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Point of View on Gravity

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We want here to put forward the point of view, that one can look at general relativity as having gravity theory as imbedded in space time (the manifold) with only the manifold structure or at a much more symmetric space for the imbedding than the usual metric space time. At least some scaling symmetry should be present in the background spacetime. The obvious examples for such a background space time is just the manifold, or 4-dimensional projective geomtry space. Such a spontaneous breakdown role for the fields of gravity will suggest that the reparametrization of the coordinate descrition of generel relativity could be genuine physical d.o.f. and it such a case would have huge fluctuations. If one had a physically existing lattice (or even a different cut off) then this lattice would fluctuate relative to our usual type coordinates. It would mean asuperposition (or mixing) of hugely different sides links in the lattice. In such a "fluctuating lattice"looking at different dimensionalities of Lagrangian density terms one would get different lattice scales, so that such a model could tolerate that there were several different fundamentalenegry scales for: See saw neutrinoes, (possibly approximate) unification of gauge couplings, Planck scale(for gravty). We predict a relation between these three "fundamental byescales".

We shall review a "derivation" by Astri Kleppe and myself of locality for such a space-time with especially scaling symmetry, as well as phenomenological finding from micro wave back gound measurements suggesting that the real world is imbedded into a projective space time.

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1. Introduction

This article is to be considered a development of the Bled Proceedings in 2023 " Deriving Locality, Gravity as Spontaneous Breaking of Diffeomorphism Symmetry"[1], but we like to stress the point that Einsteinian gravity is rather naturally comming out by having the metric tensor being just fields the non-zero value of which break the symmetry deformations by scaling in different directions, or some similar symmetry. - What may be really the point is that we even like in our picture to also think about that the metric tensor with upper indices could be degenerate or zero even.(Offen I took myself in thinking as if $g^{\mu\nu}$ could not possibly be zero or have zero determinant)-

In the search for a theory beyond the physics we know today we have long attempted "Random Dynamics", which consists of asking:

Are there some laws of nature, that **can be derived** from other ones in some (e.g. low energy-) limit ? If so, we should leave the derivable law out of the fundamental model, and assume the ones needed in derivation as **a step more fundamental**.

The example today: **Locality** $\Leftarrow$ **Diffeomorphism symmetry** (this derivation Astri Kleppe and myself [3] already claim to have performed/ proven)

So: Assume that **diffeomorphism symmetry** or something similar - e.g. projective geometry symmetry, or simplectomorphic symmetry - is a very(or just more) fundamental principle, while **we do not assume locality as fundamental** at the same level.

In the future we shall of course attempt **also to derive diffeomorphism symmetry** from something else, and that might e.g. be by assuming some "large amount" of symmetry acting on a space. Actually Masao Ninomiya and myself have stressed, that an infinite space acted upon in an sharply 3-transitive (see these concepts "sharply 3-transitive" below in section 4) way is already close to be the projective line[14].

1.1 Philosophic Speculative Introduction, Plan

Let us argue a bit looking at the present work as seeking a theory for gravity, behind or beyond gravity:

- Introduction (we do not know what is behind gravity)
- Argument for a geometry with local scale and projective symmetry:
  - Astri Kleppe and I could derive **locality** of the effectively resulting action. We take it, that, since we can get locality out from starting from a symmetry postulate, that has the whole symmetry of the manifold, **without putting in explicitely locality**, then it means that we can avoid the extra assumption, i.e. we have a simplification, if we assume such a symmetry like the whole diffeomorphism symmetry.
  - Now we only get gravity after a background field is assumed, like $g^{\mu\nu}(x)$. Actually unless we have such a field non-zero in vacuum, we get no propagation, i.e. the different space time points do not get connected. So you may say: We need as a very abstract need for physical theory, that there is connection between the different space time events. Then we must have some field like $g^{\mu\nu}(x)$ or some vierbeins or the like, that can do the

same job of introducing propagations of the field through space time. This is not really a derivation of gravity, but it is a little bit in that direction in as far as this argument present a **need** for gravity.

In usual theories like generel relativity one could say, that we just have introduced gravity because of phenomelogical need, one simply has known for long effects of gravity, and thus we need it. Here I seek to say: we need something less specific, propagation of particles, and then because we have for other reasons(the beauty of deriving locality) assumed too much symmetry, we have come to need some fields, which of the most obvious type looks just like gravity fields. So we came to assume too much symmetry so that gravity could not be avoided for **other than simply phenomelogical reasons of seeing gravitational forces**.

In this point of view the gravity fields represent the fields giving the certain needed spontaneous break down of the too much assumed symmetry.

This might make us think that it (the gravity) is less fundamental, but of course it has be something, which can be constructed from something, which is in the fundamentaltho-ery. But it is presumably easiest to find theories in which you have a lot of fields with various transformation properties under the symmetries at the stage or level of funda-mentallity we discuss. Then one can say: Oh, there are probably also some vierbeins or metric tensor fields and with imposing a symmetry like diffeomorphism symmetry or projevtive symmetry (of projective space time)then the lowest dimension Lagrangian term must like it is in usual gravity theory be the Einstein Hilbert Lagrangian density. Psykologically this way of looking at it might make us a bit less keen on looking at the effects of the fundamental theory, we look for, to think somuch on gravity, because after all the gravity was just the presumably composite field breaking the too much assumed symmetry.

That is to say, we could use this way of thinking as an encouragement not to look for that we should have to bother with all the terrible topologically complicated space times, which are almost unavoidable in quantum gravity. They are in so huge amounts that they would be pretty hard to treat mathematically.

- Projective geoemetry (a possibility beyond gravity)

  Rather we might take the here put forward suggestion of looking at gravity as being imbedded into the manifold. The manifold is there at the most fundamental level, at first we could say. But then we may get the hope, that the terrible topological forms of manifolds might be avoided by modifying a bit the theory by replacing the manifold by a topologically less terrible structure with sufficiently similar symmetrys, for which we could at least approximately still derive the locality principle. A proposal of a very nice structure behaving similarly to the manifold but with much simpler topological form are the **projective spaces.**

  So the speculative model, which we might propose here, is, that we live in a back ground space time, which is a projective space. The structure of such a space time is so simple that seen from small scale it is like a flat space, and thus has the promissing feature af providing a kind of explanation for how flat space time is in practice.

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- A further hope brought by the projective space idea, is that there is hope to characterize the projective space by defining it by means of its symmetry properties. Indeed Masao Ninomiya and myself recently brought attention to that the projective line (= the projective space of dimension 1) were close to come out by assuming a space to be sharply 3-transitively transformed under a group. Just assumming such 3-transitivity you begin to see structures like the field (the field of real numbers say) without putting in the concept of a field directly. Remember that the manifold is defined by means of coordinates, which are differentable with respect to each other, and thus you define manifolds mathematically in a way already using the field-concept as wellknown. If the dream could come through of defining the background space time (now assumed to be a projective space time) without explicitely putting in our real numbers, but rather getting it out somewhat similar to studies of 3-transitive actions of groups, then it would be very nice and suggestive on us being on the right track for the fundamental theory, because we could then claim: we did not even put in our real numbers, no they came out instead!

  Even though this requirement 3-transitivity only leads to 'almost fields" and not completely to the real numbers field as one is accustomed to use in the physical geometry, it is till much better than to have to put the fields in completely from outside.

- A phenomenological support of projective goemetry.

  Then what could in the long run, if it works out, really support the idea of a background projective space, would be, if what we shall present below in section 3 is really working out and we have a phenomelogical observational indication, that we indeed live in a world imbedded in a projective space time.

- Conclusion:

  A very optimistic Random Dynamics dream, might look like this: Almost whatever a very complicated mathematical structure would be like, it would if it is very big unavoidably have some similarity of some parts with some other parts. Such a similarity - presumably approximate only - would naturally be expressed by some approximate **symmetry**, which of course in the mathematical language means that there is some group G acting on the compicated structure being the world, say X, just as talked about in section 4. Then it is needed to find out which properties of such a system a group acting on a set/structure is most likely to be the type relevant for such an attempt of a theory of everything.

  With some empirical support I and Don Bennet found, that what characterizes the Standard Model group $S(U(2) \times U(3))$ (which is gauge group in O'Raifeartaigh.s sense [11] ("Group Structure of Gauge theories",University Press Cambridge (1986)), namely that the representation of the acting gauge group could be found to be essentially in volume the smallest possible compared to a volume constructed for the group itself[7–10]. Taking this to say " The group shall be so large compared to the structured set it acts on as possible (in Nature of fundamental physics)". Note now that that the n in the (sharp) n-transitivity of a group action roughly means, that the group is the nth cross product power of the set X on which it acts. In fact you can say that the group has been brought in correpondance to the cross product

$X \times X \times \cdots \times X$ (with n factors X="the structured set"). Really a manifold or a projective space are spaces with relativly a lot of symmetries, so there is good hope to find, that such a big group compared to the object acted upon could favour just the imbedding spaces, we suggest in this article, such as projective space or manifold.

But then since gravity is needed for getting propagation and the same big group compared to the representaiion or almost the same/the analogon the space acted upon, then the Standard model gauge group could also come from such principle. Taking it that the fermion representations and the Higgs representation in the Standard Model are indeed among the **smallest** faithful ones, there is not much in our present knowledge about the physical laws, which would not be almost unavoidable in the here sugested system.

1.2 Out line of Paper

1.3 Parts

We have made a division into two parts plus the introduction and conclusion:

First genuine part **I. Attempt to T.O.E.** is an introduction to some of the concepts we use like projective geometry and an action not a priori local, but only invariant under say the projective group. If the reader wants to take our work as an attempt to put forward how we could potentially imagine our work as a not so believable theory of everything T.O.E. he or she may start with this first part.

If, however, the reader is sufficiently familiar with projective geometry or simply take the fundamental model as being an imbedding into a manifold without any more structure than just the manifold one, such reader might begin at part **II. Explaining Locality** in which we review the old work by Astri Kleppe and myself of deriving, that the action must effectively be local in the sense of being the usual type of integral over products of fields all from the same point in space time. Also small lacks in the derivation of the full locality are considered.

1.4 The Sections

Part I., Attempt of T.O.E

The next section 2 is then mainly a review of projevtive geometry, which we consider a promissing candidate for a space to replace the full manifold. It may not lead to quite as perfect locality as the manifold, but after all phenomenological quantum field theories, as we know them and use them, have deviations from locallity at very short distencies, if they shall not be divergent(at least in many regularizations), so if we get that in our model, it might be an advantage.

In the section 3 we deliver a very speculative phonomenological arguement for that we should indeed live in a space time inbedded in a projective space based on small deviations from the standard cosmological model for the lowest i modes in the WMAP etc. data.

In the next section 4 we review the mathematical concept of a group action acting n-transitively.

In section 5 we suggest that one should excercise finding the effective action - the now local one - which obeys the symmetries in our model left over, so that we can give at least an idea about have one by a bit more work might see that essentially the usual einstein general relativity comes out of the model, in this article we have mainly left this for the reader, but we hope to come through in another article.

Next in section 6 we put forward the problem, that we cannot get a world with propagation of fields/particles without a spontaneous breakdown by having a non-zero $g^{\mu\nu}$-field (with upper indices) in vacuum. In the subsection 6.1 we give a new way of making dimensional reduction in the spirit of the world being imbedded into the manifold (this is the manifold always used in generel Relativity) or some projective space, which is similar to the manifold. In the subsection6.2 we again review the point of the gravity being needed for propagation.

In section 7 we speculate that such a slight breaking of locality might give us the hope of obtaining an ultravioletly cut off theory in spite of that these manifold or projective space-times are a priori having infinitely small distances on the same footing as the large ones.

In secton 8 we put forward the remark that our model of the imbedding into a projective space time might give some hope for being able to explain the appearance of the huge almost flat space time volumes, we find phenomenologically; and in section 9 we deliver the speculation of characterising a projective space as a set/space on which a group acts in an especially strong way. To say sharply n-transitively is connected with problems in as far as there are no true more than 3-transitive infinite spaces.

Part II. Explaining Locality:

In the first section of part II , i.e. section 10 we shall present and discuss the mentioned theorem of Astri Kleppes and myself, in which we get locality without putting it in, while the detailed proof of this theorem will be put into section 11 In this section 11 we give the real proof of our derivation of the principle of locality together of course with the statement of the "mild" assumptions, such as the analyticity - as a function**al** - of the otherwise so general action, that it is **not** by assumption local.

Now it is often the most interesting about Random Dynamics derivetions, that they do not succeed completely, and also the derivation of locality is only partly succesful. In section 13 we thus tell that one of the results from this not quite succesfullness of the locality derivation is, that we obtain an idea to derive (with in addition only very "mild" assumptions) an old postulate of ours called " Multiple Point Criticallity Principle" (=MPP).

In section 14 we conclude and resume the article.

Part I., Attempt of "T.O.E."

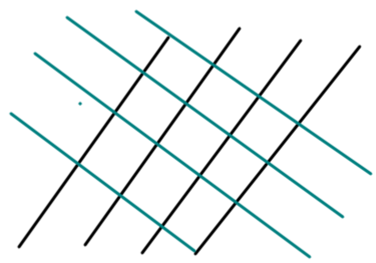
Since our suggested "theory of everything model" contains in it the idea of imbedding space time in a projective space time, we shall begin by a little review of the projective geometry. A priori imbedding of spacetime into another space time sounds to ourselves not attracktive, because it seems to introduce a lot of too speculative stuff by having a whole geometry behind the final and observable one. But for consolation one should think about that the projective geometry is a geometry with much less structure, so that this "imbedding" is actually rather that one first introduce some features of geometry and then put in some fields which bring some more - in this case one puts in the metric tensor or some vierbeins and thereby get the metric. Had we used instead of the projective space a manifold structure, there would have been absolutely no extra speculation in the space in which we imbed than needed anyway. With the projective space there is little bit extra speculation but the projective space might be nice to characterize from its symmetries.

2. Projective

Projective geometry you can think of as being the usual Euklidean geometry with some of the concepts removed, especially distances and the values of angles are removed; rather only those properties which are still the same, if one makes a projection, as one does in a photograph or a painting, are considered in the projective geometry. The space in projective geometry has so to speak less structure than in the Euklidean geometry. One might think of manifolds as having lost even more structure, since there is on the manifold not even any concept of straight line or plane etc. (when one has geodetic line - a replacement for the straight lines - in Riemann spaces, it is strongly based on the metric tensor field). In projective space there are concepts of lines and planes and in higher dimensional projective spaces also flat three dimensional (hyper) planes etc. By a smart trick of adding to the Euklidean space a plane (in 3 dimensions), or a line (in 2 dimensions), or a point (in one dimension "at infinity", one achieves, that the rules lines and planes etc. crossing each other becomes very simple.

In fact: **In Plane Projective Geometry All (different) Lines Cross in a Point**.

Formally this is achieved by simply defining a bundle of parallel lines to be called a "point at infinity" belonging to the "line at infinity":



Bundles of parallel lines are identified with points on the line at infinity. So parallel lines cross there.

Really one makes a description of the d-dimesnional projective space by taking a $d + 1$ dimensional vector space and identifuing the rays (i.e. sets of vectors proportinal to each other. A class of such non-zero to each other proportional vectors is called a ray) with the points in the d-dimensional projective space. The lines in the projective space are then identified with the two dimensional subspaces of the vector space, and the projective plans with the three-dimensional subspaces, and so on.

It is thus possible to make projective spaces corresponding to different fields(**R**, **C**, fields of numbers counted modulo some prime p, p-addic fields, and so on) in as far as one can have vector spaces with different field.

In this article we are interested in using the real field **R** only.

Since a vector space over the real numbers **R** of dimension $d + 1$ can be mapped linearly into and onto itself by the group of transformations $L(d+1, \mathbf{R})$ and we can normalize the transformation mappings to have determinant unity, without it influencing the mappings of the rays (which we remember are the point in the projective space of dimension d), we see that the projective space of dimension d, called $PG(d, \mathbf{R}) = \mathbf{P}_d(\mathbf{R})$, has a group of symmetries, group of projective transformations, $SL(d+1, \mathbf{R})$.

One may say that this group of symmetries $SL(d+1, \mathbf{R})$ is very large for high d compared to the 'size" of the space in the sense, that for almost any two ordered set of $d+2$ points in the d-dimensional projective space one can find a symmetry element from the group $SL(d+1, \mathbf{R})$ that maps one of the ordered sets into the other one. Had it been quite generally true, we could have called it, that the group $SL(d+1, \mathbf{R})$ acted $(d+2)$-transitively on the projective space of dimension d. When we must say that it is only true for almost any two ordered sets, it is because, if the points in one of the ordered sets lie in a hyperplane (of dimension $d-1$) it is not true, and the situation is more complicated. But it is non-generic for such points to lie in a hyperplane, so thinking of non-generic as meaning of propability 0 we can say, that it is almost certainly as if the symmetry group acted $d+2$ transitively.

In fact it is not difficult to show, that given the two ordered sets of $d+2$ point each there is **only one element** in the group of symmetries $SL(d+1, \mathbf{R})$ that does the job of mapping the one ordered set into the other one - except for the nul-space cases of the points lying in same hyperplane as mentoned above -; this property one calls that the group acts sharply $(d+2)$-transitively.

2.1 Weak Argumentation for Projective Space in Nature

Our model or speculation that the space time in Nature should be imbedded in a projective space-time of dimension $d = 4$ or higher is really not supported on much, but let us mention, that our best argument probably is, that, if we have a fundamental space-time with the symmetry of the type of the projective space-time or simply as the manifold without further structure, which has as symmetry group a group of diffeomorphisms, then there is hope that a **derivation of locality** (as we shall see below in Astri Kleppes and mine theorem)is useable in the manifold case exactly, but in the projective space time likely approximatively in some way, too.

But we can also provide another bad argument: Don Bennett and I found that the Standard Model **Group** (where the concept of group is included and not only the Lie algebra, which is usually what is said to charaterize a Yang mills theory; see [11]) could be characterized as getting the best value in a game we defined, which could be said crudely: The Standard Model group is that group among all the Lie groups that relative to a size of the group G transform a faithful representation, on which it acts the **least**[7–9].

If we take groups represented on spaces much smaller than crudely speaking the group itself as what for some reason Nature should like, then we could still very crudely claim, that high n n-transitive action should be prefered by Nature. So high dimensional projective space should be not so bad from this point of view.

Astri Kleppe and I study for the moment symmetry groups of graphs hoping to get some understanding as to, if there could be any sense of this argument.

It must be admitted, that there are also problems for the idea that the space time in Nature should be imbedded into a projective space:

If one should want to have a conventional Big Bang, one could perhaps hope that imbedding could be made so as to arrange it, but the projective space itself has no singularities, so for such an imbedding the most nice would rather be a bounsing universe[19] in which there is no singularity but rather first a shrinking universe which stop shrinking at a finite size and then grow again, and

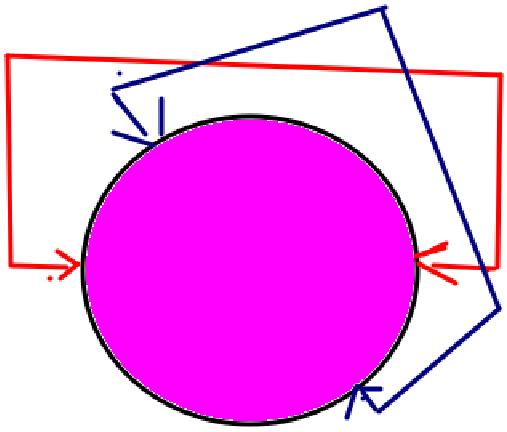


Figure 1: This figure shall illustrate that there is in the infinite line in a projective plane only **one** point representing both a direction and the opposite direction. Looking at the picture figure 2 you just have to imagine that both point in horizon corresponding to the directions of the straight railroad are identified as the same point at infinity in the projective geometry.

preferably even at some point shrink again. Preferable a cloed or circular time should be there is projective space-time ibedding because the topology of the projective space is actually compact,and so a time with two oreven worse one infinite ending would seemingly not be so welcome.

Actualy I have with Keiichi Nagao [18] a work supporting an idea of bouncing universe, but we better just stop here saying our model with projective geometry is only an attempt to look at one possibilty.

3. Phenonenological Evidence for World being a Projective Space-time

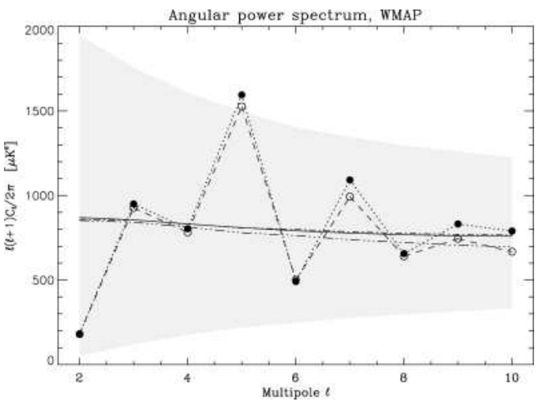
Infinite far out points in opposite direction identified in projective geometry We here stress, that since in projective geometry a bundle of Parallel lines is considered **only one point on the line or plane or etc. at infinity** there is no distingtion between the point at infinity in one direction along the bundle of parallel lines and the point in the opposite direction on the infinite plane (or watever for other dimensions than 3). There is **only** one point at infinity for each bunch of parallel lines.

This is illustrated on the figure 1 by the arrows pointing to identified directions so to speak.

If somehow our universe really were a projective space, then you might see the same object in the two opposite directions. That would give of course a correlation of e.g. the radiation comming from two opposite almost infnities. They would fluctuate in similar way because of being the same point on the infinite plane (in three dimensions)

Lowest l WMAP fluctuations

Figure 2: On this figure from Wikipedia you see how the projection onto a photograph bring a bunch of parallel lines - a rail road - to “cross each other at the horizon. So the horizon is really the line at infinity, but one shall remember that the two ends of the railroad are identified to one point in the infinite line in the projective geometry (remeber the infinite line points were identified with bunches of parallel lines (in 2 dimensions)



The analysis of the microwave background radiation is typically done by resolving the fluctuation of the temperature as function of the point on the sky into a description expanded on spherical harmonics. Thus one presents e.g. the size of such fluctuation connected to the various spherical harmonics, which are marked by $(l.m)$. We shall have in mind that the even l spherical harmonics have the same value in exact opposite directions, while the odd l ones have just opposite values in opposite directions.

On the figure we have the experimentally found fluctuations as function of l (averaged over m) for the first few lowest l's.

Remarkable:**Even** l **fluctuatons are relatively low, while the odd** l **ones are relatively high**.

We know that microwave backgound radiation comes from 13.7 millird light years away, so if the universe should really be a projective space, the infinite plane or infinite three-space, if we think of the fourdimensional space time as projective, should be not much further away than 13.7 millard light years, if we should be able to observe it.

If Projective Space "seen" in CMB-fluctuations, then Universe Not Much bigger than Visible Universe

The just shown:

- Y_{lm}-proportinal modes in temperature variation over sky with **even** l have **lower** fluctuation.

- Y_{lm}-proportional modes in temperature variation over sky with **odd** l have **higher** fluctuation.

if taken seriously implies that **the visible universe edge is not very far from where there is the identification of the diametrically opposite points** (on say the infinite line). So Universe would not be so huge as the very accurate flatness would indicate!

3.1 Correcting for my mistake

Post Talk Slide: Naively you expect the opposite!

I got very confused and chocked, when I persented the foregoing slides, because naivly thinking of a three-dimensional Projective space:

Since the points in opposite directions are **related, in fact the same region,** the **even l** spherical harmonics, should show a **big** fluctuation, because they add together by continuity across the infinite plane related regions, and thus counts really the same fluctuation in two ends as statistically independent and thus over estimate the fluctuation. Oppositely weighting with an **odd l**

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spherical harmonic you add with a relative minus sign two close by (across the infinite plane) region contributions and should get approximately zero. Thus the fluctuations for **odd l should be small.**

Thus my chock: The "evidence" I had believed, had the wrong sign!

We Forgot the Time Direction...

We should not have looked for opposite points in the purely 3-dimensional space,

Rather than looking **for the opposite point on the infinite 3-space we should look for opposite points also in the time direction i.e. in the 4-dimensional space-time.**

If we take that what should be Big Bang is rather just the narrowest point, in some sort of bouncing universe[19, 21] then we can at least speculate to have a mirror symmetry in this the narrow universe region. Big Bounce models were endorsed on largely aesthetic grounds by cosmologists including Willem de Sitter, Carl Friedrich von Weizsäcker, George McVittie, and George Gamow (who stressed that "from the physical point of view we must forget entirely about the precollapse period").[20] To be definite - and to have a good bouncing universe model to match into the projective geometry - let us take the model in the midle time we have De Sitter space time with (miraculously) empty space, so that the whole model is approximately time reversal invariant. The inflation period in the crunching era (before the midle being a replacement for the usual big bang) which is really a deflation is truly a De Sitter model - if sufficiently empty - and goes as a true De Sitter space time from contracting to expanding, and then we are already in the usual inflation period, afterthat comesrehaeting and so on. The only "new physics" needs then only to be not so likely initial condition that we have a completely (miraculously) matter free De Sitter spacetime. At least this is the modelweallude to on the figure by the smoothed off neck between the two clocks on the figure 3.

Let us consider this time usually taken as big bang to be rather a center / origo of the space time, in the sense, that we consider the opposite ends of lines extending from this big-bang like region and claim that because of a three-space at infinity having the fluctuations are approximately the same in the two ends of a line through this big-bang region.

Imagine Imbedding a Bouncing Universe Space-time into a 4-dimensional Projective space, filling it out

This drawing is very symbolic, but of course one can see that the two **directions** corresponding to the same point on the infinite 3-space, now have opposite times too. That is to say that one of them are in the prebigbang time (which probably do not exist, but the projective space has no singular start so it goes extremely badly with the big bang theory; so to have any chanse with projective space-time we better give up big-bang).

In Projective Space No Singularities like Big Bang, so better Bounce

In the Projective space(s) all points are symmetric with each other (trasitivity of the grouop action) and thus no point-singularities, so that a **big bang start would not fit well into the projective spacetime**.

Therefore we rather imagine here a **big bounce model** in which there is contracting universe before it reexpands, although such a model has rather many problems with second law of termodynamics, and with how an about to crunch universe can get its contraction turned to an expansion. But an **empty De Sitter space can bounce**, so we might postulate that in the midle of times the universe is an empty De Sitter space.

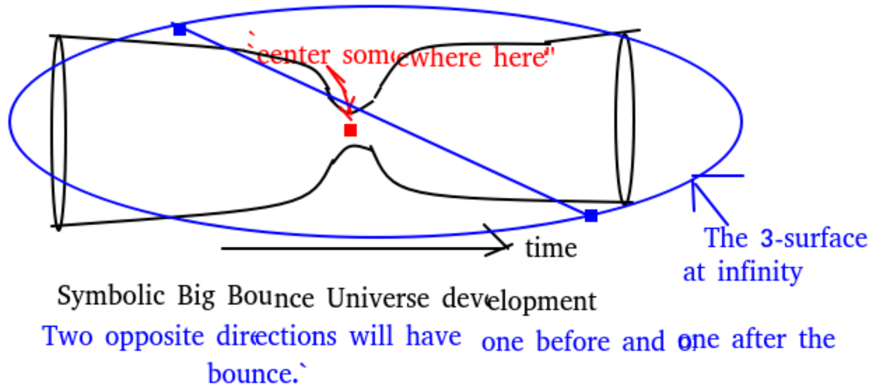


Figure 3: This a bit symbolic figure shouldillustrate an approximatively time-inversion symmetric bouncing universe model with an empty (miraculuosly) De Sitter Universe in the midle of times. This is the two time reflected usual cosmology development describing "clocks"(or glasses). Then there is in addition an ellipse around symbolizing the three-surface at infinity. The scew line across is to tell that on such a three space at infinity there is as we stressed above only one point to represent two opposite directions; so that looking at opposite directions - here in four-space youlook at the same things on the three space at infinity.

So two opposite time directions in respectively the expanding and the chrunching halfs of the space time.

Inflaton Field goes up to top of Potential to Stand there as Long as possible to get preferably slowest roll

Let us imagine as our model to get at all a reasonable imbedding into the projective space, that we have a bounsing in which the inflaton field is on the top of a hill and stands there relatively long.

Then to preferably achieve a long inflation, locally the inflaton field in the time when anti-inflation goes into inflation the inflaton field runs up the potential hill and stops very close to the top of the potential. Then it falls down again, first extremely slowly and then unavoidably faster.

Opposite Point Identified as a single point(event) in the 3-space at infinity, are oppsite in both space and time.

If an event at the recombination era at 370000 years after the big bounce is supposed to be sufficiently far out to be close to a point on the 3-space at infinity, then this approximating point at infinity is identified with a point at infinity in the opposite direction in space, meaning on the sky, but it shall also be opposite in time.

The latter presumably means it shall be on the chrunching time sector (by the chrunching time sector we mean the era in which universe gets smaller and smaller) if we have looked at a search the point opposite to a point in the expanding time sector.

Speculate biggest fluctuation in the Time of reaching the Potential Peak

Let us further assume that the largest fluctuation in the inflaton field comes from the **time** at which the inflaton just reaches the peak of the potential varies randomly from region to reagion in space.

If we have points that are relatively far from each other such fluctuation of the times of peaking should be strongly fluctuationg.

Now notice: If it happens early from the point of view of the expanding universe then it happens also early by the point of view of the chrunching universe, but that has for this compared our expanding universe the opposite effect, because of the time reversal.

Prediction of the Sign-inversion by the time reflection not detail dependent

So whether late or early gives larger or smaller CMB radiation, then the time reflected point will always give the opposite to the not timereversed one. So we will get the odd l speherical harmonics get the biggest fluctuation, and the evenones the smallest!

Conclusion on CMB-fluctuation Prediction

Assuming:

- Bouncing Universe
- Time of Inflaton field Reaching the Peak most important for the fluctuations in the CMB radiation.(The meamning is here that the quantity fluctuating the most is the exact moment at which the turning point of the inflaton field is reached in the various regions of space.)
- The Chrunching Universe behave approximately Time reversal invariant to the expanding one.

we get:

The odd l shperical harmonic modes shall have the largest fluctuations, while the **even l one the smallest,** contrary to the intuition forgetting the time to be also reflected and via timereveral can give the opposite sign.

3.2 Statistics of the CMB deviation for small l

We must admit, that there are only ca. 2 standard deviations from also the low l fluctuation observations agreeing with the staistical model, so there is only two standard deviations to build the story about the projective space on. So it is very weakly supported.

Only 2 s.d. from statistically understood low l modes

In spite of the statistical significance of the observed even-odd assymmetry we used to support Projective geometry is only $\sim 2s.d.$ theorists sought to explain these low l fluctuations, e.g. R. Mayukh et al, [16] by Superstring excitations, and by "Punctuated inflation"[17].

3.3 Bouncing Universe Not so Easy

Think we must add that such a bouncing universe although beloved by especially workers on loop-gravity is really in great trouble because of the entropy that would have to have been abnormously low further and further back in time. One might seek to avoid this by expanding the universe not from a full universe but from a tiny bit [19], but if we e.g. want the approximately time reversal invariant model,which would fit best with the projective geometry space time, then the most hopefull way would be to have some new physics in stead of the second law of thermodynamics, so that we do not have that entropy cannot decrease. Actually we for some time worked with a model allowing for such an idea that the initial conditions are not necessarily given only in the beginning, but that the initial conditions are given e.g. by minimizing some functional - in our model the "imaginary part of the action" $S_I(history)$ - of the history, so that in this way a special history is selected for the development of the universe. If the most decisive contributions in the functional comes from the period around the time when universe is small it would look after that time as if the earlier times were very well organized, while before that time -in our picture in the chrunching time - it would be the future that looked most organized. Formally of course that would look as if the future was organized while the past chotic. But of course if poeple in the chruching time just inverted their time concept, they would have everything just as us, and they could in the inversed time picture have the entrpy increase as we. A name for our speculation of this type is "the complexaction theory" and we have wotked on ti with Masao Ninomiya[22] (formulating complex action), and developped it with Keiichi Nagao[23](showing that it would not be much observed except for these initial conditions comming out).

4. Trasitivity

Because we have in mind that at least a one-dimensional projective space - the projective line - is to a large extend characterized by transforming n-transitively with n=3 for the projective line and that this high n n-transitivity couldbe used as a speculative reason for using projective geometry as a starting point for understanding areason behind the geometry of Nature, we want to here review a bit the concept of n-transitivity and the related concept of a group G acting on a set X.

On Transitivity and Action of a Group on a Set

When a group G acts on a space X

$$\alpha : G \times X \rightarrow X \tag{1}$$

$$\text{denoting } \alpha(g, x) = gx \tag{2}$$

$$\text{so if } gx_1 = x_2, \tag{3}$$

it means the group element g brings the element $x_1 \in X$ into $x_2 \in X$, then we say G **acts n-transitively provided there for any two ordered sets of n different points in X, $(x_1, x_2, ..., x_n)$ and $(y_1, y_2, ..., y_n)$ exist a group element $g \in G$ such that**

$$gx_i = y_i \textbf{ for all } i. \tag{4}$$

We say it is sharply n-transitive, when this g is unique.

d-dimensional projective space has a symmetry group acting almost(!) $(d+2)$-transitively

Examples:

- Under the action of diffeomorphisms on a manifold the action is n-transitive for any integer n; but is far from being sharply n-transitive. (There is indeed a theorem generalized by J. Tits and M. Hall, who proved that there are no infinite sharply n-transitive groups for n≥ 4.See e.g.[15])
- In d-dimensional projective space $PS(d, \mathbf{R})$ the symetry group acts essentially $(d + 2)$-transitively, but not truly so, because the immage of points say on a line remains on a line. Only the projective line $PS(1, \mathbf{R})$ is truly 3-transitive.

- Euclidean spaces are only (1-)transitive under their symmetry. It is the translation group that acts transitively. When the group conserves the length of line there can be no even 2-transitivity.

Think about removing or permuting this:

5. Repeat

Repeating Argument using Action

Instead of looking at the equation of motion we could ask, if we could make an action

$$S[fields] = \int \mathcal{L}_{\mu\nu\rho\sigma}(x) dx^\mu \Lambda dx^\nu \Lambda dx^\rho \Lambda dx^\sigma, \tag{5}$$

which is invariant under our symmetry having locally generallinear symmetry and at the same time can describe a propagation of some fields.

If a field ϕ shall not be determined locally by the other fields, but appear in equation(s) with derivatives, there must be a derivative acting on ϕ i.e. say $\partial_\mu \phi$ occuring in the Lagrangian density $\mathcal{L}_{\mu\nu\rho\sigma}$; but with what to contract the lower index μ on $\partial_\mu\phi$? To some field with an upper curled index like a vierbein V_a^μ or a $g^{\mu\nu}$? Yes but if we work in vacuum and there were no spontaneous break down of the symmetry these fields would be zero.

Continuing repeating Derivation of Need for Spontaneous Breaking of Locally General linear symmetry

Looking for making

$$S[fields] = \int \mathcal{L}_{\mu\nu\rho\sigma}(x) dx^\mu \Lambda dx^\nu \Lambda dx^\rho \Lambda dx^\sigma, \tag{6}$$

invariant under the symmetry, but still with fields propagating even with vacuum not breaking the symmetry. (We shall show you cannot find such an action.)

Can it help to let the $\partial_\mu\phi$ combination be contracted with a dx^μ to give it a chance to propagate? In fact

$$dx^\mu * \frac{\partial\phi(x)}{x^\mu} = d\phi(x) \tag{7}$$

is a total derivative. If you now wanted to make the term second order in the ∂_μ, you would use yet another of the factors in the measure

$$d^d x = dx^1 \Lambda dx^2 \Lambda \cdots \lambda dx^{n-1} \lambda dx^n, \tag{8}$$

and the second order term would be like

$$\frac{\partial\phi}{\partial x^\mu} dx^\mu \Lambda \frac{\partial\phi}{\partial x^\nu} dx^\nu = 0 \text{for same } \phi \text{ in the two factors.} \tag{9}$$

or different ϕ's, ϕ_a and ϕ_b

$$\frac{\partial\phi_a}{\partial x^\mu} dx^\mu \Lambda \frac{\partial\phi_b}{\partial x^\nu} dx^\nu = d\phi_a \Lambda d\phi_b \text{ a toal derivative.} \tag{10}$$

Couple it directly to the dx^μ's?

Seems not to give a propagating equation of motion usual type.

Let us here also remark that to get such an integral over all space time as we discuss, one also has to include a quantity that transforms as the usually known $\sqrt{g}$ the determinat of the metric tensor with lower indices. It is e.g.

$$\int \sqrt{g} d^d x = \text{“}4 - volume'' \tag{11}$$

$$\int \frac{\partial \phi}{\partial x^\mu} * \frac{\partial \phi}{\partial x^\nu} g^{\mu\nu} \sqrt{g} d^d x \quad , \tag{12}$$

which are meaningful reparametrization invariant space time integrals, while there is no meaningful 4-volume of the manifold nor of the projective space time **without the $\sqrt{g}$ or something replacing it.** The projective space has no size in itself. Also for that you need a spontaneous breakdown.

In reality is what we need for being able to get a propagation for a scalar ϕ the combined quantity $g^{\mu\nu}\sqrt{g}$ where then g is determinant of the metric with lower indices being the inverse of the one of the upper index.

6. The Propagation Problem, Need for Gravity

Let us stressagin our main point on need for gravity:

If there were no spontaneous breakdown, so that in vacuum all fields ϕ had zero expectation value, then there would be **too much locality**, superlocality (if we as we shall argue for below can derive locallity):

There would not be place for useful derivatives in the Lagrangian density $\mathcal{L}(x)$, because ∂_μ could only be contracted to the $dx^\mu \Lambda dx^\mu \Lambda \cdots dx^\rho$, but that gives only an action which is a boundary integral only (integral of total derivative).

So a gravity field with vacuum expectation value is needed (smells like deriving gravity as needed at least).

Viewing Gravity from reparametrisation Invariant Fundamental Theory

Gravity or Physics with propagation, needs a breaking of scale invarince since the equation of motion

$$g^{\mu\nu}\partial_\mu \partial_\nu \phi = 0 \tag{13}$$

needs an upper index $g^{\mu\nu}$ for being reparametrization invariant.

A non-zero $g^{\mu\nu}$ represent a spontaneous break down of a symmetry involving say scaling or reparametrizations. Similar ideas by [2].

The theorem: Spontaneous breaking of Reparametrization Symmetry Needed

Theorem: *If a theory with reparametrization invariance is not spontaneously broken - meaning the vacuum is totally reparametrization invariant - then propagation in this world is impossible.*

(In this theorem we have without mentioning assumed locality, otherwise propagation would hardly makesense; but anyway in the next part, part II, we shall derive locatlity from reparametrisation symmetry,so in that light it may hardly be needed to mention this assumption.)

The speculative suggestion: If **reparametrization** transformations constitute a **fundamental symmetry**, there would be no waves going from one point to another; so only to the extend that

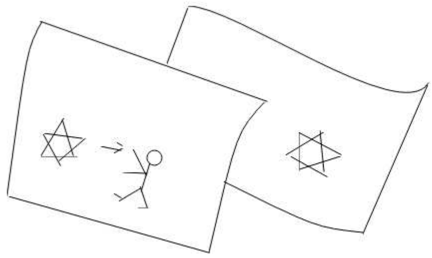


Figure 4: This picture should be conceived of as perspectivly showing a three dimensional space to represent a higher than 4 dimensional space - analogous to the 9+1 dimensional space time in superstring theory -, the fundamental space-time. Then because of the non-maximal rank of the metric tensor $g^{\mu\nu}(x)$ field there is only possible propagation along some lower dimensional surfaces imbedded in the high dimensional one. To illustrate we have drawn almost parallel such 4-dimensional universe-pieces (in the perspective as if they were 2dimensional), with a star and a man on to show it is such surfaces that are the realworld we know.

vacuum has some **breaking** of the symmetry of **reparametrization** we can get propagation. So not much interesting physics could go on without this spontaneous breaking. **Gravity-like fields** - basically $g^{\mu\nu}$ - non-zero in vacuum are **needed**.

6.1 New Way of Reducing Dimensionality, use Degenerate $g^{\mu\nu}$

Side remark on Dimensional Reduction in the Spontaneous Breakdown

Even if dimension d of the fundamental space were high, we could have that the rank of the $g^{\mu\nu}$ tensorfield in vacuum be lower. In that case the world in which we could propagate would be of the lower dimension.

You shall see this idea of having spontaneous breaking of some symmetry like a projective symmetry group or general reparametrization invariance by a metric tensor ($g^{\mu\nu}(x)$) being non-zero as an alternative to the wellknown ideas of reducing the dimension of a space time,such as compactifying the extra dimensions, or having a brane on which we live, and which the fields we are represented by cannot leave.

6.1.1 Spinors

But say spinor-fields would - you might think - anyway have to have numbers of components matching the fundamental dimension, but alas: There is no spinor representation of the general linear group which the transformation group of the tangent space for the symmetries, we care for in this talk! So can a model with reparametriztion invariance as fundamental symmetry have spinors at all?

Well, even in ordinary generel relativity, we know that the spinor fields are indeed w.r.t. curved space indices scalars - they only have the so caled flat indices, which are really only eneumerations of various vierbein fields - so they are indeed **scalars**. This means that without the vierbeins the fermion fields could not propagate. So realistically we have to think of the breaking of the too large symmetry comes by means of vierbein fields rather than by a true $g^{\mu\nu}(x)$ alone. In fact the most likely model is presumably that there is bound state combination of the vierbeins making up the $g^{\mu\nu}(x)$.

On the figure 6.1 is seen a couple of layers imbedded in the manifold; a couple of layers with no communication possible between them, in the case of degenerate $g^{\mu\nu}$*, i.e determinant =0*

If in vacuum the rank of the upper-index metric tensor/matrix is lower than the dimensionality of the manifold, then there appears surfaces on which signals can propagate, but from surface to surface it cannot.

We imbed Gravity and the rest into Spacetime WITHOUT metric, as Just Manifold or Projective Space or some Noncommutative phase space,etc.

The idea of imbedding the gravitational manifold from general relativity into an imbedding space is an old one, see e.g. [6]. But Sheikin and Paston imbed the general relativity space into a flat metric space time. In the present work we are interested in **imbedding into a geometrical space**, which have **no metric** but rather has such symmetries that locally it is part of the symmetry that a small neighborhood can be deformed and scaled up or down in size, so that a metric would be forbidden by the symmetry and at best be allowed as spontaneous symmetry breaking. The space in which to imbed in our present work is rather thus thought upon as either a **pure manifold** with no further structure **or a projective space-time**.

In spaces with local linear deformation like: Reparametrization invariant or Projective space, No Signature distinguishing Minkowski and Euclidean metric say

It is the $g^{\mu\nu}(x)$ that has the signature - in physical world 3+1 -, so without $g^{\mu\nu}$ Minkowski and Euklidean spaces are the same.

Our Point of View: Start with a locally Scaling and deformation Containing Symmetry.

Having in mind our work with Astri Kleppe[3, 4] of **deriving locality of the theory from a reparametrisation invariance of an extemely genaral action a priori not local** (see Part II) - thus it will not leave a measure over space time $\sqrt{g}d^4x$ invariant - we suggest to assume a symmetry involving - at least locally - such a reparametrisation invariance to be assumed fundamentally. But then it should be broken spontaneously meaning that the vacuum state turn out **not** to have the full reparametrization invariance or the full projective invariance but breaks some of it at least scaling symmetry and the order parameters are e.g. the metric tensor or some vierbeins so that this vacuum breaking down of the symmetry can cause gravity.

That is to say: We want to assume either reparametrisation invariance or something like that, and **after that** hopefully derive or understand gravity and locality.

Geometries with (local) scale and deformation symmetry.

Examples of how you can have local deformation symmetry:

- **Full Reparametrization Symmetry** This is the symmetry of coordinate shifts in the General relativity.
- **Projective geometry space** The symmetry of the projective space is a smaller group than that of general relativity. (I am personally especially attached to projective geometry, because I made my living from teaching it for 6 years.). M. Ninomiya and me[14].
- **Symplectomorphic invariant space.**

By local scale symmetry we mean that there are symmetries, so that in the tangent space to any point we have symmetry under scaling up by any (real) factor this tangent space.

In the present work we really want to have **locally not only scale invariance but invariance under any real linear transformation** $GL(d,\mathbf{R})$.

Starting Point: Locally General Linear Transformation Symmetry

Our starting assumption - in this work - is that there are such symmetries assumed that for every point on the manifold you must have symmetry under general linear transformations in the tangent space:

$$(dx^1, dx^2, ..., dx^d) \rightarrow ((Adx)^1, (Adx)^2, ..., (Adx)^d) \quad (14)$$
$$= (A^1_\mu dx^\mu, A^2_\mu dx^\mu, ..., A^d_\mu dx^\mu) \quad (15)$$
$$\text{for any real matrix} A^\mu_\nu \in \mathbf{M_{d\times d}}\text{(in the curled indices).} \quad (16)$$

Having such a symmetry will **not** immediately be enough for guaranteeing that we for a general functionally analytic action $S[fields]$ shall get locallity[3, 4]. For instance our favorite example of a space with such symmetries not being truly the diffeomorphism symmetric manifold with only manifold sturture, the projective spaces do allow some terms in the Tayleor expansion of an apriori non-local which are explicitely depending on fields at severalpoints and thus not local,not even local in the weak formwe reach for the only manifold space-time. We might as an example think about a one dimensional projective line; in such a space one can for four points construct the anharmonic ration which is a projective invariant quantity. By multiplying together one or more fields at four different points and further with some function of the anharmonic function for these points one could construct an obviously a priori non-local term for the action, bu a term which is anyway projective transformaation symmetric. However, in the one dimesional case considered here the term had already four factors; in 4 dimensions a corresponding term would - except for terms only defined on common line or plane etc. - need 7 factors. So if we have assumed approriate restrictions on the highest number of fields multiplied together that we accept shall be included, then there is till hope for making a theorem like Astri Kleppes and mine work even for projective geometry.

6.2 Resume propagation

Propagation Requires Breaking of the Locally General Linear Symmetry Usually the propagation of particles in say free approximation is given by a D'alembertian equation of motion

$$(\Box + m^2)\phi = 0 \quad (17)$$

but to have local general linear transformation invariance:

$$(g^{\mu\nu}\partial_\mu\partial_\nu + m^2)\phi = 0 \quad (18)$$

we need the $g^{\mu\nu}$!

If such a $g^{\mu\nu}$ is non-zero in vacuum, we have a **spontaneous break down** of the symmetry, because the $g^{\mu\nu}$ field transform non-trivially under the local general linear tranformations.

So we only can propagate (normally) any particles, provided we break (spontaneously) this locally general linear symmetry!

6.3 Many Point Systems, Anhamonic Ratio, Fluctuating Lattice

As already toldthere are many concepts,which do not find any place in say the projective geometry: E.g. the concept of distannce, we have to get from an extra spontaneous break down, because it is not there a priori. Similarly there is no usual type of ordering of the points on a line (there is a cyclic ordering,abit analogous but with three point to give a cyclic ordering).

So one could speculate if there could be in the projective space(-time) something like a lattice? Since we have in the d dimensional projective space symmetries that act $d+2$ transitively, and thus all sets of $d+2$ points are equivalent under the symmetry and cannot have properties that are different from any other set of $d+2$ points, except if some of these points are aligned, so that they are exceptional and the $d+2$ transitivity does not work for them.

But a very large number of points - bigger than $d+2$ - can without any use of exception be in a specialconfiguration,and thus "lattice like structures", by which we just a setof point organized in some way allowed by the projective geometry symmetry can indeed be defined.

To give an example of such an organization in a projective space meaningful way - meaning here a projective transformation invariant way - we shall for simplicity consider the projective line, i.e. the case $d = 1$, and then we shall introduce the cpncept of the anharmonic ratio.

6.3.1 Anharmonic Ratio

We have mentioned that the projective d-dimensional space has a symmerty group $SL(d+1, \mathbf{R})$, and thus the projective line, which we for simplicity now shall look at, has the transformation group $SL(2, \mathbf{R})$ also called the Møbius group. As the projective line is only 3-transitively transformed under this group there is in fact infiitely many classes of ordered sets of 4 points that cannot be transformed into each other. In fact these classes of ordered sets of 4 points are distiguished from each other by means of a real number called the anharmonic ratio of the four points.

Although one in principle should be able to describe the anharmonic ratio only with concepts of meaning in the projective geometry, one very often think that one has introduced the distance concept, and define this anharmonic ratio -or cross ratio or double ratio - by

$$(A, B; C, D) = \frac{AC \cdot BD}{BC \cdot AD} \tag{19}$$

for four points A, B, C, and D on a projective line. The orientation of the pieces of line as AC etc determines the sign of the symbols AC etc. The important thing is that making projective transformations of the four points does not change this anharmonic ratio.

6.3.2 Many Point Structures

One can imagine that one can specify the relative placement of a large (larger than $d+2$) number of points in the d-dimensional projective space by means of anharmonic ratios, and thus make a multipoint figure, something for large number of points reminiscent of a lattice.

Let us as an example see that one even can specify in a projective geometry meaningfull way a configuration of points on the projective line, which at first looks regular in a similar way to a genuine lattice.

Before presenting the example let us remind the reader that the projective line topologically is a circle, so we can very naturally draw the projective line as a circle. On the circle model for the

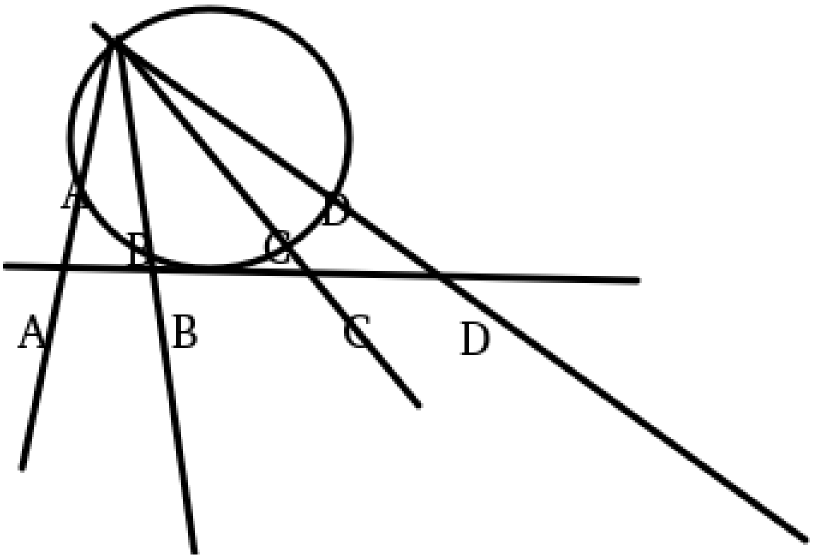


Figure 5: Here we show how a projection from an arbitrary point E on a circle to a tangent to that circlecan be used to define - actually E-dependent correspondence between the tangent with an extra point at infinity considered the projective line and the circle. The concept of anharmonic ratio for four point of the circle turns, however, out not todepend on the point E used for the projection identifying the circle with the tangent asprojective line. The formula for the anharmonic ratio can be calculated to be (20). We used the same letter for the pairs of identified points on respectively the tangent and the circle.

projective line we have a slightly different definiton of the anharmonic ratio, now in terms of the angles θ_A representing the point A, θ_B the point B and so on. The anharmonic ratio in the circle formulation is[25]

$$(A, B; C, D) = \frac{\sin((\theta_C - \theta_A)/2) * \sin((\theta_D - \theta_B)/2)}{\sin((\theta_C - \theta_B)/2) * \sin((\theta_D - \theta_A)/2)}. \tag{20}$$

As an example we could claim that we have n points and that we count them cyclically modulo n so that the point P_q and P_{q+n} are the same point. We could as the example put them equidistantly on the circle (although distance, we remember, is not a concept in the projective geometry), so that each point is followered by the next $\frac{2\pi}{n}$ radian after it. For four successive points, see figure 6 in the chain of the n points the anharmonic ratio is

$$(A, B; C, D) = \frac{\sin(2\pi/n) * \sin(2\pi/n)}{\sin(\pi/n) * \sin(3\pi/n)} \tag{21}$$

But our point is just that one can describe by means of appropriately chosen anharmonic ratios for a large number of points asystem of points some in one configuration looks much like a lattice of point as say the Wilson-lattice. It should be had in mind that by a projective transformation we could easily shrink all the points in such a set into a very small region. Such shrinking into a very small region is what typically will happen if you make 'random" projective tranformation on some configuration. Since the Haarmeasure for the projective group is not normalizable a "random" projective transformation is strictly speaking not welldefined concept, but if you imagined making

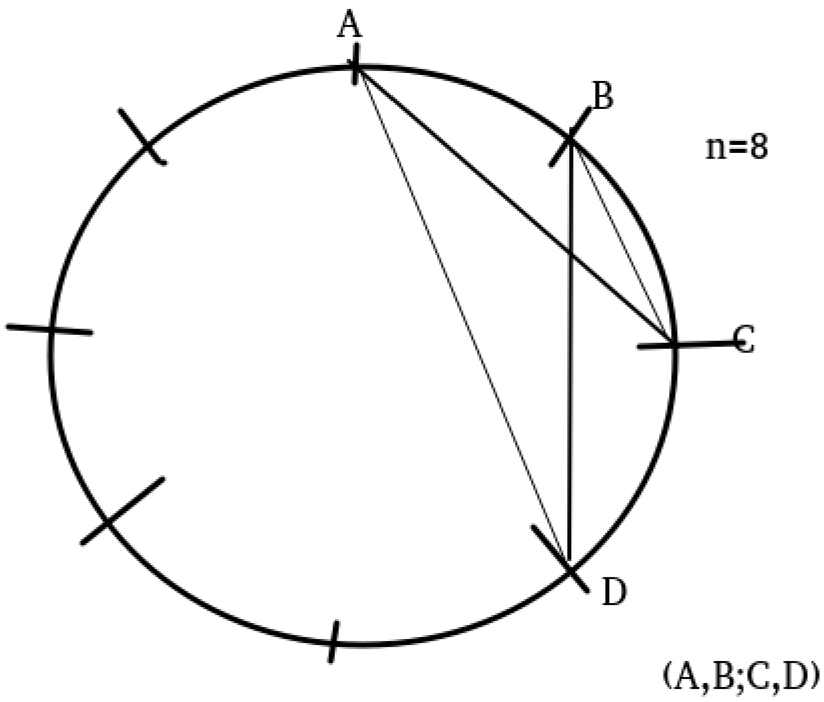


Figure 6: We here drawn a little lattice on a projective line represented as circle. The anharmonic ratio for the four "first" points in the lattice is calculated in (21).

it sensible by some sort of cut off/regularization it should be so that by far the most likely was to transform a finite set of points into a divergently small corner.

When we as here described could specify what we would call a "lattice" in the sense of the lattices one uses to work with QCD in the $d = 1$ case in terms of the anharmonic ratio and thus in terms of the projective geoemetry concepts, then we can rather easily extend such a method of describing a "lattice" in higher dimensional projective spaces. We could namely use the anharmonic ratio for points assumed to lie on lines and construct a higher dimensional lattice by means of one-dimensional ones.

What we want to conclude is that "lattices" in projective space is not excluded, although they would be somewahat easy to deform somewhat jointless. For instance you could not make the link orthogonal to other links because orthogonality does not makesense in pure projective geometry.

6.3.3 Gravitational Field Fluctuation and Fluctuating Lattice

Now we shuold consider such a possible "lattice" of many points in the projective space in which we imbed the Riemanian space, from the point of view of the Riemanian space (i.e. the spacewith the metric comming from the spontaneous break down). I.e. we shall consider the "lattice"described in projective space from the point of view of an observer living in the metric space time. We must of course imagine that the spontaneous breaking gravitational fields at least approximately get to be governed by a an Einstein-Hilbert-action so that it gives them the usual Einstein equation dynamics. It is actually rather easy to argue, that taking the usual reparametrization symmetry for the Lagrangian density to be the lowest dimensional Lagrangian density, it can be, leads to Einstein-Hilbert one. Have now in mind, that the usual reparametrization symmetry when you look for local actions is actually also imposed by the subsymmetry of the projective symmetry with one point fixed, because the tangent space to a point in the projective space from the projective symmetries is imposed a similar symmetry as what diffeomorphism symmetry imposes of tangent space transformations.

In this way such an imbeded gravity model is rather easily becomming the phenomenologically good Einstein general relativity gravity in pracsis.

But now think about that the vierbeins or the metric $g^{\mu\nu}(x)$ field being transformed by a reparametrization will not change the action. That is to say that there will be **enormous quantum fluctuation** of the $g^{\mu\nu}(x)$ or the vierbeins **corresponding to reparametrizations**.

Let us note that when we think of the gravity fields as order parameters for a spontaneous breakdown and then find that with the main term in the Lagrangian at least has the reparametrization symmetry approximately in as far as it is the Einstein-Hilbert action, then we have to take the gauge degrees of freedom as true variables physically. It would be very strange that it should from the outset have been a reparametrization invariance for those - perhaps even conposite fields - that shall make up the vierbeins or metric-. Possibly even this reparametrization symmetry for the vierbiens or $g^{\mu\nu}(x)$ is only approximate, in as far it is only the first lowest dimension term that becomes just the Einstein-Hilbert.

So we must make us the picture, that the coordinates specified to some restriction on the coordinates connecting them to the metric would fluctuate wildly compared to points in the space - the projectivespace - in which we consider here the world imbedded.

Concerning the "lattices"we talked about, and which could possibly be constructed in the projective space, the just mentioned fluctuation of by a coordinate restriction defined coordinate sytem would fluctuate wildly compared to such a lattice. Or we could say it oppositely: The "lattice" would fluctuate wildly relativeto a generel relativetiy coordinate system specified by some relations involving the metric, such as fixing some distances etc., but the space time we usually use is coordinates specified mainly relative to the metric. If the Riemann space is close to being flat we usually take it that the distance along the coordinate axes is following the distance in the sense that the coordinates run with unit speed w.r.t. the metric.

If we use this kind of usual coordinates, then we should conceive it, that potential lattices constructed from the projective space anharmonic ratio etc. would fluctuate wildly comparedto our coordinates.

6.3.4 Fluctuating Lattice

The above argument for that a lattice related to or defined from the imbedding space - the projective space say - would fluctuate wildly compared to our usual type of coordinates means, that if we should like in the type of model of the present work to imagine a physically existing lattice theory, then it would have to be with **wildly fluctuating lattice** compared to a usual type of coordinate system!

In other words our model - with the gravity comming in by spontaneous breaking - points to or supports a fluctuating lattice model.

Now recently [24] I published a work seeking to revive minimal $SU(5)$ grand unification, but **not taking the $SU(5)$ symmetry as more than an accidental symmetry** in the first approximation of a lattice theory with the smaller group the Standard Model Group, $S(U(2) \times U(3))$, only. In this work [24] the difference between the unification (approximately) scale and the Planck scale (gravity scale) gets even further from each other than in supersymmetry GUT theories.

So there appears a strong need in that work for having the fundamental energy scale for gravity and for approximate unification orders of magnitude different. In that work I proposed to achieve such a difference in scales by taking a "fluctuating lattice", i.e. that the in that paper truly existing lattice is supposed to fluctuate compared to usual coordinates wildly, so that in some superposition components somewhere the link is much smaller than in some other places or other components of the superposition.

But this means, that if you look for a quantity like a Yang-Mills coupling comming from a lattice term having the link length a to the -4th power and compare with a quantity like the Einstein Hilbert Lagrangian density going instead comming with the -6 th power of a, then you do not have to get the same average value of a dominating in the different cases. This would be a potential mechnism for making different fundamental lattice scales the relevant one for different interactions / purposes (i.e. say different for unification and for gravity).

We find - only very weak - pheomenological support for this fluctuating lattice idea, but as said above in the model imbedding our spacetime into e.g. projective space with the spontaneous breaking gravity, such a fluctuating lattice gets unavoidable, if we shall have a lattice at all.

We are hoping soon to publish an article, in which we as the main point compare the three wellknown (and even a fourth I shall ignore here) scales

- The **sew-saw-neutrino scale** (only very crudely)obtainable from neutrino oscillation data.
- The (minimal and approximate) grand unification $SU(5)$ scale.
- The **Planck scale**, which is of course really the scale associated to the gravitational constant G.

by finding that they are rougly lying on a logarithmic energy scale with differences as corresponding to their coupling dimensionalities. Since the see-saw -mass term has dimension of mass (to the first power) - Majorana neutrino Lagrange density -, and the Yang-Mills-coupling(fine structure constant inverted) is dimensionless (mass to zeroth power) and the Planck scale Einstein-Hilbert Coupling has dimesnion $mass^{-2}$,we shall predict, that the ratio of the Planck scale over the unification scale for the approximate $SU(5)$ shall be the square of the unification scale over the see-saw one.

Using as [24] $5.13 * 10^{13} GeV$ for the unification scale and $10^{11} GeV * 10^{\pm 5}$ for the see-sawscale, and Planck scale $1.9 * 10^{19} GeV$ we get for the square root

$$\sqrt{\frac{1.9 * 10^{19} geV}{5.13 * 10^{13} GeV}} = 6.0 * 10^{2} GeV \quad (22)$$

$$\text{which divided into } 5.13 * 10^{13} GeV \text{ gives} \quad 8.4 * 10^{11} GeV, \quad (23)$$

which is very close to estimated see-saw scale $10^{11} GeV * 10^{\pm 5}$.

Let us remark that even, if we let the very gravity fields - the vierbeins or $g^{\mu\nu}(x)$ - be given by variables on a lattice lying in the projective space and determined by anharmonic ratios fixing most of itsstructure, we still would get the lattice to fluctuate relative to usual type of **metric determined** coordinate choices.

What matters is rather whether we take all the degrees of freedom, of the vierbeins say, to be physical, so that the reparametrization-gauge degrees of freedom are real physical degrees of freedom, so that they can fluctuate.

7. cut off

So we can have a lattice cut off if we wish.

One shall note that in pure manifold space-time there is no structure useful for making a lattice in the same way as we argued above wecould use the anharmonic ratio.So if one wishes a cut off of some lattice type the projective space could be said to be slightly preferential.

8. Huge

The Hugeness of the Universe ?

Dirac wondered about the huge numbers of order 10^{20}, that e.g. the age of the universe is of the order of $(10^{20})^3$ time the Planck time.

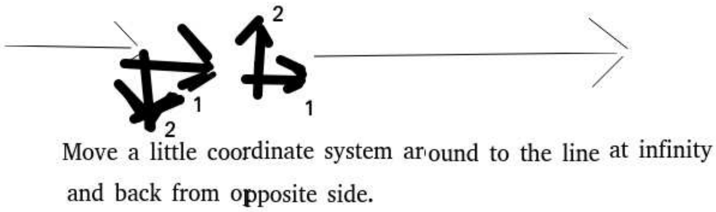
Assuming a **projective space** background for our space time could in an a priori unexpected way enforce the existence of very - infinitely - extended space time reagion(s)!

Argument goes:

- The projective space of even dimension is non-orientable.
- That enforces a hyper-surface, on which the $g^{\mu\nu}$ is is of rank one less - say for normal rank 4 it has 3 there.
- But then there $g_{\mu\nu} = \infty$.
- Appraoching this degeneracy surface the volume relative to the coordinates grow so much that an infinite universe in space and time pops out.

Non-orientability of Even Dimensional Projective space

Most easily seen in the even dimension $d = 2$.



.**The**

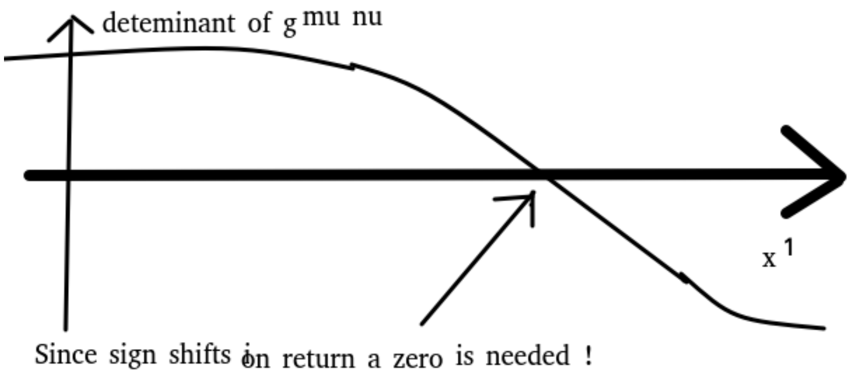
Needed $g^{\mu\nu}(x)$ must be degenrate along a 3-surface

The determinant $det(g^{\mu\nu})$ cannot avoid a zero surface of dimension 3 in a 4 dimensional projective space. The sign of this determinant namely represents an orientation.

Write it the coordinates chosen locally x^1, x^2, x^3, x^4 and in a certain order say 1,2,3,4. Then

$$\begin{aligned} \text{If } det(g^{\mu\nu}) &> 0, \text{ orientation is that of ordered coordinates} \\ \text{If } det(g^{\mu\nu}) &< 0, \text{orientation is opposite coordinates in their order} \end{aligned} \tag{24}$$

.**Think of the determinat $det(g^{\mu\nu})$ followed around to infinite line and back the other way**



We really needed upper index $g^{\mu\nu}$, so it must be "fundamentally" an effective (?) field

But the lower index ones $g_{\mu\nu}$ could just a definiton of an inverse.

I.e. the $g_{\mu\nu}$ with lower indices would just be the defined as the inverse

$$g_{\mu\nu} = (-1)^{\mu+\nu} \frac{det g^{\cdot\cdot}|_{\text{left out}_{\mu\nu}}}{det(g^{\cdot\cdot})} \tag{25}$$

So when $det(g^{\mu\nu}) = 0$ (genericly) all matrix elements of $g_{\mu\nu}$ go to infinity. And so near by all distances between the point in the projective space become huge.

It is the evendimensional $d = 2, 4, \ldots$ projective spaces which have the property of being non-orientable,while the odd dimensional ones $d = 1, 3, \ldots$ are orientable.

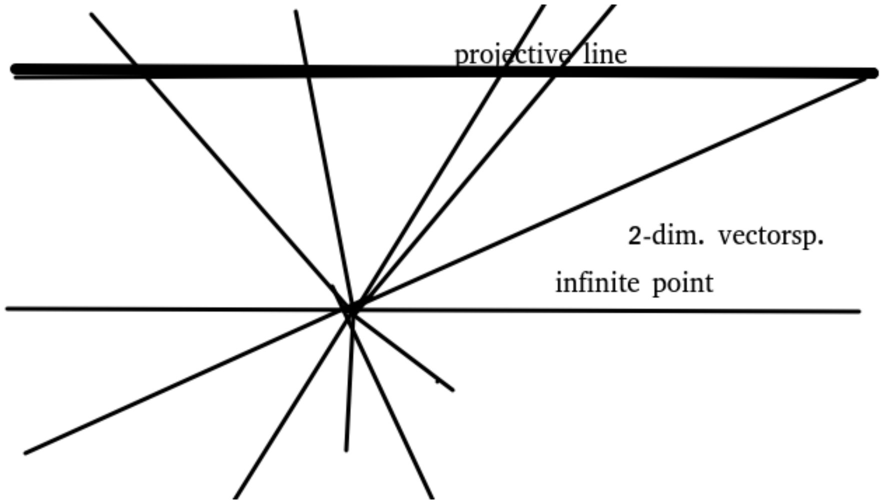


Figure 7: Here we illustrate with the case of the dimension of the projective space being $d = 1$, so that the vector space to use to construct the $d = 1$ projective space, meaning the projective line, is $d + 1 = 2$ dimensional, here the plane. The lines radiating out from the origo are classes of vectors equivalent under the equivalence relation, that $\vec{v} \sim \vec{y}$ if and only if there exists a real number r such that $\vec{v} = r\vec{y}$. We only define the equivalence relation $\sim$ for non-zero vectors and only take non-zero vectors in the classes. The classes are called rays.

9. Characterising Projective Space by Symmetry ?

Characterization of Projective line as 3-transitive

In Bled talk 2022 I presented a work with Masao Ninomiya[14], in which we showed that requiring for a group acting on space X in sharply 3-transitive way, essentially led you to the projective line (= a one dimensional projective space.)

Hope to somehow characterize projevtive spaces by some form of n-transitivity (may be next years talk?) **Projective line**

The projective line is the real axis extended with one point at infinity. **Projective space of d dimension as set of Rays in Vector space of $d + 1$**

II. Explaining Locality.

10. Astri Kleppes and mine Theorem:

10.1 Setting the Scene for Deriving Locality

Before we start telling in detail about the theorem by Astri Kleppe and me, that we can derive "locality" from mainly reparametrization invariance, let us define, what we here mean by 'locality":

Definiton: We call it that the (field) theory obeys locality, if the Action S in terms local field $\phi(x)$ is of the form

$$S = \int \mathcal{L}(x) d^4x, \quad (26)$$

where $\mathcal{L}(x)$ only depends on fields $\phi(x) = A_\mu(x), g^{\mu\nu}(x), etc.$ at the space-time point x and their derivatives up to finite order.

Idealy - and ignoring what is at first to be ignored - we derive this property of the laws of nature from the reparametrization invariance known from generel relativity.

But to appreciate that it is of course crucial, that the reader can at all imagine a theory or a world that did not have locality, let us remark:

We below just think of such possibilities of non-local theories by writting down a symbol $S[\phi]$ for an action and saying that **we do not assume locallity**. Now this means that, there are typically products present of all possible fields from completely different places (mixed up). That would mean that whenever we do at one point, it would woble fields and thus objects everywhere in the universe, and that we oppositely would be pushed and touched and disturbed by whatever happens wherever in the universe and that typically immediately (or even backward in time, or long time forward in time). The world of this type would probably be so hard to get an ordered picture of, that science at all might be very hard.

We should have in mind, that it is because the local form of the action, which we have phenomelogically, two states of the Universe only deviating in one little neighborhood can only in the future give rise to difference between the two states developping, which will have to continuously spread from the little region, where the difference started. If we have the usual type of reltativistic theories then the difference between such two states, only deviating in a narrow region, world states could only devlop with at most speed of light. (But of course if we had a non-local action there would be no such restriction usually.)

So now we set the scene by talking about such an action being as actions allways a functional of some fields, which for simplicity are all kept under the same field symbol $\phi(x)$ and which a priori is of the type with everything interacting with everything everywhere, because it is not local. But here comes then the assumptions we shall make instead of the locality - and then see how much locality they lead to in our derivation ?:

10.2 Meeting the Theorem by Astri and Myself

Even taking an action $S[\phi]$ depending on many fields defined over a space-time manifold **not to be a priori local at all** but only to obey

- 1. $S[\phi]$ is Taylor-expandable as a **functional**,
- 2. It is **symmetric** under the **diffeomorphism symmetry**,
- 3. We observe it only with so long wave lengths that only products of fields up to some limited dimension is observed,

then the effectively observed theory will have a weak form of locallity, in the sense, that the action will be observed as one of the form

$$S[\phi] \quad = \quad F(\int \mathcal{L}_1(\phi(x))d^dx, ..., \int \mathcal{L}_n(\phi(x))d^dx), \tag{27}$$

meaning the action functional $S[\phi]$ would be a **function** of a series of usual local action integrals $\int \mathcal{L}_i(\phi(x))d^dx$, but presumably not itself of this form.

The field $\phi(x)$ is a common name and can stand for fields with many different transformation properties under the diffeomorphism symmetry

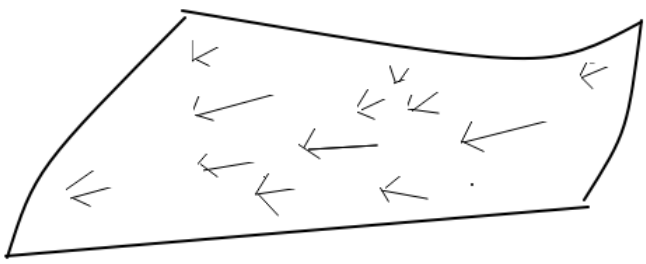


Figure 8: Just to illustrate by a picture we have drawn here a symbolic picture of a manifold by a two-dimensional example. The edges around it is just to draw something, but is misleading in the sense that we have in mind aclosedmanifold preferably,so that there are no edges. The arrows should illustrate a smoothly varying field of arrows,and each arrow should be an arrow from a point to its image under a diffeomorphism.

The field $\phi(x)$ is just short hand for any of the many fields we know (or do not even know) like

$$\phi(x) = g^{\mu\nu}(x), V_a^\mu(x), \varphi(x), A_\mu(x), \sqrt{g}, ... \tag{28}$$

or even combinations(products) of them with their various transformation properties under diffeomorphism symmetry.

With diffeomorphisms we think of bijective maps of the manifold on which the theory is defined onto itself having the property of being many times continuous differentiable.(see figure 8).

They are smooth deformations of the manifold M, say.

A Diffeomorphism transforms bijectively the Manifold and is continuos differentiable (some number of times)

We can likely (almost) do with less than full diffeomorphism symmetry

We believe the argument for locallity (which I still owe you) could go through with similar symmetry such as:

- A projective space time with as symmetry the projective maps of this space onto itself. (We shall give the reader a reminder about projective spaces in section 2.)
- The symplectimorphisms of a non-commutative space-time with a symplectic structure on it.(The symplectomorphisms are maps preserving the/a symplectic structure defined on the space. [5])
- Of course a true manifold with its diffeomorphisms is o.k.

But a Minkowski space time **with a distance** between two points that cannot be varied by the symmetry of the goemetry would **not** be suitable for our derivation of locality.

Essential is that you by the symmetry can move one point around even keeping another point fixed, so that the **only** kept information on the relative position of a pair of points is, if they coincide or not.

We did NOT get full/true Locality out: Only $S[\phi]$ of the Form $F(\int \mathcal{L}_1(x)d^dx, ..., \int \mathcal{L}_n(x)d^dx)$. (Here F is an ordnary function, not a functional, i.e. it is function of a number of (real) numbers; wile of course the action is a function*al*.)

With Random Dynamics derivations you are often **not quite** successful as here:

- **True locality:** $S[\phi] = \int \mathcal{L}(x)d^dx$.
- **But Only got:** $S[\phi] = F(\int \mathcal{L}_1(x)dx, ..., \int \mathcal{L}_n(x)dx)$

But that is precisely **interesting**, because then the suggestion is that nature may only have the **not quite** successful form of e.g. locality:

In fact it is suggested: **The coupling constants such as the fine structure constant or the Higgs mass or the cosmological constant "knows" about what goes on far away.**

Since the theorem did not truly lead to locally by the action being simply of the forn $S = \int \mathbf{L}(\phi(x), \partial_\mu\phi(x), ...)d^4x$ it means that even after imposing the diffeomorphism symmetry etc. assumed for the theorem, then we can still push and influence the whole world; but now with the special form $S = F(\int \mathbf{L}_1(\phi(x), \partial_\mu\phi(x), ...)d^4x, ..., \int \mathbf{L_n}(\phi(x), \partial_\mu\phi(x), ...)d^4x)$, we can only influence the world far away **via the coupling constants**, the fine structure constants, the Higgs mass, etc. and that only partly backward in time, arranged so that these couplings are translational invariant, and thus the same at all times. That is to say that if there were not full locality, but only the weak form procided by our theorem, then choices and decissions and what you do would influence the fine structure constant, say. But this will be very difficult to really check, because we do not know, what it would have been, if you had done something else than what you really did. Only if we could see some so remarkable numbers for some of the coupling constants so that we by some impressive coincidence could say that the values carried signs indicating that such a lack of locallity model was behind, could we come to support the model from that lack of full locallity. But precisely because the model with not-full-locallity is so hard to verify or disprove we may accept it as having **sufficient locallity to be acceptable as being possibly true**.

The Coupling Constants "knowing" about Remote Happenings is an Advantage

That the effective coupling constants or the cosmological constant etc. depend on integrals like $\int \mathcal{L}_i(x)d^dx$ gives at least **hope for solving finetuning problems** such as:

Why is the cosmological constant so phantastically small - in say Planck units -?

Now we can at least hope it is so small to make universe big or flat...**But if the coupling constant (say cosmological constant) did not "know" about the remote things (in space or in time), it could not adjust to it.**

Another fine tuning problem is: Why is the Higgs mass and thereby the weak scale so small compared to Planck scale or unification scale (if there were unification)?

11. Locality Derivation Argument

Reviewing our " Derivation of Locality"

Let us review Astri Kleppes and HBN's [3, 4] "derivation" of locality under the assumptions of **diffeomorphism symmetry** (invariance under reparamterizations) for a very general action S not being a priori local but rather only having the diffeomorphism symmetry and being **Taylor expandable** in **local fields**:

$$S[\phi] = \sum_n \frac{1}{n!} \sum (\text{or integral}) \frac{\partial}{\partial\phi(x_1)} \cdots \frac{\partial}{\partial\phi(x_n)} S[0]\phi(x_1)\cdots\phi(x_n),$$

where here ϕ stands for very general fields, possibly with many indices, and S **is an a priori non-local action.**

("Mild assumptions": Taylor-exapandability, Finite order Lagrange term only observed (low energy))

Setting for the Derivation of Locality of the Action $S[\phi]$

The field $\phi(x)$ can be so general, that it can stand for all the fields, we know, or do not know yet

$$\phi(x) = A_\mu(x), g^{\mu\nu}(x), \psi_\alpha(x), \ldots \tag{29}$$

x is a coordinate point, but in the spaces like manifold, or projective space, there is always a coordinate choice needed.

The action $S[\phi]$ is a functional of the fields ϕ and is assumed

- Taylor expandable (functional Taylor expansion)
- But **not assumed local**, since it is the point to derive/prove **locality**

The Taylor Expansion for Functional in Integral form

The **functional Taylor expansion** in the more functional notation (i.e. without imagining a lattice cut off say):

$$S[\phi] = \sum_n \frac{1}{n!} \int \frac{\delta}{\delta\phi(x_1)} \cdots \frac{\delta}{\delta\phi(x_n)} S[0]_{\mu\cdots\nu}\phi(x_1)\cdots\phi(x_n) dx_1^\mu \cdots dx_n^\mu.$$

Here the $\frac{\delta}{\delta\phi(x)}$ means functional derivative

The Crucial Point: All Points can by Symmetry (Difeomorphism symmetry) be brought into Any Other one, Trasitivity

When there is no distance a priori in the just manifold with diffeomorphism symmetry or the projective space, you at least, if you do not go to higher order interaction with several fields multiplied, **a field at one point will interact the same way with fields at any other point, except the very point itself.** Thus you either get interactions between all points, or interaction of the fields at the same point, i.e. locality.

Spelling a bit out the Functional Taylor Expansion

The **functional Taylor expansion** in the more functional notation:

$$S[\phi] = \sum_n \frac{1}{n!} \int \frac{\delta}{\delta\phi(x_1)} \cdots \frac{\delta}{\delta\phi(x_n)} S[0]_{\mu\cdots\nu}\phi(x_1)\cdots\phi(x_n) dx_1^\mu \cdots dx_n^\mu.$$

The symbol $\delta/\delta\phi(x)$ means functional derivative. (It is essentially partial derivative, but with a normalization so that it goes with being integrated over the space-time variable X_p by d^4x_i rather than just summed)

To split the telling a little bit up let us first tell what would happen, if we did not allow the functional derivative like $\frac{\delta}{\delta\phi(x_1)}\cdots\frac{\delta}{\delta\phi(x_n)}S[0]_{\mu\cdots\nu}$ to have distribtuion values meaning here delta-functions. But let us imediately tell that we think it is o.k. to have delta-functions in such functional derivatives. In fact we would get a divergens like that in a delta-function just by rewritting an ordinary derivative to a functional one, with normalization of the latter.

- **Only normal functions:** If we did not allow δ-functions so that we could get no contribution from cases where two of space-variables x_i and x_j say coincide, and if there were a symmetry like a translaton $x_i \rightarrow x_i + a$ for all vectors a the terms in the functional Taylor expansion could only depend on $\phi(x_j)$ via the intergal $\int \phi(x_j)dx_j$. So the whole taylor expanded action would be a function of such integrals $\int \phi(x)dx$. (for the type of symmetries we want to assume one could not even construct such integrals, except if $\phi(x)$ transform as a density-like $\sqrt{g}$)

  In the **functional Taylor expansion**:

$$S[\phi] = \sum_n \frac{1}{n!}\int \frac{\delta}{\delta\phi(x_1)}\cdots\frac{\delta}{\delta\phi(x_n)}S[0]_{\mu\cdots\nu}\phi(x_1)\cdots\phi(x_n)dx_1^{\mu}\cdots dx_n^{\mu}.$$

  we can **without delta functions** $\delta(..)$ only get the Taylor expanded (action) $S[\phi]$ to depend on integrals of the type

$$\int \phi(x)dx^{\mu} \quad \text{, which are invariant under the symmetries} \quad (30)$$

$$\text{(of a manifold say)} \quad (31)$$

  Denoting the fields among the $\phi(x)$-fields, which we remember were short notation for many different fields, which have transformations under diffeomorphisms as $\sqrt{g}$ by $\sqrt{g}_1$, $\sqrt{g}_2$, ψ_a,... $\sqrt{g}_n$ we would write this without delta-function result:

  The action must be of the form

$$S[\phi] = F(\int \sqrt{g}_1(x)d^4x, \int \sqrt{g}_2(x)d^4x, ..., \int \sqrt{g}_n(x)d^4x). \quad (32)$$

- **Allowing simple delta-function $\partial()$:** But if we allow δ-functions - in the functional derivatives of the functional to be exapanded $S[\phi]$, then one can get integrals allowed involving products of fields $\phi(x^{\mu})$ taken at the **same** point.

  What can occur, if we allow the δ-functions ?

  If we allow the $\delta(...)$ functions and require the symmetry of the diffeomorphism of the manifold, the integrals on which the being Taylor-expanded quantity $S[\phi]$ can depend, must be integrals **symmetric under the prescribed (diffeomorphism) symmetry** of **the form**

$$S_i[\phi] = \int \mathbf{L}_{i\ \mu\nu\rho\kappa}(x)dx^{\mu}\Lambda dx^{\nu}\Lambda\cdots\Lambda dx^{\kappa} \quad (33)$$

where now the $\mathbf{L}_i(\phi(x), ..., \phi(x))_{\mu\nu\rho\sigma}$ is a product of several of the $\phi(x)$ fields at the **same** point $x = x^\mu$. If these arguments for the $\phi(x)$ are not at the same point, then you can by the diffeomorphism symmetry transform them around separately and the integral would be required to **factorize** into integrals each only having fields from **one point** (kind of locallity).

- **Even Derivatives of delta functions∂_μ :** It may be more tricky to argue for that we even shall include derivatives of delta-functions as possible functional Taylor expansion coefficients, because points infinitesimally close to a given point also can by the tranformations be moved a bit around; but if you indeed can compensate for it by appropriate transformations of some of the fields represented as $\phi(x)$ but which actually has upper or lower indices. So when we can make by also involving derivatives of delta-functions contracted combinations,we are allowed to have them too. It is of course in such cases where one can get the transformations of the infinitesimal transformations of infinitesimally seperate points to compensate that one gets after the integration involved in the Taylor expansion expression derivatives acting on the field into the various Lagrangian density-like expressions $\mathbf{L}_{i;\mu\nu\cdot\rho}(x)$ for the various integral-expressions on which the function F depends.

  So with the derivatives of delta-functions we can get into these expressions on which the function F depends, which you get when you in the usualway look for a reparmetrization invariant action.

The $S_i[\phi] = \int \mathbf{L}_{i\ \mu\nu\rho\kappa}(x)dx^\mu \Lambda dx^\nu \Lambda \cdots \Lambda dx^\kappa$ **are ordinary local Actions, but...**

We did **not** derive that the action functional we discussed $S[\phi]$ was of the form $S_i[\phi] = \int \mathcal{L}_{i\ \mu\nu\rho\kappa}(x)dx^\mu \Lambda dx^\nu \Lambda \cdots \Lambda dx^\kappa$, but **only that it could only depend on the fields via such integrals**. So rather only the expanded action is a **function** of such integrals.

We derived a form of the Diffeomorphism Invariant Taylor Expandable Action $S[\phi]$ as a Function $F(S_1[\phi], S_2[\phi], ..., S_n[\phi])$ of usual *local* action integral.

Indeed the terms in the functional Taylor expansion will be of the form that groups of factors are at the same point inside the groups, and that then these point are integrated over all the space(manifold). Denoting the possible integral over local field combinations

$$S_i[\phi] \quad = \quad \int \mathcal{L}_{i\ \mu\nu\rho\kappa}(x)dx^\mu \Lambda dx^\nu \Lambda \cdots \Lambda dx^\kappa \tag{34}$$

we get the **form**

$$S[\phi] \quad = \quad F(S_1[\phi], ..., S_n[\phi]). \tag{35}$$

This form was studied by my student Stillits[13]

But this was not Really Locality!

The form, which we derived from the diffeomorphism invariance

$$S[\phi] \quad = \quad F(S_1[\phi], ..., S_n[\phi]) \tag{36}$$

is not truly local; we should have had a linear combination

$$S[\phi] \quad \overset{=}{wanted} \quad a_1 S_1[\phi] + ... + a_n S_n[\phi]. \tag{37}$$

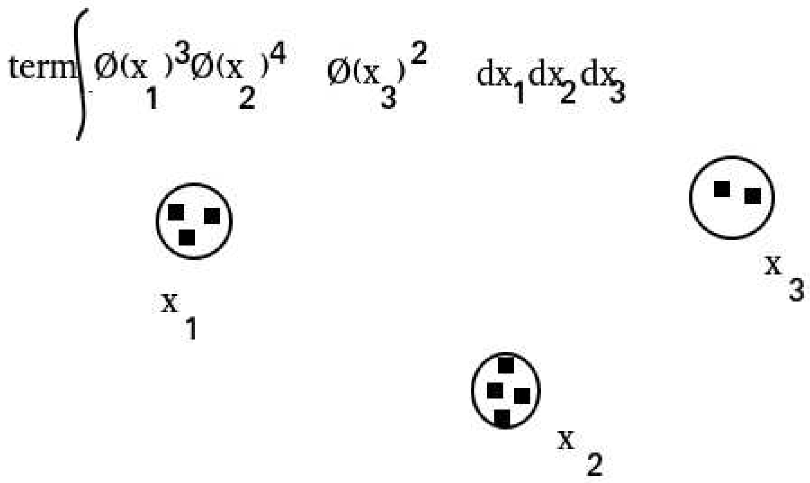


Figure 9: A typical term in the Taylor expansion for our functional $S[\phi]$ is an integral over the whole space-time of some fields $\phi(x)$ with a coefficient of the form $\frac{\delta}{\delta\phi(x_1)} \cdots \frac{\delta}{\delta\phi(x_n)} S[0]_{\mu\cdots\nu}$. This coefficientis typically a sum of many terms which are constant as functions of the variables except that there are in the various term a variable number of delta-functions. The figure concerns one such term for a coefficient depending at first on 9 *"the dimenssion" number of coordinates. But one of the several terms in this of $n = 9$ depending 6 times the dimension delta function factors, so that after integration it becomes the only 3 times the number of dimensions integral written on the figure. The figure illustrates by the small circles in which regions the contribution to this term comes in the sense that the plane is used toillustrate depending on for which point the corresponding space- time variables for that one of the 9 points.So the figure is somewhat symbolic in that sense only; We illustrate how a 9*4 dimensional integral with delta-functions can give a 3*4 dimensional one.

However, if we construct the **equation of motion** by putting the functional derivative of (38) to zero, we get the wanted form (37), as if the action had been a simple local one, an integralover all space time. The effective Lagrangian would just be some linear combinations of the Lagrangian-like functions found in the true full action S.

Equations of Motion got Already Local, but...

The equations of motion for an action of the form - derived from the diffeoemorphism symmetry

$$S[\phi] = F(S_1[\phi], ..., S_n[\phi]) \tag{38}$$

becomes

$$0 = \frac{\delta S[\phi]}{\delta\phi(x)} \tag{39}$$

$$= \sum_i \frac{dF}{dS_i}|_\phi * \frac{\delta S_i[\phi]}{\delta\phi} \tag{40}$$

$$= \frac{\delta}{\delta\phi} \sum_i \frac{dF}{dS_i}|_\phi * S_i[\phi] \text{ considering } \frac{dF}{dS_i}|_\phi = a_i \text{ constants}$$

We can say the equation of motion becomes of the form (37) provided we can count the coefficients as constants, and they are because the translationalsymmetry not broken by the gractity fields constants as a function of time. It is only that depend on some principle way on what goes on in the world, but since one in pracsis fit the couplings and just use their constancy in time and space, this dependence on what goes on the world is a rather meta-physical prediction; we shall though in next section about the MPP claim that we have a little sign that the type of theory we got to is indeed right.

The coefficients $\frac{dF}{dS_i}|_\phi$ do depend on the fields ϕ, but integrated over all time and all space. Effectively these coefficients

$$a_i = \frac{dF}{dS_i}|_\phi \tag{41}$$

to the various possible local actions $S_i[\phi]$ **do depend on the fields** ϕ but since they depend via integrals over all space time, we can in pracsis take them as constants. Indeed they are the **coupling constants** which we just fit to experiments. But it means that our lack of completing the derivation of locallity means:

The coupling constants - say fine structure constant etc. - depends on huge integrals over space time, although composed in a way which depends on the fundamental non-local action, which we do not know (yet?).

We did not get full locallity! Coupling constants depend on all space-time Only if we consider coefficients $a_i = \frac{dF}{dS_i}{}_\phi$ as constants did we get, that **the Lagrangian density only depends on the fields in the point you write this Lagrangean density**, and that is practical locality, **but we did not get locallity for the coupling constants in the sense that they with our derivation "know about" what goes on all over space and time, including even future.**

. This suggests that the question of what the **initial conditions** should be at least in principle needs an extra discussion.

In principle there is a back reation for any choice of initial condition, because it influences the couplings depeding on the development of it also to far future.

12. Would Projective Space-time Harm

Since we so much speculate on that the fundamental geometry instead of pure manifold structure could be a projective space,it would be nice a moment to think about how our derivation of locality would be challenged. For simplicity let us just consider the $d = 1$ dimensional projective space-time, or we shaould rather in this case have only spaceor time. Since we have projective in geometry the concept of the anharmonic ratio as already described in section 6.3.1, an action contribution that were an integral over four points of some fields multipliedby a function of theanharmonic ratio of the four points would be allowed. It would namely be invariant under the projective transformations. But such a term would at first not seem local!

Now, however, let us think about how an integral over all sets of four points with a fixed or given anharmonic ratio would look. Actually it is divergent, but the divergent contribution comes from cases where the configuration of the four points with the given anharmonic ratio is very small seen from the point of view of some (by spontaneous breaking achieved) metric. Either one might seek to repair the divergent Haar meamsure for the projective transformation group work in some only intutiv way, but at least the anharmonic ration is seen to be scale invariant relative to s simple metric, so that just simplifying by thinking of a Haar measure for a scaling group, you can argue: If I just concentrate on scaling with respectto a point, and seek a Haar measure for this group of scalings, then it would be a constant density measure in the logarithm of the size. Such a flat density in the logarithm diverges towards very large figures and very small ones. In projective space we should somehow compactify the very large figures as concentrated around the point at infinity, while the very small diverge towards the small size.

This kind of considerations might suggest that it is really very "few"(in the sense of small compare to divergent) configurations of four points with given anharmonic ratio that is not either so big that it is away at the point at infinite or so small that it is nolonger a thread to the locality in pracsis.

But of course our derivation of locality do not work strictly speaking if the projective space replacesthe manifold; but nevertheless the locality breaking terms tend to get their main contribution from configurations for the points at which the fields comming in are defines, which are effectively small in extension if one takes seriously divergent terms. This of course means that the non-local effects from having a projective space are suggested to be very small; approximately we still should get locality.

Further we should remark that to have any term breaking the locality due tosing the projective-geometry youneeded in 1 dimension four points at which to use the fieldvalues; but that means the lagrangian term youconstruct becomes a priori a product of at least four fields. It seems easily to make the condition of low dimensionality of the Lagragian density troublesome. If we go to say 4 spacetime dimensions the number of fields in the product needed(in the generic case) goes up to 6 field factors. It does not look so promissing for a usual type of renormalizable term.

You can see that the present speculation is that going from full diffeomorphism symmetry to only projective geometry symmeries will approximately still give locality in the same way as our theorem says.

13. MPP

Prediction of Severel Degenerate Vacua ("Mulitple Point Criticallity Principle")

This great importance of integrals over all space time of the fields could very easily lead to limitations for such overall space time integrals.

Such specification of a non-local (even in time) is analogous to **extensive quantities** in thermodynamics:

If you specify them you risk to put your system into a phase transition point.

If you specify several of them you easily end up with several phases in a ballance.

Here the analogues of the **intensive quantities** like temperature and chemical potentials are the coupling constants, which with our incomplete localliy derivation depend on what goes on or will go on or has gone on in the universe.

When Ice and Water in equilibrium Temperature 0^0

When one has the situation of slush - that there is both water and ice - then ones knows the temperature must be zero, cold but not extremely cold.

In the analogous way we have what we called **the Multiple Point Criticalallity Principle** when in space time one has several vacua in ballance taken to mean, that they have the same energy density. To find a good argument for this suggested principle we speculated that the some integrals of the type $S_i[\phi]$ got specified values analogously to fixing extensive quamtities in chemistry. Then it could easily be that the specified quantities could only be realized when there were indeed several (vauum) phases anlogous to the some specified combinations of a number of mols water, total energy and volume could enforce there to be sluch and may be even water wapor and the temperature and pressure could be enforced to be at the triple point.

With the Action derived from our locality derivation involving strongly the many integrals over all space time, one could easily imagine that by some way of getting a selfconsistent solution it could turn out that several of these integrals get so restricted, that one has such a situation similar to the slush one, that it was needed to have several phases of vacuum in space-time. And if they should be in equilibrium by having the same energy densities say, then coupling constants might end up in some critical point where the phases could coexist.

At least we can say, that since such integrals, as $S_i[\phi]$, appear in the form we argued for, seeking consitent solutions for the equations of motions could easily lead to restrictions. In fact to get consistent solution to not quite local equations of motion is not at all trivial. The time development, namely,influences the values of the integrals over space and time, thus influencing values for the effective couplings.

So it is not unlikely, that our not quite local action would lead to our earlier proposed **multiple point criticallity principle.**

This would be a success, if we could get the Multiple point criticallity principle out as extra premium from the attampt to derive locallity, because Colin Froggatt and I claim to have PREdicted the mass of the Higgs boson before the Higgs boson was found experimentally, by means of the multiple point criticallity principle. In fact in the article[26] Phys.Lett. B368 (1996) 96-102 following an arXiv-article submitted in Nov. 1995, we publiched the Higgs-mass prediction $135GeV \pm 9GeV$ also seen on the following painting:

We PREdicted the Higgs Mass by Several Balancing Vacua (MPP) before the Higgs was

found

The painting of me together with the Danish finance minister - whom I only met many years later - were printed in the 90's much before the Higgs was observed in LHC (=Large Hadron Collider) with 3 σ in 2012 and finally estblished in 2014. Nevertheless you can see the mass of the Higgs particle written as $135GeV \pm 10GeV$ (only the 1 is hidden behind Mogens Lykketofts head), but in our article in Phys. Lett. we have the 135 ± 10 GeV. (The measured mass turned out 125 GeV).

This we like to take as a support for the multiple point criticallity principle, and thus if this could be a consequence of the incompletely local action form for that even this form is being a little supported.

Actually it would be rather impossible to see how such phases with same energy density could come about in a world with complete locallity.

If namely one vacuum did not appear before after some time in the universe development - and that must be so because there were so hot in the beginning, that there were no vacuum proper anywhere - then how could any coupling constant or the Higgs mass adjust to make such a vacuum obtain a special value for its energy density say, when the vacuum had yet never existed ? At least it looks that some "non-locality" of this type must exist: Higgs mass or other parameters in the theory such as coupling constants and the cosmological constant must have been informed from the beginning about e.g. vacuum properties of vacua first existing long after.

This type of locality, which should preferably not be there to not give us mysteries of the type, how the couplings knew about vacua being degenerate say, is precisely the type of locality, we did **not** mannage to derive. We derived what is known right, but not the type that we would prefer not to be right, namely that coupling constants should not be influenced by the future either.

14. Conclusion.

- **Part I** Inspired from that we believe we could derive locallity from essentially reparametrization invariance (see part II), we put up a point of view that there is most fundamentally a back ground geometry which is just a manifold at first without further structure; then gravity comes in by having non-zero $g^{\mu\nu}(x)$ fields, or actually prefeably some vierbeins $V_a^\mu(x)$, being looked at as a spontaneous breakdown of the much higher symmetry of the pure manifold, all reparametrizations. This is meant to have psykological suggestion: We shall not stress gravity so much in building the great theory of everything, but just get the other and easier interactions right, and then we may hope for just seeing gravity popping out rather easily as a spontaneous break down ?!

But we stressed that such a spontaneous breaking representing gravity is strongly **needed**, because we without it will not be able to propagation of the (other) particles.

It also were suggested as an almost equally good model to instead to hav a projective space-time instead of the first mentioned possibility the pure manifold structure. It would only give some approximate locallity, but sufficient.

Then we delivered an extremely speculative phenomenological argument, that favoured the projective space-time. Indeed the projective space time contains as one of its deviations from usual geometry a "hyperplane at infinity", which is formally the collection of its "points", which are the classes of parallel lines (in usual Minkowski spacetime). Thus a single point on the "infinite hyperplane" is "seen" in two opposite directions, thus causing correslations due to the projective space-time (if true) in e.g. mricrowave back ground radiation. Including in a very speculative way that we considerthe wholespace-time and not only space as a projective space, we arrive at the rather simple qualitative result based on this projectivehypotesis:

For small l mainly the fluctuation in the intensity of the CMB should fluctuate more l being odd than for even l.

As you can read above the sign is a bit tricky - and was chocking for me - and requires too speculative assumptions.

We discuss also that a lattice model should be possible with a projective geometry. But our spontaneousbreaking based gravity would tend to fluctuate so much that we are led towards a "fluctuating lattice". A "fluctuating lattice" is an idea I recently published [**?**] and that can allow for the fundamental scale of the lattice as observed can depend on the dimensionality in powers of energy say $[m^{dim}]$ of the coeffient of the relavnt termin the lagrangian density. Most importantly you can take three different offers on the market for the fundamental scale:

- **The See-saw neutrino (mass) scale**
- **an approximate minimal $SU(5)$ unification scale**
- **Planck scale (= scale for quantum gravitation)**

and put their values as scales on alogarithmic axis. Then the differences between two of these three scale-values (which are somewhat depending on detailed models especially the see-saw scale) will be proportional to the difference in dimemsion of the coefficient to the "relevant term" in the Lagrangian density. (The coefficient to a see saw neutrino mass term has dimension [m], the unification relevant $(F^{mu\nu})^2$ Lagrange term is dimensionless [1] and the gravitational one has coefficent $[m^{-2}]$. Thus we predict from "fluctuating lattice" which in turn is favoured by gravity by spontaneous breaking

$$\frac{\mu_u}{\mu_{see-saw}} = \sqrt{\frac{\mu_{Pl}}{\mu_U}}, \tag{42}$$

$$\text{where } \mu_U = \text{“approximate (minimal) unification scale”} = 5.13 * 10^{13} GeV, \tag{43}$$

$$\mu_{Pl} = \text{“Planck scale} = 1.9 * 10^{19} GeV \tag{44}$$

$$\mu_{see-saw} = \text{“see-saw mass scale”} = 10^{11\pm 5} GeV, \tag{45}$$

and it fits quite well.

Smaller points in part I

We mentioned that the fact that a projective space in even dimensions is not orientable, could a priori be bad for our model,but might also enforce some zero along a three-dimensional manifold in the metric tensor with upper indices $g^{\mu\nu}(x)$ and that could enforce some very extended regions in space-time which could also be considered an advantage.

- **Part II** A major motivation for the suggestions of the present talk were the theorem by Astri Kleppe and myself, that we derived: **Principle of locality, that the action effectively is an integral over a local Lagrange density - only depending on fields defined at a single space-time point -.**

  From: **Diffeomorphism symmetry i.e. a manifold** and some milder assumptions, Taylor expandability, keeping only low dimensional terms.

  The locality-derivation were, however, not quite successful:

- A couple of not quite succeeding results turned out promissing:
  - Only getting a function form $S[\phi] = F(S_1[\phi], ..., S_n[\phi])$, where the $S_i[\phi]$ are truly local actions, meant: couplings could depend on what goes on all over and at all times, we could likely get our several vacua with same energy density as a consequence of this tiny lack of getting full locallity.
  - Would get superlocality and thus no propagation, unless we have some spontaneous breaking of diffeomorphism symmetry down to a metric space time, by **gravity fields**

  A phenomenological relation: We mentioned, that the occurence in the total action $S[\phi]$ of the integrals $S_i[\phi]$ over all space time of various (as Lagrangian densities usefull) combinations of fields easily can give rise to, that some of these integrals or combinations of them get essentially fixed for consistency reason and thus our derivation of locality with the lack of it being full in fact can induce the old speculation of ours of “mulitple point critically principle” of several vacua being degenerate. This principle had in fact a success of PREdicted the Higgs mass before it were observed.

Speculative Conclusion

Fundamentally we have for some reason a projective space or a manifold with diffeomorphism invariance - in any case a space-time with symmetry group acting in a practically n-transitive way with a high n - but then either a field $g^{\mu\nu}(x)$ or some corresponding vierbein fields $V_a^\mu(x)$ (also with upper curved indices) get non-zero in the vacuum. This makes possible propagation of waves/particles along the direction of the subspace of the tangent space spanned by this $g^{\mu\nu}(x)$ or these vierbeins $V_a^\mu(x)$ (Here we allowed for the possibility that the metric tensor with upper indices could be a degenerate matrix, so that the dimension of the space-time spanned could be lower than that of the manifold or the projective space in which the space time spanned get imbedded.) . So at the end the end **the Einstein general relativity four-space is imbedded into the more fundamental general manifold or projective space.**

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