

Recent results on hyperon decays at BESIII

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With the large datasets from e^+e^- annihilation at the J/ψ and $\psi(3686)$ resonances collected at the BESIII experiment, multi-dimensional analyses leveraging polarization and entanglement can provide new insights into the production and decay properties of hyperon-antihyperon pairs. Recent studies at BESIII have revealed significant transverse polarization of the (anti)hyperons in the decays J/ψ or $\psi(3686) \rightarrow \Lambda\bar{\Lambda}, \Sigma\bar{\Sigma}$, and $\Xi\bar{\Xi}$.

The decay parameters for the most common hadronic weak decay modes have been measured. Thanks to the non-zero polarization, the decay parameters for hyperons and antihyperons could be determined independently for the first time. Comparing these decay parameters allows for precise tests of direct $\Delta S = 1$ CP violation, complementing studies conducted in the kaon sector.

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1. Introduction

One of the fundamental mysteries in physics is the overwhelming abundance of matter compared to antimatter in the universe. According to current models, the Big Bang should have created equal amounts of both. The observed asymmetry is thought to have arisen through a process called baryogenesis [1], which requires the violation of charge conjugation (C) and combined charge-parity (CP) symmetry.

However, the amount of CP violation predicted by the Standard Model (SM) is insufficient to explain the observed matter-antimatter imbalance. This suggests the possibility of enhanced CP violation beyond the SM, which could provide crucial insights into the missing antimatter.

Direct CP symmetry tests can be conducted using polarized hyperons by comparing the angular distributions of hyperon and anti-hyperon decay products. Since CP-violating effects are expected to be small, high precision and large data samples are required. These precise CP tests can be performed in processes such as $e^+e^- \rightarrow J/\psi, \psi' \rightarrow B\bar{B}$. The BESIII collaboration [2], having collected the world's largest data sample from electron-positron annihilation, allows for stringent precision tests of CP symmetry. The current analyses are based on $1.3 \cdot 10^9 J/\psi$ and $4.5 \cdot 10^7 \psi'$ events, with more recent datasets comprising $10^{10} J/\psi$ and $3 \cdot 10^9 \psi'$ events.

2. Hyperon decays

The main decay modes of ground-state hyperons are weak $\Delta S = 1$ transitions, resulting in a baryon and a pseudoscalar meson. Historically, these decays played a key role in revealing the pattern of parity violation in weak interactions [3]. Today, they are essential for searching for CP violation signals in the baryon sector and for determining spin polarization in hadronic reactions involving hyperons.

In the weak decay of a spin-1/2 parent baryon (B) to a spin-1/2 daughter baryon (b) and a pion - such as $\Lambda \rightarrow p\pi^-$ or $\Xi^- \rightarrow \Lambda\pi^-$ - the parity-even amplitude produces a final state in the p -wave, while the parity-odd amplitude leads to a final state in the s -wave. These amplitudes, denoted as P and S respectively, can be described by two independent decay parameters [4]:

$$\alpha_D = \frac{2\text{Re}(S * P)}{|S|^2 + |P|^2} \quad \text{and} \quad \beta_D = \frac{2\text{Im}(S * P)}{|S|^2 + |P|^2}, \quad (1)$$

where $|S|^2 + |P|^2$ is the normalisation of amplitudes. The parameters provide the real and imaginary part of the interference term between the amplitudes. The experimentally motivated parameter is ϕ_D , $\phi_D \in [-\pi, \pi]$, which is related to the rotation of the spin vector between mother and daughter baryons. For $\Xi^- \rightarrow \Lambda\pi^-$ decay with polarized cascade the ϕ_D parameter can be determined using the subsequent $\Lambda \rightarrow p\pi^-$ decay which acts as a polarimeter. The relation between β_D and ϕ_D parameters is $\beta_D = \sqrt{1 - \alpha_D^2} \sin \phi_D$. The decay parameter α_D , $\alpha_D \in [-1, 1]$, can be determined from the angular distribution asymmetry of the b baryon in the B baryon rest frame. The distribution is given as

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\Omega} = \frac{1}{4\pi} (1 + \alpha_D \mathbf{P}_B \cdot \hat{\mathbf{n}}), \quad (2)$$

where \mathbf{P}_B is the B baryon polarization vector and $\hat{\mathbf{n}}$ is the direction of the b baryon momentum in the B baryon rest frame. In the CP-conserving limit the hyperon-antihyperon average values can

be defined as $\langle \alpha_D \rangle = (\alpha_D - \bar{\alpha}_D)/2$ and $\langle \phi_D \rangle = (\phi_D - \bar{\phi}_D)/2$. Experimentally, two independent CP-violation tests based on comparison of the decay parameters in the hyperon and anti-hyperon processes are possible [5]

$$A_{\text{CP}}^D = \frac{\alpha_D + \bar{\alpha}_D}{\alpha_D - \bar{\alpha}_D} \quad \text{and} \quad \Phi_{\text{CP}}^D = \frac{\phi_D + \bar{\phi}_D}{2}. \quad (3)$$

The two-body hyperon decay can be described using decay matrices $a_{\mu\nu}^D$, representing the transformations of the spin operators (Pauli matrices) σ_μ^B and σ_ν^b defined in the B and b baryon rest frames, respectively [6]:

$$\sigma_\mu^B \rightarrow \sum_{\nu=0}^3 a_{\mu\nu}^D \sigma_\nu^b. \quad (4)$$

The elements of such 4×4 matrices are parameterized in terms of the decay parameters α_D and ϕ_D and depend on the helicity angles.

3. Production of baryon-antibaryon pairs and joint angular distributions

Thanks to their relatively large branching fractions [7] and low hadronic background, the processes $e^+e^- \rightarrow J/\psi, \psi' \rightarrow B\bar{B}$ are ideal for determining hyperon decay properties and conducting CP-violation tests. Two analysis approaches can be employed: an exclusive method, where both the baryon and anti-baryon decay chains are fully reconstructed, and an inclusive method, where only one decay chain - either the baryon or the anti-baryon - is reconstructed.

The significance of single-step weak decays, such as $\Lambda \rightarrow p\pi^-$, lies in the fact that both the Λ and $\bar{\Lambda}$ are produced with transverse polarization. This polarization, along with spin correlations between the baryon and anti-baryon, allows for a simultaneous determination of the decay parameters α and $\bar{\alpha}$ [8], which are essential for CP symmetry tests.

To describe the baryon-antibaryon pair production in electron-positron annihilation including the two-body sequential decay processes a modular approach [6] can be used. The general expression for the joint density matrix of the $B\bar{B}$ pair is:

$$\rho_{B\bar{B}} = \sum_{\mu, \nu=0}^3 C_{\mu\nu} \sigma_\mu^B \otimes \sigma_\nu^{\bar{B}}, \quad (5)$$

where a set of four Pauli matrices $\sigma_\mu^B(\sigma_\nu^{\bar{B}})$ in the $B(\bar{B})$ rest frame is used and $C_{\mu\nu}$ is 4×4 real matrix representing polarizations and spin correlations. It describes the spin configuration of the entangled hyperon-antihyperon pair in their respective helicity frames. The coefficients $C_{\mu\nu}$ depend on the angle θ between the positron and baryon B . The structure of the $C_{\mu\nu}$ 4×4 matrix can be represented by polarization vector

$$P_y(\theta) = \frac{\sqrt{1 - \alpha_\psi^2} \sin \theta \cos \theta}{1 + \alpha_\psi \cos^2 \theta} \sin(\Delta\Phi) \quad (6)$$

and spin correlations $C_{ij}(\theta)$. The α_ψ and $\Delta\Phi$ are two real parameters describing the angular distributions of the baryon-antibaryon pair production. The complete joint angular distribution of

$J/\psi \rightarrow B\bar{B}$ with a single-step decay of hyperon and anti-hyperon is

$$\mathcal{W}(\xi; \omega) = \sum_{\mu, \nu=0}^3 C_{\mu\nu} a_{\mu 0}^D a_{\nu 0}^{\bar{D}}. \quad (7)$$

The vector $\xi = (\theta, \theta_b, \varphi_b, \bar{\theta}_b, \bar{\varphi}_b)$ represents a complete set of helicity angles with the complete global parameter vector $\omega = (\alpha_\psi, \Delta\Phi, \alpha_D, \bar{\alpha}_D)$.

The production and the two-step decays in the $e^+e^- \rightarrow J/\psi \rightarrow \Xi^-\bar{\Xi}^+$ are described by a nine-dimensional vector of the helicity angles, $\xi = (\theta, \theta_\Lambda, \varphi_\Lambda, \theta_p, \varphi_p, \bar{\theta}_\Lambda, \bar{\varphi}_\Lambda, \bar{\theta}_p, \bar{\varphi}_p)$. The structure of the nine-dimensional angular distribution is determined by eight global parameters $\omega_\Xi = (\alpha_\psi, \Delta\Phi, \alpha_\Xi, \phi_\Xi, \alpha_\Lambda, \bar{\alpha}_\Xi, \bar{\phi}_\Xi, \bar{\alpha}_\Lambda)$ [6].

4. Experimental measurements

4.1 $e^+e^- \rightarrow J/\psi \rightarrow \Lambda\bar{\Lambda}$

A recent update from BESIII reports polarization measurements in the process $e^+e^- \rightarrow J/\psi \rightarrow \Lambda\bar{\Lambda} \rightarrow p\pi^-\bar{p}\pi^+$, based on a dataset of approximately $3.2 \cdot 10^6$ candidates [9]. A clear polarization-dependent signal was observed in the decay directions of both Λ and $\bar{\Lambda}$. The phase between helicity-flip and helicity-conserving transitions was measured to be $\Delta\Phi = (43.11 \pm 0.24 \pm 0.46)^\circ$, leading to a transverse polarization P_y reaching a maximum of 25%. The value of the Λ decay parameter $\langle\alpha_\Lambda\rangle$ was found to be $0.7542 \pm 0.0010 \pm 0.0020$, which shows a 17% deviation from the world average of $\alpha_\Lambda = 0.642 \pm 0.013$, established 40 years ago [10]. Additionally, the CLAS experiment [11], which re-analyzed spin data from the process $\gamma p \rightarrow \Lambda K^+$, reported a value of $\alpha_\Lambda = 0.721 \pm 0.006 \pm 0.005$. Although closer, this result is still inconsistent with the BESIII measurement, which presents a discrepancy that requires further investigation.

4.2 $e^+e^- \rightarrow J/\psi, \psi' \rightarrow \Sigma^+\bar{\Sigma}^-$

The decay parameter α_Σ for the process $\Sigma^+ \rightarrow p\pi^0$ was historically based on experiments from 50 years ago [12–14] until the BESIII measurement in 2020 [15]. A similar measurement using $\Sigma^+ \rightarrow n\pi^+$ was recently conducted [16]. The process $e^+e^- \rightarrow J/\psi, \psi' \rightarrow \Sigma^+\bar{\Sigma}^-$ is useful for studying quantum entangled spin correlations, as the large $|\alpha_\Sigma|$ enhances sensitivity. The $\alpha_{J/\psi}$ parameter is negative, while $\alpha_{\psi'}$ is positive, with opposite phase signs between helicity amplitudes.

CP-odd observables, $A_{\text{CP}}^{\Sigma^+} = -0.004 \pm 0.037 \pm 0.010$ and $A_{\text{CP}}^{\Sigma^0} = -0.080 \pm 0.052 \pm 0.028$, were measured for the first time and align with Standard Model predictions [17]. Additionally, the radiative hyperon decay $\Sigma^+ \rightarrow p\gamma$ was studied at an e^+e^- collider for the first time [18]. No CP violation signal was found, and the branching fraction of $\Sigma^+ \rightarrow p\gamma$ is lower than the world average by 4.2σ . The decay asymmetry parameter $\alpha_\gamma = -0.651 \pm 0.056 \pm 0.020$ is consistent with the world average within 1.1σ [19].

4.3 $e^+e^- \rightarrow J/\psi \rightarrow \Xi\bar{\Xi}$

The analysis results for the process $J/\psi \rightarrow \Xi^-\bar{\Xi}^+ \rightarrow (\Lambda \rightarrow p\pi^-)\pi^-(\bar{\Lambda} \rightarrow \bar{p}\pi^+)\pi^+$ have been recently published [20]. The study measured the weak decay parameters for $\Xi^- \rightarrow (\Lambda \rightarrow p\pi^-)\pi^-$ along with production-related parameters α_ψ and $\Delta\Phi$. Comparing the decay parameters for baryons

and anti-baryons enabled three independent tests of CP symmetry: A_{CP}^{Ξ} , A_{CP}^{Λ} and Φ_{CP}^{Ξ} , with asymmetries for Ξ decay measured for the first time [20].

The BESIII result for $\langle\phi_{\Xi}\rangle$ shows similar precision to the HyperonCP result [21], with $\phi_{\Xi} = -0.042 \pm 0.016$ rad; however, the two values differ by 2.6 standard deviations. The measurement of $\langle\phi_{\Xi}\rangle$ leads to a determination of the strong phase difference $\delta_P - \delta_S$ of $(-4.0 \pm 3.3 \pm 1.7) \cdot 10^{-2}$ rad, which is consistent with heavy-baryon chiral perturbation theory predictions of $(1.9 \pm 4.9) \cdot 10^{-2}$ rad [17]. The weak phase difference $(\xi_P - \xi_S) = (1.2 \pm 3.4 \pm 0.8) \cdot 10^{-2}$ rad aligns with the Standard Model predictions of $(1.8 \pm 1.5) \cdot 10^{-4}$ rad [17]. This analysis represents one of the most precise tests of CP symmetry for strange baryons and the first direct measurement of the weak phase for any baryon.

A new result has been published this year regarding the charged $\Xi^{-}\bar{\Xi}^{+}$ pair process with anti-neutron in the final state, along with its charge conjugate channel [22]. The findings from this final fit are consistent with earlier measurements from the proton-antiproton final state channel [20].

The ratio of decay asymmetry parameters $\alpha_0/\alpha_{\Lambda}$ for the decay $\Lambda \rightarrow n\pi^0$ to that of $\Lambda \rightarrow p\pi^{-}$ was determined to be $0.877 \pm 0.015_{-0.010}^{+0.014}$, and for the anti-hyperon, $\bar{\alpha}_0/\bar{\alpha}_{\Lambda} = 0.863 \pm 0.014_{-0.008}^{+0.012}$. This ratio is significantly less than unity, with a deviation of more than 5σ , indicating the presence of the $\Delta I = 3/2$ transition in Λ decay for the first time.

To investigate the relationship between protons and neutrons, two additional tests were conducted. First, if CP is conserved, the product of the decay asymmetry parameters $\alpha_{-}\alpha_{\Xi}$ and $\alpha_{+}\bar{\alpha}_{\Xi}$ should be equal. Additionally, the ratios of helicity angular distributions for different nucleons in the final states, defined as $R(\cos\theta_i, \cos\theta_{\bar{i}}) = (1 + \alpha_{-}\alpha_{\Xi}\cos\theta_i)/(1 + \alpha_{+}\bar{\alpha}_{\Xi}\cos\theta_{\bar{i}})$ with $i = \{p, n\}$, should be flat and equal to unity. The accuracy of the CP symmetry test is enhanced by using the isospin average for $R_1 = (1 + \alpha_{\Lambda}\alpha_{\Xi}\cos\theta)/(1 + \bar{\alpha}_{\Lambda}\bar{\alpha}_{\Xi}\cos\theta)$, where $\cos\theta$ represents the helicity angle of the nucleon and $\alpha_{\Lambda} = (2\alpha_{-} + \alpha_0)/3$.

In a similar manner, if there is no $\Delta I = 3/2$ transition in Λ decay, then α_{-} should equal α_0 , resulting in flat ratios for nucleons and anti-nucleons: $R(\cos\theta_n, \cos\theta_p) = (1 + \alpha_0\alpha_{\Xi}\cos\theta_n)/(1 + \alpha_{\Lambda}\alpha_{\Xi}\cos\theta_p)$ and $R(\cos\theta_{\bar{n}}, \cos\theta_{\bar{p}}) = (1 + \bar{\alpha}_0\bar{\alpha}_{\Xi}\cos\theta_{\bar{n}})/(1 + \bar{\alpha}_{\Lambda}\bar{\alpha}_{\Xi}\cos\theta_{\bar{p}})$. The accuracy of the $\Delta I = 1/2$ rule tests can be improved using the average of the decay symmetry parameters for hyperon and anti-hyperon to define $R_2 = (1 + \langle\alpha_0\rangle\langle\alpha_{\Xi}\rangle\cos\theta)/(1 + \langle\alpha_{\Lambda}\rangle\langle\alpha_{\Xi}\rangle\cos\theta)$.

To illustrate the CP symmetry test and the $\Delta I = 1/2$ rule, two ratios of helicity angular distributions for different nucleons and anti-nucleons in the final states are presented in Fig. 1. The ratios obtained from fitting events in bins of $\cos\theta$ are in good agreement with the global curves for R_1 (Fig. 1(a)), indicating that the nearly flat distribution of R_1 is consistent with CP conservation. Conversely, the sloping distribution of R_2 (Fig. 1(b)) suggests the presence of the $\Delta I = 3/2$ transition contribution in Λ decay.

The ratio of $\Delta I = 3/2$ to $\Delta I = 1/2$ transitions in S -wave was determined to be $0.0349 \pm 0.0017_{-0.0013}^{+0.0012} \pm 0.0047$, while in P -wave it was measured at $-0.0752 \pm 0.0078_{-0.0062}^{+0.0067} \pm 0.0044$ [23]. The first uncertainties are statistical, the second are systematic, and the third arise from input parameters. The S -wave ratio is consistent with $K \rightarrow \pi\pi$ results within uncertainty, while the P -wave ratio, measured for the first time, differs from that in the S -wave. This measurement constrains lattice QCD [24] and the dual QCD approach [25] to better understand the $\Delta I = 1/2$ rule.

Recent studies have focused on the decay process $J/\psi \rightarrow \Xi^0\bar{\Xi}^0 \rightarrow (\Lambda \rightarrow p\pi^{-})\pi^0(\bar{\Lambda} \rightarrow \bar{p}\pi^{+})\pi^0$ [26]. This research includes the first measurement of the polarization of Ξ^0 in sequential

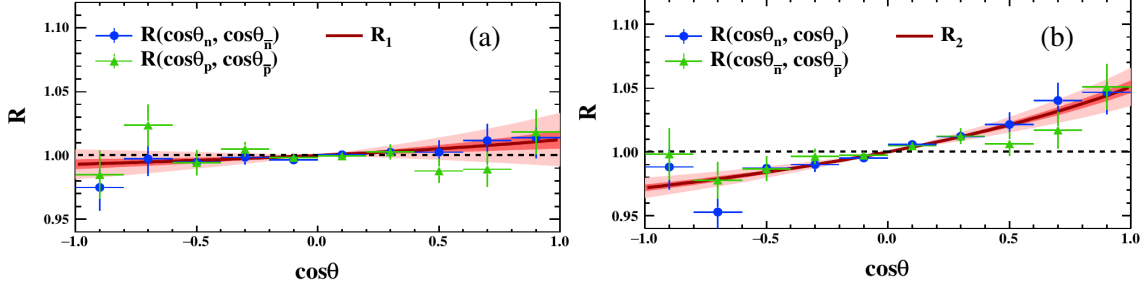


Figure 1: The ratios of helicity angular distributions for different nucleons in the final states, (a) $R(\cos\theta_p, \cos\theta_{\bar{p}})$ and $R(\cos\theta_n, \cos\theta_{\bar{n}})$ as well as (b) $R(\cos\theta_n, \cos\theta_p)$ and $R(\cos\theta_{\bar{n}}, \cos\theta_{\bar{p}})$ versus $\cos\theta$. The dots with errors are determined by independent fits for each $\cos\theta$ bin of the corresponding nucleons. The solid curves in red with 1σ (red) and 3σ (pink) statistical uncertainty bands show the results of the simultaneous fit. The dashed curves in black show the CP-conserving and no $\Delta I = 3/2$ transition expectations.

decays and provides significantly improved measurements of all decay parameters for Ξ^0 , Λ , and their charge conjugated decays, enhancing precision by more than an order of magnitude compared to previous studies [9, 27].

The weak phase difference result was found to be $(\xi_P - \xi_S) = (0.0 \pm 1.7 \pm 0.2) \cdot 10^{-2}$ rad. Additionally, two independent tests for CP symmetry yielded the results $A_{CP} = (-5.4 \pm 6.5 \pm 3.1) \cdot 10^{-3}$ and $\Delta\Phi_{CP} = (-0.1 \pm 6.9 \pm 0.9) \cdot 10^{-3}$ rad. These findings contribute to a deeper understanding of cascade hyperon decays and CP violation.

4.4 $e^+e^- \rightarrow J/\psi, \psi' \rightarrow \Sigma^0 \bar{\Sigma}^0$

The processes $e^+e^- \rightarrow J/\psi, \psi' \rightarrow \Sigma^0 \bar{\Sigma}^0 \rightarrow (\Lambda \rightarrow p\pi^-)\gamma(\bar{\Lambda} \rightarrow \bar{p}\pi^+)\gamma$ have been analyzed using the complete data samples collected with the BESIII detector [28]. This study represents the first test of strong CP symmetry in the decays of Σ^0 hyperons, measuring the decay parameters $\alpha_{\Sigma^0} = -0.0017 \pm 0.0021 \pm 0.0018$ and $\bar{\alpha}_{\Sigma^0} = 0.0021 \pm 0.0020 \pm 0.0022$.

Additionally, for the first time, transverse polarizations of Σ^0 hyperons in J/ψ and ψ' decays were observed to point in opposite directions. This behavior was also noted in the decay of Σ^+ and $\bar{\Sigma}^-$ hyperons with a $p\pi^0$ final state [15], but not in the decays of charged and neutral Ξ hyperons [20, 26].

Theoretical predictions [29] suggest that this discrepancy arises from a difference in the relative phase between S - and D -waves. The phase difference between the two neutral Σ pairs from J/ψ and ψ' is similar to that between the charged Σ pairs, both being approximately equal to π . If this observation is not coincidental, further investigation into the underlying mechanisms will be warranted.

5. Summary and Outlook

Hyperons serve as a powerful diagnostic tool for studying the strong interaction and fundamental symmetries. Exclusive measurements of polarized and entangled hyperon-antihyperon pairs provide access to information that is challenging or impossible to obtain through other processes. Recent studies by the BESIII collaboration have examined the structure and decay of the single-strange Λ

hyperon with unprecedented precision, including the first direct measurement of the weak phase difference using multi-strange Ξ hyperon decay. Ongoing research into sequentially decaying charmed baryons aims to disentangle strong and weak decay phases. With a world-record data sample of $10^{10} J/\psi$ and $3 \cdot 10^9 \psi'$ events, these studies have the potential to elevate hyperon physics to a new level.

In 2024-2025, the BEPCII will be upgraded to optimize the energies at the charmed baryon-antibaryon pair threshold, enabling the collection of additional data and the study of highly polarized hyperons, which will originate from charmed baryon decays.

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