804398 [hep-ph]].

[9] C. Vafa, *The string landscape and the swampland*, [arXiv:hep-th/0509212 [hep-th]].

[10] A. Makridou, *Swampland program: cobordism, tadpoles, and the dark dimension*, PhD Thesis (2023) doi:10.5282/edoc.32315

[11] E. Palti, *The swampland: introduction and review*, Fortsch. Phys. **67**, no.6, 1900037 (2019) doi:10.1002/prop.201900037 [arXiv:1903.06239 [hep-th]].

[12] M. van Beest, J. Calderón-Infante, D. Mirfendereski and I. Valenzuela, *Lectures on the Swampland Program in string compactifications*, Phys. Rept. **989**, 1-50 (2022) doi:10.1016/j.physrep.2022.09.002 [arXiv:2102.01111 [hep-th]].

[13] M. Graña and A. Herráez, *The Swampland Conjectures: A bridge from quantum gravity to particle physics*, Universe **7**, no.8, 273 (2021) doi:10.3390/universe7080273 [arXiv:2107.00087 [hep-th]].

[14] N. B. Agmon, A. Bedroya, M. J. Kang and C. Vafa, *Lectures on the string landscape and the swampland*, [arXiv:2212.06187 [hep-th]].

[15] H. Ooguri and C. Vafa, *On the Geometry of the String Landscape and the Swampland*, Nucl. Phys. B **766**, 21-33 (2007) doi:10.1016/j.nuclphysb.2006.10.033 [arXiv:hep-th/0605264 [hep-th]].

[16] S. J. Lee, W. Lerche and T. Weigand, *Emergent strings from infinite distance limits*, JHEP **02**, 190 (2022) doi:10.1007/JHEP02(2022)190 [arXiv:1910.01135 [hep-th]].

[17] I. Basile, D. Lüst and C. Montella, *Shedding black hole light on the emergent string conjecture*, [arXiv:2311.12113 [hep-th]].

[18] A. Bedroya, R. K. Mishra and M. Wiesner, *Density of States, Black Holes and the Emergent String Conjecture*, [arXiv:2405.00083 [hep-th]].

[19] D. Lüst, E. Palti and C. Vafa, *AdS and the Swampland*, Phys. Lett. B **797**, 134867 (2019) doi:10.1016/j.physletb.2019.134867 [arXiv:1906.05225 [hep-th]].

[20] Y. Li, E. Palti and N. Petri, *Towards AdS distances in string theory*, JHEP **08** (2023), 210 doi:10.1007/JHEP08(2023)210 [arXiv:2306.02026 [hep-th]].

[21] E. Palti and N. Petri, *A positive metric over DGKT vacua*, [arXiv:2405.01084 [hep-th]].

[22] L. A. Anchordoqui, I. Antoniadis, D. Lüst and S. Lüst, *On the cosmological constant, the KK mass scale, and the cut-off dependence in the dark dimension scenario*, Eur. Phys. J. C **83**, no.11, 1016 (2023) doi:10.1140/epjc/s10052-023-12206-2 [arXiv:2309.09330 [hep-th]].

[23] E. Perlmutter, L. Rastelli, C. Vafa and I. Valenzuela, *"A CFT distance conjecture*, JHEP **10** (2021), 070 doi:10.1007/JHEP10(2021)070 [arXiv:2011.10040 [hep-th]].

[24] F. Baume and J. Calderón-Infante, *On higher-spin points and infinite distances in conformal manifolds*, JHEP **12** (2023), 163 doi:10.1007/JHEP12(2023)163 [arXiv:2305.05693 [hep-th]].

[25] H. Ooguri and Y. Wang, *Universal Bounds on CFT Distance Conjecture*, [arXiv:2405.00674 [hep-th]].

[26] S. Demulder, D. Lüst and T. Raml, *Topology change and non-geometry at infinite distance*, [arXiv:2312.07674 [hep-th]].

[27] Q. Bonnefoy, L. Ciambelli, D. Lüst and S. Lüst, *Infinite Black Hole Entropies at Infinite Distances and Tower of States*, Nucl. Phys. B **958** (2020), 115112 doi:10.1016/j.nuclphysb.2020.115112 [arXiv:1912.07453 [hep-th]].

[28] M. Lüben, D. Lüst and A. R. Metidieri, *The Black Hole Entropy Distance Conjecture and Black Hole Evaporation*, Fortsch. Phys. **69** (2021) no.3, 2000130 doi:10.1002/prop.202000130 [arXiv:2011.12331 [hep-th]].

[29] N. Cribiori, M. Dierigl, A. Gnecchi, D. Lüst and M. Scalisi, *Large and small non-extremal black holes, thermodynamic dualities, and the Swampland*, JHEP **10** (2022), 093 doi:10.1007/JHEP10(2022)093 [arXiv:2202.04657 [hep-th]].

[30] M. Delgado, M. Montero and C. Vafa, *Black holes as probes of moduli space geometry*, JHEP **04** (2023), 045 doi:10.1007/JHEP04(2023)045 [arXiv:2212.08676 [hep-th]].

[31] R. Angius, J. Huertas and A. M. Uranga, *Small black hole explosions*, JHEP **06** (2023), 070 doi:10.1007/JHEP06(2023)070 [arXiv:2303.15903 [hep-th]].

[32] G. Dvali, *Black holes and large N species solution to the hierarchy problem*, Fortsch. Phys. **58**, 528-536 (2010) doi:10.1002/prop.201000009 [arXiv:0706.2050 [hep-th]].

[33] G. Dvali and M. Redi, *Black hole bound on the number of species and quantum gravity at LHC*, Phys. Rev. D **77**, 045027 (2008) doi:10.1103/PhysRevD.77.045027 [arXiv:0710.4344 [hep-th]].

[34] G. Dvali and D. Lüst, *Evaporation of Microscopic Black Holes in String Theory and the Bound on Species*, Fortsch. Phys. **58** (2010), 505-527 doi:10.1002/prop.201000008 [arXiv:0912.3167 [hep-th]].

[35] G. Dvali and C. Gomez, *Species and Strings*, [arXiv:1004.3744 [hep-th]].

[36] G. Dvali, C. Gomez and D. Lüst, *Black Hole Quantum Mechanics in the Presence of Species*, Fortsch. Phys. **61** (2013), 768-778 doi:10.1002/prop.201300002 [arXiv:1206.2365 [hep-th]].

[37] D. van de Heisteeg, C. Vafa, M. Wiesner and D. H. Wu, *Moduli-dependent Species Scale*, [arXiv:2212.06841 [hep-th]].

[38] N. Cribiori, D. Lüst and G. Staudt, *Black hole entropy and moduli-dependent species scale*, Phys. Lett. B **844**, 138113 (2023) doi:10.1016/j.physletb.2023.138113 [arXiv:2212.10286 [hep-th]].

[39] A. Castellano, A. Herráez and L. E. Ibáñez, *The emergence proposal in quantum gravity and the species scale*, JHEP **06** (2023), 047 doi:10.1007/JHEP06(2023)047 [arXiv:2212.03908 [hep-th]].

[40] D. van de Heisteeg, C. Vafa and M. Wiesner, *Bounds on Species Scale and the Distance Conjecture*, Fortsch. Phys. **71** (2023) no.10-11, 2300143 doi:10.1002/prop.202300143 [arXiv:2303.13580 [hep-th]].

[41] N. Cribiori, D. Lüst and C. Montella, *Species entropy and thermodynamics*, JHEP **10** (2023), 059 doi:10.1007/JHEP10(2023)059 [arXiv:2305.10489 [hep-th]].

[42] R. Blumenhagen, A. Gligovic and A. Paraskevopoulou, *The emergence proposal and the emergent string*, JHEP **10** (2023), 145 doi:10.1007/JHEP10(2023)145 [arXiv:2305.10490 [hep-th]].

[43] N. Cribiori and D. Lüst, *A Note on Modular Invariant Species Scale and Potentials*, Fortsch. Phys. **71** (2023) no.10-11, 2300150 doi:10.1002/prop.202300150 [arXiv:2306.08673 [hep-th]].

[44] J. Calderón-Infante, M. Delgado and A. M. Uranga, *Emergence of species scale black hole horizons*, JHEP **01** (2024), 003 doi:10.1007/JHEP01(2024)003 [arXiv:2310.04488 [hep-th]].

[45] D. van de Heisteeg, C. Vafa, M. Wiesner and D. H. Wu, *Species scale in diverse dimensions*, [arXiv:2310.07213 [hep-th]].

[46] A. Castellano, A. Herráez and L. E. Ibáñez, *On the Species Scale, Modular Invariance and the Gravitational EFT expansion*, [arXiv:2310.07708 [hep-th]].

[47] A. Castellano, I. Ruiz and I. Valenzuela, *A Universal Pattern in Quantum Gravity at Infinite Distance*, [arXiv:2311.01501 [hep-th]].

[48] A. Castellano, I. Ruiz and I. Valenzuela, *Stringy Evidence for a Universal Pattern at Infinite Distance*, [arXiv:2311.01536 [hep-th]].

[49] I. Basile, N. Cribiori, D. Lüst and C. Montella, *Minimal black holes and species thermodynamics*, [arXiv:2401.06851 [hep-th]].

[50] A. Kehagias, D. Lüst and S. Lüst, *Swampland, Gradient Flow and Infinite Distance*, JHEP **04** (2020), 170 doi:10.1007/JHEP04(2020)170 [arXiv:1910.00453 [hep-th]].

[51] D. De Biasio and D. Lüst, *Geometric Flow Equations for Schwarzschild-AdS Space-Time and Hawking-Page Phase Transition*, Fortsch. Phys. **68** (2020) no.8, 2000053 doi:10.1002/prop.202000053 [arXiv:2006.03076 [hep-th]].

[52] D. De Biasio and D. Lüst, *Geometric flow of bubbles*, Nucl. Phys. B **980** (2022), 115812 doi:10.1016/j.nuclphysb.2022.115812 [arXiv:2201.01679 [hep-th]].

[53] D. M. Velázquez, D. De Biasio and D. Lüst, *Cobordism, singularities and the Ricci flow conjecture*, JHEP **01** (2023), 126 doi:10.1007/JHEP01(2023)126 [arXiv:2209.10297 [hep-th]].

[54] D. De Biasio, J. Freigang, D. Lüst and T. Wiseman, *Gradient flow of Einstein-Maxwell theory and Reissner-Nordström black holes*, JHEP **03** (2023), 074 doi:10.1007/JHEP03(2023)074 [arXiv:2210.14705 [hep-th]].

[55] D. De Biasio, *On-Shell Flow*, [arXiv:2211.04231 [hep-th]].

[56] D. De Biasio, *Geometric flows and the Swampland*, [arXiv:2307.08320 [hep-th]].

[57] Q. Bonnefoy, L. Ciambelli, D. Lüst and S. Lüst, *The swampland at large number of space-time dimensions*, JHEP **07** (2021), 009 doi:10.1007/JHEP07(2021)009 [arXiv:2011.06610 [hep-th]].

[58] D. De Biasio, J. Freigang and D. Lüst, *Geometric Flow Equations for the Number of Space-Time Dimensions*, Fortsch. Phys. **70** (2022) no.1, 2100171 doi:10.1002/prop.202100171 [arXiv:2104.05621 [hep-th]].

[59] I. Antoniadis, C. Bachas, D. C. Lewellen and T. N. Tomaras, *On supersymmetry breaking in superstrings*, Phys. Lett. B **207**, 441-446 (1988) doi:10.1016/0370-2693(88)90679-X

[60] I. Antoniadis, "A Possible new dimension at a few TeV," Phys. Lett. B **246** (1990), 377-384 doi:10.1016/0370-2693(90)90617-F

[61] N. Cribiori, D. Lüst and M. Scalisi, *The gravitino and the swampland*, JHEP **06**, 071 (2021) doi:10.1007/JHEP06(2021)071 [arXiv:2104.08288 [hep-th]].

[62] A. Castellano, A. Font, A. Herraez and L. E. Ibáñez, *A gravitino distance conjecture*, JHEP **08**, 092 (2021) doi:10.1007/JHEP08(2021)092 [arXiv:2104.10181 [hep-th]].

[63] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, *The String landscape, black holes and gravity as the weakest force*, JHEP **06**, 060 (2007) doi:10.1088/1126-6708/2007/06/060 [arXiv:hep-th/0601001 [hep-th]].

[64] D. Harlow, B. Heidenreich, M. Reece and T. Rudelius, *Weak gravity conjecture*, Rev. Mod. Phys. **95** (2023) no.3, 3 doi:10.1103/RevModPhys.95.035003 [arXiv:2201.08380 [hep-th]].

[65] R. Penrose, *Gravitational collapse: The role of general relativity*, Riv. Nuovo Cim. **1**, 252-276 (1969) doi:10.1023/A:1016578408204

[66] A. Higuchi, *Forbidden mass range for spin-2 field theory in de Sitter space-time*, Nucl. Phys. B **282** (1987), 397-436 doi:10.1016/0550-3213(87)90691-2

[67] I. Antoniadis, *Strings and extra dimensions: Mass hierarchies and experimental signatures*, EPJ Web Conf. **70**, 00004 (2014) doi:10.1051/epjconf/20147000004

[68] G. Dvali and C. Gomez, *Quantum information and gravity cutoff in theories with species*, Phys. Lett. B **674**, 303-307 (2009) doi:10.1016/j.physletb.2009.03.024 [arXiv:0812.1940 [hep-th]].

[69] S. Hannestad and G. G. Raffelt, *Supernova and neutron star limits on large extra dimensions reexamined*, Phys. Rev. D **67**, 125008 (2003) [erratum: Phys. Rev. D **69**, 029901 (2004)] doi:10.1103/PhysRevD.69.029901 [arXiv:hep-ph/0304029 [hep-ph]].

[70] J. G. Lee, E. G. Adelberger, T. S. Cook, S. M. Fleischer and B. R. Heckel, *New test of the gravitational $1/r^2$ law at separations down to 52 μm*, Phys. Rev. Lett. **124**, no.10, 101101 (2020) doi:10.1103/PhysRevLett.124.101101 [arXiv:2002.11761 [hep-ex]].

[71] L. A. Anchordoqui, *Dark dimension, the swampland, and the origin of cosmic rays beyond the Greisen-Zatsepin-Kuzmin barrier*, Phys. Rev. D **106** (2022) no.11, 116022 doi:10.1103/PhysRevD.106.116022 [arXiv:2205.13931 [hep-ph]].

[72] N. T. Noble, J. F. Soriano and L. A. Anchordoqui, *Probing the Dark Dimension with Auger data*, Phys. Dark Univ. **42** (2023), 101278 doi:10.1016/j.dark.2023.101278 [arXiv:2306.03666 [hep-ph]].

[73] H. Itoyama and T. R. Taylor, *Supersymmetry restoration in the compactified $O(16) \times O(16)$-prime heterotic string theory*, Phys. Lett. B **186** (1987), 129-133 doi:10.1016/0370-2693(87)90267-X

[74] H. Itoyama and T. R. Taylor, *Small cosmological constant in string models*, FERMILAB-CONF-87-129-T.

[75] I. Antoniadis and C. Kounnas, *Superstring phase transition at high temperature*, Phys. Lett. B **261** (1991), 369-378 doi:10.1016/0370-2693(91)90442-S

[76] Q. Bonnefoy, E. Dudas and S. Lüst, *On the weak gravity conjecture in string theory with broken supersymmetry*, Nucl. Phys. B **947** (2019), 114738 doi:10.1016/j.nuclphysb.2019.114738 [arXiv:1811.11199 [hep-th]].

[77] C. Branchina, V. Branchina, F. Contino and A. Pernace, *Does the cosmological constant really indicate the existence of a dark dimension?*, [arXiv:2308.16548 [hep-th]].

[78] C. Branchina, V. Branchina, F. Contino and A. Pernace, *Dark dimension and the effective field theory limit*, [arXiv:2404.10068 [hep-th]].

[79] L. A. Anchordoqui, I. Antoniadis, N. Cribiori, D. Lüst and M. Scalisi, *The Scale of Supersymmetry Breaking and the Dark Dimension*, JHEP **05**, 060 (2023) doi:10.1007/JHEP05(2023)060 [arXiv:2301.07719 [hep-th]].

[80] J. H. Schwarz, *Comments concerning a hypothetical mesoscopic dark dimension*, [arXiv:2403.12899 [hep-th]].

[81] E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, Nucl. Phys. B **471**, 135-158 (1996) doi:10.1016/0550-3213(96)00190-3 [arXiv:hep-th/9602070 [hep-th]].

[82] T. Banks and M. Dine, *Couplings and scales in strongly coupled heterotic string theory*, Nucl. Phys. B **479**, 173-196 (1996) doi:10.1016/0550-3213(96)00457-9 [arXiv:hep-th/9605136 [hep-th]].

[83] I. Antoniadis and C. Bachas, *Branes and the gauge hierarchy*, Phys. Lett. B **450**, 83-91 (1999) doi:10.1016/S0370-2693(99)00102-1 [arXiv:hep-th/9812093 [hep-th]].

[84] E. Caceres, V. S. Kaplunovsky and I. M. Mandelberg, *Large volume string compactifications, revisited*, Nucl. Phys. B **493**, 73-100 (1997) doi:10.1016/S0550-3213(97)00129-6 [arXiv:hep-th/9606036 [hep-th]].

[85] Y. B. Zel'dovich and I. D. Novikov, *The hypothesis of cores retarded during expansion and the hot cosmological model*, Soviet Astron. AJ (Engl. Transl.), **10**, 602 (1967)

[86] S. Hawking, *Gravitationally collapsed objects of very low mass*, Mon. Not. Roy. Astron. Soc. **152**, 75 (1971)

[87] B. J. Carr and S. W. Hawking, *Black holes in the early Universe*, Mon. Not. Roy. Astron. Soc. **168**, 399-415 (1974)

[88] B. J. Carr, *The primordial black hole mass spectrum*, Astrophys. J. **201**, 1-19 (1975) doi:10.1086/153853

[89] B. Carr and F. Kuhnel, *Primordial black holes as dark matter: Recent developments*, Ann. Rev. Nucl. Part. Sci. **70** (2020), 355-394 doi:10.1146/annurev-nucl-050520-125911 [arXiv:2006.02838 [astro-ph.CO]].

[90] A. M. Green and B. J. Kavanagh, *Primordial Black Holes as a dark matter candidate*, J. Phys. G **48** (2021) no.4, 043001 doi:10.1088/1361-6471/abc534 [arXiv:2007.10722 [astro-ph.CO]].

[91] P. Villanueva-Domingo, O. Mena and S. Palomares-Ruiz, *A brief review on primordial black holes as dark matter*, Front. Astron. Space Sci. **8**, 87 (2021) doi:10.3389/fspas.2021.681084 [arXiv:2103.12087 [astro-ph.CO]].

[92] E. Bagui *et al.* [LISA Cosmology Working Group], *Primordial black holes and their gravitational-wave signatures*, [arXiv:2310.19857 [astro-ph.CO]].

[93] B. J. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, *New cosmological constraints on primordial black holes*, Phys. Rev. D **81**, 104019 (2010) doi:10.1103/PhysRevD.81.104019 [arXiv:0912.5297 [astro-ph.CO]].

[94] S. Clark, B. Dutta, Y. Gao, L. E. Strigari and S. Watson, *Planck constraint on relic primordial black holes*, Phys. Rev. D **95**, no.8, 083006 (2017) doi:10.1103/PhysRevD.95.083006 [arXiv:1612.07738 [astro-ph.CO]].

[95] W. DeRocco and P. W. Graham, *Constraining primordial black hole abundance with the Galactic 511 keV line*, Phys. Rev. Lett. **123** (2019) no.25, 251102 doi:10.1103/PhysRevLett.123.251102 [arXiv:1906.07740 [astro-ph.CO]].

[96] R. Laha, *Primordial black holes as a dark matter candidate are severely constrained by the Galactic Center 511 keV γ-ray line*, Phys. Rev. Lett. **123** (2019) no.25, 251101 doi:10.1103/PhysRevLett.123.251101 [arXiv:1906.09994 [astro-ph.HE]].

[97] B. Dasgupta, R. Laha and A. Ray, *Neutrino and positron constraints on spinning primordial black hole dark matter*, Phys. Rev. Lett. **125** (2020) no.10, 101101 doi:10.1103/PhysRevLett.125.101101 [arXiv:1912.01014 [hep-ph]].

[98] C. Keith and D. Hooper, *511 keV excess and primordial black holes*, Phys. Rev. D **104** (2021) no.6, 063033 doi:10.1103/PhysRevD.104.063033 [arXiv:2103.08611 [astro-ph.CO]].

[99] S. Mittal, A. Ray, G. Kulkarni and B. Dasgupta, *Constraining primordial black holes as dark matter using the global 21-cm signal with X-ray heating and excess radio background*, JCAP **03** (2022), 030 doi:10.1088/1475-7516/2022/03/030 [arXiv:2107.02190 [astro-ph.CO]].

[100] R. Laha, J. B. Muñoz and T. R. Slatyer, *INTEGRAL constraints on primordial black holes and particle dark matter*, Phys. Rev. D **101** (2020) no.12, 123514 doi:10.1103/PhysRevD.101.123514 [arXiv:2004.00627 [astro-ph.CO]].

[101] J. Berteaud, F. Calore, J. Iguaz, P. D. Serpico and T. Siegert, *Strong constraints on primordial black hole dark matter from 16 years of INTEGRAL/SPI observations*, Phys. Rev. D **106** (2022) no.2, 023030 doi:10.1103/PhysRevD.106.023030 [arXiv:2202.07483 [astro-ph.HE]].

[102] M. Korwar and S. Profumo, *Updated constraints on primordial black hole evaporation*, JCAP **05** (2023), 054 doi:10.1088/1475-7516/2023/05/054 [arXiv:2302.04408 [hep-ph]].

[103] R. A. Allsman *et al.* [MACHO], *MACHO project limits on black hole dark matter in the 1-30 solar mass range*, Astrophys. J. Lett. **550**, L169 (2001) doi:10.1086/319636 [arXiv:astro-ph/0011506 [astro-ph]].

[104] P. Tisserand *et al.* [EROS-2], *Limits on the MACHO content of the Galactic halo from the EROS-2 survey of the Magellanic clouds*, Astron. Astrophys. **469**, 387-404 (2007) doi:10.1051/0004-6361:20066017 [arXiv:astro-ph/0607207 [astro-ph]].

[105] K. Griest, A. M. Cieplak and M. J. Lehner, *Experimental limits on primordial black hole dark matter from the first 2 yr of Kepler data*, Astrophys. J. **786**, no.2, 158 (2014) doi:10.1088/0004-637X/786/2/158 [arXiv:1307.5798 [astro-ph.CO]].

[106] M. Oguri, J. M. Diego, N. Kaiser, P. L. Kelly and T. Broadhurst, *Understanding caustic crossings in giant arcs: characteristic scales, event rates, and constraints on compact dark matter*, Phys. Rev. D **97**, no.2, 023518 (2018) doi:10.1103/PhysRevD.97.023518 [arXiv:1710.00148 [astro-ph.CO]].

[107] H. Niikura, M. Takada, S. Yokoyama, T. Sumi and S. Masaki, *Constraints on Earth-mass primordial black holes from OGLE 5-year microlensing events*, Phys. Rev. D **99**, no.8, 083503 (2019) doi:10.1103/PhysRevD.99.083503 [arXiv:1901.07120 [astro-ph.CO]].

[108] D. Croon, D. McKeen, N. Raj and Z. Wang, *Subaru-HSC through a different lens: Microlensing by extended dark matter structures*, Phys. Rev. D **102**, no.8, 083021 (2020) doi:10.1103/PhysRevD.102.083021 [arXiv:2007.12697 [astro-ph.CO]].

[109] L. A. Anchordoqui, I. Antoniadis and D. Lüst, *Dark dimension, the swampland, and the dark matter fraction composed of primordial black holes*, Phys. Rev. D **106**, no.8, 086001 (2022) doi:10.1103/PhysRevD.106.086001 [arXiv:2206.07071 [hep-th]].

[110] S. B. Giddings and S. D. Thomas, *High-energy colliders as black hole factories: The End of short distance physics*, Phys. Rev. D **65**, 056010 (2002) doi:10.1103/PhysRevD.65.056010 [arXiv:hep-ph/0106219 [hep-ph]].

[111] F. R. Tangherlini, *Schwarzschild field in n dimensions and the dimensionality of space problem*, Nuovo Cim. **27** (1963), 636-651 doi:10.1007/BF02784569

[112] R. C. Myers and M. J. Perry, *Black holes in higher dimensional space-times*, Annals Phys. **172**, 304 (1986) doi:10.1016/0003-4916(86)90186-7

[113] P. C. Argyres, S. Dimopoulos and J. March-Russell, *Black holes and submillimeter dimensions*, Phys. Lett. B **441** (1998), 96-104 doi:10.1016/S0370-2693(98)01184-8 [arXiv:hep-th/9808138 [hep-th]].

[114] L. A. Anchordoqui, I. Antoniadis and D. Lüst, *More on black holes perceiving the dark dimension*, [arXiv:2403.19604 [hep-th]].

[115] L. A. Anchordoqui, J. L. Feng, H. Goldberg and A. D. Shapere, *Black holes from cosmic rays: Probes of extra dimensions and new limits on TeV scale gravity*, Phys. Rev. D **65** (2002), 124027 doi:10.1103/PhysRevD.65.124027 [arXiv:hep-ph/0112247 [hep-ph]].

[116] S. W. Hawking, *Black hole explosions*, Nature **248** (1974), 30-31 doi:10.1038/248030a0

[117] S. W. Hawking, *Particle creation by black holes*, Commun. Math. Phys. **43** (1975), 199-220 [erratum: Commun. Math. Phys. **46** (1976), 206] doi:10.1007/BF02345020

[118] L. Parker, *Probability distribution of particles created by a black hole*, Phys. Rev. D **12** (1975), 1519-1525 doi:10.1103/PhysRevD.12.1519

[119] R. M. Wald, *On particle creation by black holes*, Commun. Math. Phys. **45** (1975), 9-34 doi:10.1007/BF01609863

[120] S. W. Hawking, *Breakdown of predictability in gravitational collapse*, Phys. Rev. D **14** (1976), 2460-2473 doi:10.1103/PhysRevD.14.2460

[121] D. N. Page, *Particle emission rates from a black hole: Massless particles from an uncharged, nonrotating hole*, Phys. Rev. D **13** (1976), 198-206 doi:10.1103/PhysRevD.13.198

[122] D. N. Page, *Particle emission rates from a black hole II: Massless particles from a rotating hole*, Phys. Rev. D **14** (1976), 3260-3273 doi:10.1103/PhysRevD.14.3260

[123] D. N. Page, *Particle emission rates from a black hole III: Charged leptons from a nonrotating hole*, Phys. Rev. D **16** (1977), 2402-2411 doi:10.1103/PhysRevD.16.2402

[124] P. Kanti and J. March-Russell, *Calculable corrections to brane black hole decay I: The scalar case*, Phys. Rev. D **66** (2002), 024023 doi:10.1103/PhysRevD.66.024023 [arXiv:hep-ph/0203223 [hep-ph]].

[125] A. Ireland, S. Profumo and J. Scharnhorst, *Gravitational waves from primordial black hole evaporation with large extra dimensions*, [arXiv:2312.08508 [gr-qc]].

[126] R. Emparan, G. T. Horowitz and R. C. Myers, *Black holes radiate mainly on the brane*, Phys. Rev. Lett. **85** (2000), 499-502 doi:10.1103/PhysRevLett.85.499 [arXiv:hep-th/0003118 [hep-th]].

[127] S. Dimopoulos and G. L. Landsberg, *Black holes at the LHC*, Phys. Rev. Lett. **87** (2001), 161602 doi:10.1103/PhysRevLett.87.161602 [arXiv:hep-ph/0106295 [hep-ph]].

[128] L. Anchordoqui and H. Goldberg, *Black hole chromosphere at the CERN LHC*, Phys. Rev. D **67** (2003), 064010 doi:10.1103/PhysRevD.67.064010 [arXiv:hep-ph/0209337 [hep-ph]].

[129] J. M. Maldacena and L. Susskind, *D-branes and fat black holes*, Nucl. Phys. B **475**, 679-690 (1996) doi:10.1016/0550-3213(96)00323-9 [arXiv:hep-th/9604042 [hep-th]].

[130] L. A. Anchordoqui, I. Antoniadis and D. Lüst, *The dark dimension, the swampland, and the dark matter fraction composed of primordial near-extremal black holes*, Phys. Rev. D (in press) [arXiv:2401.09087 [hep-th]].

[131] T. Han, G. D. Kribs and B. McElrath, *Black hole evaporation with separated fermions*, Phys. Rev. Lett. **90** (2003), 031601 doi:10.1103/PhysRevLett.90.031601 [arXiv:hep-ph/0207003 [hep-ph]].

[132] E. Gonzalo, M. Montero, G. Obied and C. Vafa, *Dark Dimension Gravitons as Dark Matter*, JHEP **11**, 109 (2023) doi:10.1007/JHEP11(2023)109 [arXiv:2209.09249 [hep-ph]].

[133] K. R. Dienes and B. Thomas, *Dynamical dark matter I: Theoretical overview*, Phys. Rev. D **85**, 083523 (2012) doi:10.1103/PhysRevD.85.083523 [arXiv:1106.4546 [hep-ph]].

[134] L. J. Hall and D. Tucker-Smith, *Cosmological constraints on theories with large extra dimensions*, Phys. Rev. D **60**, 085008 (1999) doi:10.1103/PhysRevD.60.085008 [arXiv:hep-ph/9904267 [hep-ph]].

[135] T. R. Slatyer and C. L. Wu, *General Constraints on Dark Matter Decay from the Cosmic Microwave Background*, Phys. Rev. D **95**, no.2, 023010 (2017) doi:10.1103/PhysRevD.95.023010 [arXiv:1610.06933 [astro-ph.CO]].

[136] J. A. P. Law-Smith, G. Obied, A. Prabhu and C. Vafa, *Astrophysical Constraints on Decaying Dark Gravitons*, [arXiv:2307.11048 [hep-ph]].

[137] G. Obied, C. Dvorkin, E. Gonzalo and C. Vafa, *Dark Dimension and Decaying Dark Matter Gravitons*, [arXiv:2311.05318 [astro-ph.CO]].

[138] C. Vafa, *Swamplandish unification of the dark sector*, [arXiv:2402.00981 [hep-ph]].

[139] K. N. Abazajian, *Sterile neutrinos in cosmology*, Phys. Rept. **711-712**, 1-28 (2017) doi:10.1016/j.physrep.2017.10.003 [arXiv:1705.01837 [hep-ph]].

[140] G. Dvali and C. Gomez, *Black hole's quantum N-portrait*, Fortsch. Phys. **61**, 742-767 (2013) doi:10.1002/prop.201300001 [arXiv:1112.3359 [hep-th]].

[141] L. A. Anchordoqui, I. Antoniadis and D. Lüst, *The dark universe: Primordial black hole ⇋ dark graviton gas connection*, Phys. Lett. B **840**, 137844 (2023) doi:10.1016/j.physletb.2023.137844 [arXiv:2210.02475 [hep-th]].

[142] L. A. Anchordoqui, I. Antoniadis and D. Lüst, *Fuzzy Dark Matter and the dark dimension*, Eur. Phys. J. C **84**, no.3, 273 (2024) doi:10.1140/epjc/s10052-024-12622-y [arXiv:2307.01100 [hep-ph]].

[143] R. N. Mohapatra, S. Nussinov and A. Perez-Lorenzana, *Large extra dimensions and decaying KK recurrences*, Phys. Rev. D **68**, 116001 (2003) doi:10.1103/PhysRevD.68.116001 [arXiv:hep-ph/0308051 [hep-ph]].

[144] K. R. Dienes, E. Dudas and T. Gherghetta, *Neutrino oscillations without neutrino masses or heavy mass scales: A Higher dimensional seesaw mechanism*, Nucl. Phys. B **557**, 25 (1999) doi:10.1016/S0550-3213(99)00377-6 [arXiv:hep-ph/9811428 [hep-ph]].

[145] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali and J. March-Russell, *Neutrino masses from large extra dimensions*, Phys. Rev. D **65**, 024032 (2001) doi:10.1103/PhysRevD.65.024032 [arXiv:hep-ph/9811448 [hep-ph]].

[146] G. R. Dvali and A. Y. Smirnov, *Probing large extra dimensions with neutrinos*, Nucl. Phys. B **563**, 63-81 (1999) doi:10.1016/S0550-3213(99)00574-X [arXiv:hep-ph/9904211 [hep-ph]].

[147] P. A. N. Machado, H. Nunokawa and R. Zukanovich Funchal, *Testing for Large Extra Dimensions with Neutrino Oscillations*, Phys. Rev. D **84**, 013003 (2011) doi:10.1103/PhysRevD.84.013003 [arXiv:1101.0003 [hep-ph]].

[148] D. V. Forero, C. Giunti, C. A. Ternes and O. Tyagi, *Large extra dimensions and neutrino experiments*, Phys. Rev. D **106**, no.3, 035027 (2022) doi:10.1103/PhysRevD.106.035027 [arXiv:2207.02790 [hep-ph]].

[149] A. Lukas, P. Ramond, A. Romanino and G. G. Ross, *Solar neutrino oscillation from large extra dimensions*, Phys. Lett. B **495**, 136-146 (2000) doi:10.1016/S0370-2693(00)01206-5 [arXiv:hep-ph/0008049 [hep-ph]].

[150] A. Lukas, P. Ramond, A. Romanino and G. G. Ross, *Neutrino masses and mixing in brane world theories*, JHEP **04**, 010 (2001) doi:10.1088/1126-6708/2001/04/010 [arXiv:hep-ph/0011295 [hep-ph]].

[151] M. Carena, Y. Y. Li, C. S. Machado, P. A. N. Machado and C. E. M. Wagner, *Neutrinos in Large Extra Dimensions and Short-Baseline ν_e Appearance*, Phys. Rev. D **96** (2017) no.9, 095014 doi:10.1103/PhysRevD.96.095014 [arXiv:1708.09548 [hep-ph]].

[152] L. A. Anchordoqui, I. Antoniadis and J. Cunat, *The dark dimension and the standard model landscape*, Phys. Rev. D **109**, no.1, 016028 (2024) doi:10.1103/PhysRevD.109.016028 [arXiv:2306.16491 [hep-ph]].

[153] L. A. Anchordoqui, I. Antoniadis, K. Benakli, J. Cunat and D. Lüst, *Searching for neutrino-modulino oscillations at the Forward Physics Facility*, Phys. Lett. B **850**, 138530 (2024) doi:10.1016/j.physletb.2024.138530 [arXiv:2308.11476 [hep-ph]].

[154] K. Benakli and A. Y. Smirnov, *Neutrino-modulino mixing*, Phys. Rev. Lett. **79**, 4314-4317 (1997) doi:10.1103/PhysRevLett.79.4314 [arXiv:hep-ph/9703465 [hep-ph]].

[155] L. A. Anchordoqui, A. Ariga, T. Ariga, W. Bai, K. Balazs, B. Batell, J. Boyd, J. Bramante, M. Campanelli and A. Carmona, *et al. The Forward Physics Facility: Sites, experiments, and physics potential*," Phys. Rept. **968**, 1-50 (2022) doi:10.1016/j.physrep.2022.04.004 [arXiv:2109.10905 [hep-ph]].

[156] J. L. Feng, F. Kling, M. H. Reno, J. Rojo, D. Soldin, L. A. Anchordoqui, J. Boyd, A. Ismail, L. Harland-Lang and K. J. Kelly, *et al. The Forward Physics Facility at the High-Luminosity LHC*, J. Phys. G **50**, no.3, 030501 (2023) doi:10.1088/1361-6471/ac865e [arXiv:2203.05090 [hep-ex]].

[157] H. Ooguri and C. Vafa, *Non-supersymmetric AdS and the Swampland*, Adv. Theor. Math. Phys. **21** (2017), 1787-1801 doi:10.4310/ATMP.2017.v21.n7.a8 [arXiv:1610.01533 [hep-th]].

[158] E. R. Harrison, *Fluctuations at the threshold of classical cosmology*, Phys. Rev. D **1**, 2726-2730 (1970) doi:10.1103/PhysRevD.1.2726

[159] Y. B. Zeldovich, *A Hypothesis, unifying the structure and the entropy of the universe*, Mon. Not. Roy. Astron. Soc. **160**, 1P-3P (1972) doi:10.1093/mnras/160.1.1P

[160] N. Aghanim *et al.* [Planck], *Planck 2018 results VI: Cosmological parameters*, Astron. Astrophys. **641**, A6 (2020) [erratum: Astron. Astrophys. **652**, C4 (2021)] doi:10.1051/0004-6361/201833910 [arXiv:1807.06209 [astro-ph.CO]].

[161] B. Ratra, *Restoration of spontaneously broken continuous symmetries in de Sitter space-time*, Phys. Rev. D **31** (1985), 1931-1955 doi:10.1103/PhysRevD.31.1931

[162] I. Antoniadis, J. Cunat and A. Guillen, *Cosmological perturbations from five-dimensional inflation*, [arXiv:2311.17680 [hep-ph]].

[163] L. A. Anchordoqui, I. Antoniadis and D. Lüst, *Anti-de Sitter → de Sitter transition driven by Casimir forces and mitigating tensions in cosmological parameters*, [arXiv:2312.12352 [hep-th]].

[164] P. A. R. Ade *et al.* [BICEP and Keck], *Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season*, Phys. Rev. Lett. **127** (2021) no.15, 151301 doi:10.1103/PhysRevLett.127.151301 [arXiv:2110.00483 [astro-ph.CO]].

[165] M. Tristram, A. J. Banday, K. M. Górski, R. Keskitalo, C. R. Lawrence, K. J. Andersen, R. B. Barreiro, J. Borrill, L. P. L. Colombo and H. K. Eriksen, *et al. Improved limits on the tensor-to-scalar ratio using BICEP and Planck data*, Phys. Rev. D **105** (2022) no.8, 083524 doi:10.1103/PhysRevD.105.083524 [arXiv:2112.07961 [astro-ph.CO]].

[166] N. Arkani-Hamed, L. J. Hall, D. Tucker-Smith and N. Weiner, *Solving the hierarchy problem with exponentially large dimensions*, Phys. Rev. D **62** (2000), 105002 doi:10.1103/PhysRevD.62.105002 [arXiv:hep-ph/9912453 [hep-ph]].

[167] N. Arkani-Hamed, S. Dubovsky, A. Nicolis and G. Villadoro, *Quantum horizons of the standard model landscape*, JHEP **06** (2007), 078 doi:10.1088/1126-6708/2007/06/078 [arXiv:hep-th/0703067 [hep-th]].

[168] C. Heymans, T. Tröster, M. Asgari, C. Blake, H. Hildebrandt, B. Joachimi, K. Kuijken, C. A. Lin, A. G. Sánchez and J. L. van den Busch, *et al. KiDS-1000 Cosmology: Multi-probe weak gravitational lensing and spectroscopic galaxy clustering constraints*, Astron. Astrophys. **646**, A140 (2021) doi:10.1051/0004-6361/202039063 [arXiv:2007.15632 [astro-ph.CO]].

[169] E. Di Valentino, O. Mena, S. Pan, L. Visinelli, W. Yang, A. Melchiorri, D. F. Mota, A. G. Riess and J. Silk, *In the realm of the Hubble tension—a review of solutions*, Class. Quant. Grav. **38**, no.15, 153001 (2021) doi:10.1088/1361-6382/ac086d [arXiv:2103.01183 [astro-ph.CO]].

[170] N. Schöneberg, G. Franco Abellán, A. Pérez Sánchez, S. J. Witte, V. Poulin and J. Lesgourgues, *The H0 Olympics: A fair ranking of proposed models*, Phys. Rept. **984**, 1-55 (2022) doi:10.1016/j.physrep.2022.07.001 [arXiv:2107.10291 [astro-ph.CO]].

[171] L. Perivolaropoulos and F. Skara, *Challenges for ΛCDM: An update*, New Astron. Rev. **95**, 101659 (2022) doi:10.1016/j.newar.2022.101659 [arXiv:2105.05208 [astro-ph.CO]].

[172] E. Abdalla, G. Franco Abellán, A. Aboubrahim, A. Agnello, O. Akarsu, Y. Akrami, G. Alestas, D. Aloni, L. Amendola and L. A. Anchordoqui, *et al.*, *Cosmology intertwined: A review of the particle physics, astrophysics, and cosmology associated with the cosmological tensions and anomalies*, JHEAp **34**, 49-211 (2022) doi:10.1016/j.jheap.2022.04.002 [arXiv:2203.06142 [astro-ph.CO]].

[173] Ö. Akarsu, J. D. Barrow, L. A. Escamilla and J. A. Vazquez, *Graduated dark energy: Observational hints of a spontaneous sign switch in the cosmological constant*, Phys. Rev. D **101** (2020) no.6, 063528 doi:10.1103/PhysRevD.101.063528 [arXiv:1912.08751 [astro-ph.CO]].

[174] Ö. Akarsu, S. Kumar, E. Özülker and J. A. Vazquez, *Relaxing cosmological tensions with a sign switching cosmological constant*, Phys. Rev. D **104** (2021) no.12, 123512 doi:10.1103/PhysRevD.104.123512 [arXiv:2108.09239 [astro-ph.CO]].

[175] O. Akarsu, S. Kumar, E. Özülker, J. A. Vazquez and A. Yadav, *Relaxing cosmological tensions with a sign switching cosmological constant: Improved results with Planck, BAO, and Pantheon data*, Phys. Rev. D **108** (2023) no.2, 023513 doi:10.1103/PhysRevD.108.023513 [arXiv:2211.05742 [astro-ph.CO]].

[176] O. Akarsu, E. Di Valentino, S. Kumar, R. C. Nunes, J. A. Vazquez and A. Yadav, Λ_s*CDM model: A promising scenario for alleviation of cosmological tensions*, [arXiv:2307.10899 [astro-ph.CO]].

[177] A. Gomez-Valent and J. Sola Peracaula, *Phantom matter: a challenging solution to the cosmological tensions*, [arXiv:2404.18845 [astro-ph.CO]].

[178] S. A. Adil, U. Mukhopadhyay, A. A. Sen and S. Vagnozzi, *Dark energy in light of the early JWST observations: case for a negative cosmological constant?*, JCAP **10** (2023), 072 doi:10.1088/1475-7516/2023/10/072 [arXiv:2307.12763 [astro-ph.CO]].

[179] N. Menci, S. A. Adil, U. Mukhopadhyay, A. A. Sen and S. Vagnozzi, *Negative cosmological constant in the dark energy sector: tests from JWST photometric and spectroscopic observations of high-redshift galaxies*, [arXiv:2401.12659 [astro-ph.CO]].

[180] G. Steigman, D. N. Schramm and J. E. Gunn, *Cosmological limits to the number of massive leptons*, Phys. Lett. B **66** (1977), 202-204 doi:10.1016/0370-2693(77)90176-9

[181] L. A. Anchordoqui, I. Antoniadis, D. Lüst, J. F. Soriano and T. R. Taylor, *H_0 tension and the String Swampland*, Phys. Rev. D **101** (2020), 083532 doi:10.1103/PhysRevD.101.083532 [arXiv:1912.00242 [hep-th]].

[182] N. Gendler and C. Vafa, *Axions in the dark dimension*, [arXiv:2404.15414 [hep-th]].

[183] D. Baumann, D. Green and B. Wallisch, *New target for cosmic axion searches*, Phys. Rev. Lett. **117**, no.17, 171301 (2016) doi:10.1103/PhysRevLett.117.171301 [arXiv:1604.08614 [astro-ph.CO]].

[184] D. Green, Y. Guo and B. Wallisch, *Cosmological implications of axion-matter couplings*, JCAP **02**, no.02, 019 (2022) doi:10.1088/1475-7516/2022/02/019 [arXiv:2109.12088 [astro-ph.CO]].

[185] F. D'Eramo, E. Di Valentino, W. Giarè, F. Hajkarim, A. Melchiorri, O. Mena, F. Renzi and S. Yun, *Cosmological bound on the QCD axion mass, redux*, JCAP **09**, 022 (2022) doi:10.1088/1475-7516/2022/09/022 [arXiv:2205.07849 [astro-ph.CO]].

[186] L. A. Anchordoqui, I. Antoniadis, D. Lüst, N. T. Noble and J. F. Soriano, *From infinite to infinitesimal: Using the Universe as a dataset to probe Casimir corrections to the vacuum energy from fields inhabiting the dark dimension*, [arXiv:2404.17334 [astro-ph.CO]].

[187] R. Gregory and R. Laflamme, *Black strings and p-branes are unstable*, Phys. Rev. Lett. **70** (1993), 2837-2840 doi:10.1103/PhysRevLett.70.2837 [arXiv:hep-th/9301052 [hep-th]].

[188] A. Bedroya, C. Vafa and D. H. Wu, *The tale of three scales: the Planck, the species, and the black hole scales*, [arXiv:2403.18005 [hep-th]].

[189] D. Larson, J. Dunkley, G. Hinshaw, E. Komatsu, M. R. Nolta, C. L. Bennett, B. Gold, M. Halpern, R. S. Hill and N. Jarosik, *et al. Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Power spectra and WMAP-derived parameters*, Astrophys. J. Suppl. **192** (2011), 16 doi:10.1088/0067-0049/192/2/16 [arXiv:1001.4635 [astro-ph.CO]].

[190] E. A. Paraskevas, A. Cam, L. Perivolaropoulos and O. Akarsu, *Transition dynamics in the Λ_sCDM model: Implications for bound cosmic structures*, [arXiv:2402.05908 [astro-ph.CO]].

[191] O. Akarsu, A. De Felice, E. Di Valentino, S. Kumar, R. C. Nunes, E. Ozulker, J. A. Vazquez and A. Yadav, Λ_s*CDM cosmology from a type-II minimally modified gravity*, [arXiv:2402.07716 [astro-ph.CO]].

[192] N. Aghanim *et al.* [Planck], *Planck 2018 results V: CMB power spectra and likelihoods*, Astron. Astrophys. **641** (2020), A5 doi:10.1051/0004-6361/201936386 [arXiv:1907.12875 [astro-ph.CO]].

[193] D. Brout, D. Scolnic, B. Popovic, A. G. Riess, J. Zuntz, R. Kessler, A. Carr, T. M. Davis, S. Hinton and D. Jones, *et al. The Pantheon+ analysis: Cosmological constraints*, Astrophys. J. **938** (2022) no.2, 110 doi:10.3847/1538-4357/ac8e04 [arXiv:2202.04077 [astro-ph.CO]].

[194] M. Asgari *et al.* [KiDS], *KiDS-1000 Cosmology: Cosmic shear constraints and comparison between two point statistics*, Astron. Astrophys. **645** (2021), A104 doi:10.1051/0004-6361/202039070 [arXiv:2007.15633 [astro-ph.CO]].

[195] R. C. Nunes, S. K. Yadav, J. F. Jesus and A. Bernui, *Cosmological parameter analyses using transversal BAO data*, Mon. Not. Roy. Astron. Soc. **497** (2020) no.2, 2133-2141 doi:10.1093/mnras/staa2036 [arXiv:2002.09293 [astro-ph.CO]].

[196] A. Bernui, E. Di Valentino, W. Giarè, S. Kumar and R. C. Nunes, *Exploring the H_0 tension and the evidence for dark sector interactions from 2D BAO measurements*, Phys. Rev. D **107** (2023) no.10, 103531 doi:10.1103/PhysRevD.107.103531 [arXiv:2301.06097 [astro-ph.CO]].

[197] A. Gómez-Valent, A. Favale, M. Migliaccio and A. A. Sen, *Late-time phenomenology required to solve the H0 tension in view of the cosmic ladders and the anisotropic and angular BAO datasets*, Phys. Rev. D **109**, no.2, 023525 (2024) doi:10.1103/PhysRevD.109.023525 [arXiv:2309.07795 [astro-ph.CO]].