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The $\Omega_c(3120)$ as a molecular state and its analogy with the $\Omega(2012)$

PoS(QNP2024)056

Natsumi Ikeno,[a,b,*] **Wei-Hong Liang**[c,d] **and Eulogio Oset**[e,c]

[a] *Graduate School of Maritime Sciences, Kobe University, Kobe 658-0022, Japan*

[b] *Department of Agricultural, Life and Environmental Sciences, Tottori University, Tottori 680-8551, Japan*

[c] *Department of Physics, Guangxi Normal University, Guilin 541004, China*

[d] *Guangxi Key Laboratory of Nuclear Physics and Technology, Guangxi Normal University, Guilin 541004, China*

[e] *Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia - CSIC, Institutos de Investigación de Paterna, Aptdo. 22085, 46071 Valencia, Spain*

E-mail: ikeno@maritime.kobe-u.ac.jp

We report theoretical results for the $\Omega_c(3120)$ state, observed by the LHCb collaboration, based on the molecular picture. The mass of the $\Omega_c(3120)$ state is reproduced from the s-wave $\Xi_c^*\bar{K}$ and $\Omega_c^*\eta$ channels. The width of the $\Omega_c(3120)$ is reproduced by introducing the d-wave $\Xi_c\bar{K}$ channel in addition to the other two channels. Furthermore, we calculate the partial decay widths of the $\Omega_c(3120)$ into $\Xi_c\bar{K}$ and $\pi\Xi_c\bar{K}$, following a discussion on the molecular nature of the $\Omega(2012)$ state. We find that the scattering length and effective range, along with the binding energy and width of the $\Omega_c(3120)$ state, can provide useful information for discussing the nature of the $\Omega_c(3120)$ state in the near future.

10th International Conference on Quarks and Nuclear Physics (QNP2024)
8-12 July, 2024
Barcelona, Spain

*Speaker

1. Introduction

The excited states of Ω_c were discovered by the LHCb Collaboration. In 2017, five states, $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$, and $\Omega_c(3119)$, were observed as narrow peak structures in the $\Xi_c K$ invariant mass distributions [1]. More recently, in 2023, two additional states, $\Omega_c(3185)$ and $\Omega_c(3327)$, were observed [2]. The Ω_c states have attracted a lot of interest from the theoretical community, and various studies have been carried out to understand the nature of these states from the quark model and hadronic molecular perspectives. In this article, we make a thorough study of the $\Omega_c(3119)$ state [3], one of these Ω_c states, which is called as $\Omega_c(3120)$ in PDG [4]. The mass and width of the $\Omega_c(3120)$ state are reported as [1]

$$\begin{aligned} M_{\Omega_c(3120)} &= 3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5} \text{ MeV}, \\ \Gamma_{\Omega_c(3120)} &= 1.1 \pm 0.8 \pm 0.4 \text{ MeV}. \end{aligned} \tag{1}$$

In the study of Ref. [5] based on the molecular perspective, the mass of the $\Omega_c(3120)$ with $J^P = 3/2^-$ is reproduced as a molecular state from mostly the $\Xi_c^* \bar{K}$ and $\Omega_c^* \eta$ channels, while the width is zero because the decay channel is not included and the width of Ξ_c^* is ignored. Therefore, in our study of Ref. [3], we introduce the d-wave $\Xi_c \bar{K}$ channel in addition to the s-wave $\Xi_c^* \bar{K}$ and $\Omega_c^* \eta$ channels to evaluate the width of the $\Omega_c(3120)$ state.

Since it is interesting to understand the nature of the hadronic state, as an example, the Belle Collaboration performed some tests on the molecular nature of the $\Omega(2012)$ state, which was reported as a new state in 2018 [6]. The $\Omega(2012)$ state with $J^P = 3/2^-$ is reproduced as a molecular state from the $\Xi^* \bar{K}$ and $\Omega\eta$ channels based on the molecular picture, e.g. [7, 8]. The experiment looked at the decay of the $\Omega(2012)$ state into $\pi\Xi\bar{K}$, a signal of the $\Xi^* \bar{K}$ component, and determined the ratio for the decay rate into $\pi\Xi\bar{K}$ versus $\Xi\bar{K}$ [9]. The recent analysis [9] concluded that the experimental data is consistent with the molecular interpretation of the $\Omega(2012)$ [7, 8].

In this article, we also discuss the $\Omega_c(3120)$ state by the analogy of the $\Omega(2012)$. We evaluate the partial decay widths of the $\Omega_c(3120)$ into $\Xi_c \bar{K}$ and $\pi\Xi_c\bar{K}$, which could be a measure of the molecular component $\Xi_c^* \bar{K}$ of the $\Omega_c(3120)$ state, since the information on the partial decay width was useful for discussion on the molecular nature in the study of the $\Omega(2012)$.

2. The $\Omega_c(3120)$ state based on the molecular picture

We assume that the $\Omega_c(3120)$ state has a spin of $J^P = 3/2^-$ as in Ref. [5] and we take into account three coupled channels to study the $\Omega_c(3120)$ state as

$$\Xi_c^* \bar{K} \ (1), \quad \Omega_c^* \eta \ (2), \quad \Xi_c \bar{K} \ (3). \tag{2}$$

The $\Xi_c^* \bar{K}$ and $\Omega_c^* \eta$ channels are in s-wave and the $\Xi_c \bar{K}$ channel is in d-wave. We solve the Bethe-Salpeter equation

$$T = [1 - VG]^{-1} V, \tag{3}$$

[1] In Ref. [2], the $\Omega_c(3120)$ width is changed to 0.60 ± 0.63 MeV. We perform the calculations with the data of Eq. (1). The conclusions do not change because our model has free parameters to fit the data.

to obtain the scattering matrix T of the coupled channels. The transition potential V for each channel is given by

$$V = \begin{pmatrix} F & \frac{4}{\sqrt{3}}F & \alpha q_{\rm on}^2 \\ \frac{4}{\sqrt{3}}F & 0 & \beta q_{\rm on}^2 \\ \alpha q_{\rm on}^2 & \beta q_{\rm on}^2 & 0 \end{pmatrix}, \tag{4}$$

with

$$F = -\frac{1}{4f^2}(k^0 + k'^0); \qquad q_{\rm on} = \frac{\lambda^{1/2}\left(s, m_{\bar{K}}^2, m_{\Xi_c}^2\right)}{2\sqrt{s}}, \tag{5}$$

with the pion decay constant $f = 93$ MeV, and k^0, k'^0 the energies of initial and final mesons respectively. The s-wave potentials between the $\Xi_c^* \bar{K}$ and $\Omega_c^* \eta$ channels are taken from Ref. [5]. The d-wave potential is introduced phenomenologically as done in Refs. [7, 8], and has two free parameters α and β to be fitted to the mass and width of the $\Omega_c(3120)$ state. The meson-baryon loop function G in Eq. (3) is given by the diagonal matrix $G = \mathrm{diag}(G_i)$. For the s-wave channels $i = \Xi_c^* \bar{K}, \Omega_c^* \eta$, the G_i is written as

$$G_i(\sqrt{s}) = \int_{|\vec{q}|<q_{\rm max}} \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_i(\vec{q})} \frac{M_i}{E_i(\vec{q})} \frac{1}{\sqrt{s} - \omega_i(\vec{q}) - E_i(\vec{q}) + i\epsilon}, \tag{6}$$

with $\omega_i(\vec{q}) = \sqrt{m_i^2 + \vec{q}^2}$, $E_i(\vec{q}) = \sqrt{M_i^2 + \vec{q}^2}$ and m_i, M_i the meson and baryon masses of the i channels. For the d-wave channel, $\Xi_c \bar{K}$, the G is written as

$$G_{\Xi_c \bar{K}}(\sqrt{s}) = \int_{|\vec{q}|<q_{\rm max}} \frac{d^3q}{(2\pi)^3} \frac{(q/q_{\rm on})^4}{2\omega_{\bar{K}}(\vec{q})} \frac{M_\Xi}{E_\Xi(\vec{q})} \frac{1}{\sqrt{s} - \omega_{\bar{K}}(\vec{q}) - E_{\Xi_c}(\vec{q}) + i\epsilon}. \tag{7}$$

The loop functions are regularized with the cut-off parameters $q_{\rm max}$ which are determined from the experimental data.

We make a fit to the experimental data of Eq. (1), the mass and width of the $\Omega_c(3120)$ state, by changing the parameters α, β and $q_{\rm max}$. The parameters we determined are

$$q_{\rm max} = 674.6\,\text{MeV}, \quad \alpha = 2.6 \times 10^{-8}\,\text{MeV}^{-3}, \quad \beta = 2.0 \times 10^{-9}\,\text{MeV}^{-3}. \tag{8}$$

With the parameters, we obtain the pole position at,

$$(3119.13 + i\,0.54)\,\text{MeV}. \tag{9}$$

The width is then given by 1.08 MeV. We can see that the mass and width we obtained are consistent with the values of the experiment. In Table 1, we show the couplings g_i and the probabilities we calculated for the $\Omega_c(3120)$ to different channels as well as those for the $\Omega(2012)$ state. In the $\Omega_c(3120)$ state, the $\Xi_c^* \bar{K}$ has the largest probability and it is around 63% and $\Omega_c^* \eta$ is around 10%. Therefore, we can say that the $\Omega_c(3120)$ is largely of molecular nature, and the $\Xi_c^* \bar{K}$ channel mostly dominates. For the comparison with the results of the $\Omega(2012)$ state, the couplings and the probabilities of $\Omega_c(3120)$ are very similar to those of the $\Omega(2012)$ state. It implies that $\Omega_c(3120)$ state is an analogous state to the $\Omega(2012)$ state.

PoS(QNP2024)056

Table 1: The couplings g_i for different channels in the $\Omega_c(3120)$ and $\Omega(2012)$, and the molecular probabilities $-g_i^2\frac{\partial G_i}{\partial\sqrt{s}}$ for the s-wave channels. The numbers of $\Omega_c(3120)$ and $\Omega(2012)$ are taken from Ref. [3] and Ref. [7], respectively.

	$\Omega_c(3120)$			$\Omega(2012)$		
	$\Xi_c^*\bar{K}$	$\Omega_c^*\eta$	$\Xi_c\bar{K}$	$\Xi^*\bar{K}$	$\Omega\eta$	$\Xi\bar{K}$
g_i	$2.06-i0.02$	$2.09-i0.01$	-0.138	$2.01+i0.02$	$2.84-i0.01$	-0.29
$-g_i^2\frac{\partial G_i}{\partial\sqrt{s}}$	0.63	0.10		0.64	0.16	

3. Partial decay width of the $\Omega_c(3120)$ into the $\Xi_c\bar{K}$ and $\pi\Xi_c\bar{K}$

Here, we consider the mechanism for the $\Omega_c(3120)\to\Xi_c^*\bar{K}\to\pi\Xi_c\bar{K}$ process as shown in Fig. 1. We evaluate the partial decay width of $\Omega_c(3120)$ state into the $\pi\Xi_c\bar{K}$ channel by the analogy with the $\Omega(2012)\to\Xi^*\bar{K}\to\pi\Xi\bar{K}$ process calculated in Ref. [8]. The mass distribution for the $\Omega_c(3120)$ decay into the three-body decay $\pi\Xi_c\bar{K}$ of the mechanism is given as

$$\frac{d\Gamma_{\Omega_c}}{dM_{\rm inv}(\pi\Xi_c)}=\frac{1}{(2\pi)^3}\frac{M_{\Xi_c}}{M_{\Omega_c}}\,p_{\bar{K}}\tilde{p}_\pi\,|t_{\Omega_c\to\pi\bar{K}\Xi_c}|^2, \tag{10}$$

with

$$p_{\bar{K}}=\frac{\lambda^{1/2}(M_{\Omega_c}^2,m_{\bar{K}}^2,M_{\rm inv}^2(\pi\Xi_c))}{2\,M_{\Omega_c}},\quad \tilde{p}_\pi=\frac{\lambda^{1/2}(M_{\rm inv}^2(\pi\Xi_c),m_\pi^2,M_{\Xi_c}^2)}{2\,M_{\rm inv}(\pi\Xi_c)}, \tag{11}$$

and we can obtain the width Γ_{Ω_c} by integrating over $M_{\rm inv}(\pi\Xi_c)$. The amplitude $t_{\Omega_c\to\pi\bar{K}\Xi_c}$ in Eq. (10) is written as

$$t_{\Omega_c\to\pi\Xi_c\bar{K}}=g_{\Omega_c,\bar{K}\Xi_c^*}\,\frac{1}{M_{\rm inv}(\pi\Xi_c)-M_{\Xi_c^*}-i\Gamma_{\Xi_c^*}/2}\,g_{\Xi_c^*,\pi\Xi_c}\,\tilde{p}_\pi, \tag{12}$$

where the coupling $g_{\Omega_c,\bar{K}\Xi_c^*}$ corresponds to the value taken from Table 1. The coupling $g_{\Xi_c^*,\pi\Xi_c}$ is given via the width $\Gamma_{\Xi_c^*}$ as

$$\Gamma_{\Xi_c^*}=\frac{1}{2\pi}\frac{M_{\Xi_c}}{M_{\Xi_c^*}}\,g_{\Xi_c^*,\pi\Xi_c}^2\,\tilde{p}_\pi^3, \tag{13}$$

and we consider the enegy dependence of the width Ξ_c^* as

$$\Gamma_{\Xi_c^*}(M_{\rm inv}(\pi\Xi_c))=\frac{M_{\Xi_c^*}}{M_{\rm inv}(\pi\Xi_c)}\left(\frac{q'}{q'_{\rm on}}\right)^3\Gamma_{\rm on}\,\theta(M_{\rm inv}(\pi\Xi_c)-m_\pi-M_{\Xi_c}), \tag{14}$$

with $\Gamma_{\rm on}=2.25$ MeV, the average width of Ξ_c^{*+} and Ξ_c^{*0} from PDG, and

$$q'_{\rm on}=\frac{\lambda^{1/2}(M_{\Xi_c^*}^2,m_\pi^2,M_{\Xi_c}^2)}{2\,M_{\Xi_c^*}},\quad q'=\frac{\lambda^{1/2}(M_{\rm inv}^2,m_\pi^2,M_{\Xi_c}^2)}{2\,M_{\rm inv}}. \tag{15}$$

As for the two-body decay width for the $\Omega_c(3120)\to\Xi_c\bar{K}$ process, we calculate

$$\Gamma=\frac{1}{2\pi}\frac{M_{\Xi_c}}{M_{\Omega_c}}\,g_{\Omega_c,\Xi_c\bar{K}}^2\,p'_{\bar{K}}, \tag{16}$$

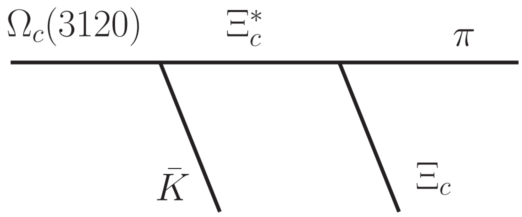

Figure 1: The $\Omega_c(3120) \to \Xi_c^* \bar{K} \to \pi \Xi_c \bar{K}$ decay process. This figure is taken from Ref. [3].

where the coupling $g_{\Omega_c,\Xi_c\bar{K}}$ is the value taken from Table 1, and $p'_{\bar{K}}$ is the $\bar{K}$ momentum for $\Omega_c \to \Xi_c \bar{K}$ decay in the Ω_c rest frame.

As a result, we obtain the three-body decay width for the $\Omega_c(3120) \to \Xi_c^* \bar{K} \to \pi \Xi_c \bar{K}$ process as

$$\Gamma_{\Omega_c \to \Xi_c \pi \bar{K}} = 0.03 \text{ MeV}, \tag{17}$$

while the two-body decay width for $\Omega_c(3120) \to \Xi_c \bar{K}$ process is found to be

$$\Gamma_{\Omega_c \to \Xi_c \bar{K}} = 0.90 \text{ MeV}. \tag{18}$$

The total width Γ_{Ω_c}, the sum of both widths, is around 1 MeV, which corresponds to the same as the central value of the experiment, Eq. (1). Thus, we obtain that the fraction of $\Omega_c(3120)$ decay into $\pi \Xi_c \bar{K}$ versus that into $\Xi_c \bar{K}$ is 3%. We find that this value is much smaller than that of the $\pi \Xi \bar{K}$ channel in the case of the $\Omega(2012)$, where the analogous fraction is around 96% The difference comes from the different bindings because of the interaction potential V for the $\Omega_c(3120)$ and $\Omega(2012)$ states. In the case of the $\Omega(2012)$ state, the diagonal terms in the V matrix are zero, while in the $\Omega_c(3120)$ state, they are finite and negative, which has extra attraction and produces a much bigger binding.

4. Scattering length and effective range for the $\Omega_c(3120)$ and $\Omega(2012)$

Since the determination of a fraction, $\Gamma_{\Omega_c \to \Xi_c \pi \bar{K}} = 0.03$ MeV, is very challenging with the present experimental errors, we calculate alternative observables here, such as the scattering length and effective range, which are nowadays determined experimentally using correlation functions. The scattering length and effective range of the different channels are written as [3]

$$-\frac{1}{a_j} = -\frac{8\pi\sqrt{s}}{2M_j} (T_{jj})^{-1}\bigg|_{\sqrt{s}_{\text{th},j}}, \tag{19}$$

$$r_{0,j} = \frac{1}{\mu_j} \frac{\partial}{\partial\sqrt{s}} \left[-\frac{8\pi\sqrt{s}}{2M_j} (T_{jj})^{-1} + ik_j \right]_{\sqrt{s}_{\text{th},j}}, \tag{20}$$

with $\sqrt{s}_{\text{th},j}$ the threshold mass of channel j, μ_j the reduced mass of channel j, and k_j the momentum of a particle of the pair in their rest frame. In Table 2, we show the results for the different channels in the $\Omega_c(3120)$ and $\Omega(2012)$ states. We can see that the values of the scattering length for the different channels in the $\Omega_c(3120)$ and $\Omega(2012)$ states are very similar. Therefore, we think that these new magnitudes, in addition to the binding energy and width of the $\Omega_c(3120)$, should be helpful to determine the nature of the $\Omega_c(3120)$ state.

Table 2: Scattering length a_j and effective range $r_{0,j}$ for the $\Xi_c^* \bar{K}$ and $\Omega_c^* \eta$ channels in $\Omega_c(3120)$ state, and for the $\Xi^* \bar{K}$ and $\Omega\eta$ channels in the $\Omega(2012)$ state. Numbers are taken from Ref. [3].

	$\Omega_c(3120)$		$\Omega(2012)$	
[fm]	$\Xi_c^* \bar{K}$	$\Omega_c^* \eta$	$\Xi^* \bar{K}$	$\Omega\eta$
a_j	$1.45 - i0.07$	$0.44 - i0.09$	$1.69 - i0.17$	$0.51 - i0.09$
$r_{0,j}$	$-0.08 - i0.01$	$0.26 + i0.01$	$-0.37 - i0.01$	$0.25 - i0.03$

5. Summary

We thoroughly studied the $\Omega_c(3120)$ state, which was reported in the LHCb experiment in Refs. [1, 2]. Based on the molecular picture, the mass of the $\Omega_c(3120)$ state with $J^P = 3/2^-$ is reproduced from the $\Xi_c^* \bar{K}$ and $\Omega_c^* \eta$ channels. The $\Xi_c^* \bar{K}$ state mostly dominates with 63%. We also reproduced the experimental width of $\Omega_c(3120)$ by introducing the $\Xi_c \bar{K}$ channel in addition to the two channels. Furthermore, we calculated the partial decay widths of the $\Omega_c(3120)$ into $\Xi_c \bar{K}$ and $\pi \Xi_c \bar{K}$, following a similar discussion on the molecular nature of the $\Omega(2012)$. Since we found that the decay width of the $\Omega_c(3120)$ into $\pi \Xi_c \bar{K}$ is much smaller compared to that of the $\Omega(2012)$ state, we calculated alternative observables, such as the scattering length and effective range. In conclusion, we believe that the information on the scattering length and effective range, together with the binding energy and width of the $\Omega_c(3120)$, could provide a more detailed information on the nature of the $\Omega_c(3120)$ state in the near future.

Acknowledgments

This work of N. I. was partly supported by JSPS KAKENHI Grant Number 24K07020.

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