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Hadron interactions using three-quark potential in a constituent quark model

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In this work, we investigate the hadron interactions using the three-quark potential in a constituent quark model. Three-quark potentials have only been studied for simple cases because it is difficult to calculate the three-color interaction matrix and determine the radial dependence of potential. In the case of a multiquark system, the three-quark color matrix can be calculated using the permutation matrix, but when there are antiquarks, a different method must be used. In order to calculate the three-quark potential, we use the commutation and anticommutation relations of SU(3) and apply it to exotic hadron configurations.

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1. Introduction

Nuclear three-body forces are well established and essential for accurately reproducing the binding energy of few-nucleon systems[1, 2]. The three-body forces among quarks have also been discussed[3], specifically the three-color matrix types proportional to SU(3) structure constants f^{abc} and d^{abc}[4]. The f^{abc} type arises from three-gluon interactions in Fig.1 a), but does not contribute to baryon mass because it is not SU(3) invariant.

The d-type does contribute to the baryon mass, with its strength constrained by the stability of the baryon mass. The effects of this interaction on the nucleon mass and on the stability of exotic states have been discussed in previous studies [5, 6]. However, this d-type factor contriutes uniformly to all the baryon states in the color singlet representation and hence does not improve the fitting results to the flavor-dependent masses.

At the same time, it is known that the quark masses required to describe the baryon spectrum differ from those used in the meson spectrum in a simple quark model[7]. Also, the naive extrapolation of the color-spin interaction in the meson and baryon systems does not seem to follow the necessary changes in the color-spin matrix elements[8]. These observations imply that additional effects need to be considered when transitioning from a two-quark to a three-quark system. The inclusion of the quark three-body interaction naturally emerges as a necessary addition.

In this work, we investigate the hadron interactions using three-quark potentials in exotic hadron configurations. In our previous work[9], we derived the three-quark potentials using the wisdom from nuclear physics, where the three-body interaction originates from two-pion exchanges with an intermediate delta[10]. If the pions and nucleons are replaced by gluons and quarks, as shown in Fig.1 b) and c), we can obtain the three-quark potential with d-type three-color factor. These interactions are crucial for understanding the hadron interactions in compact configurations.

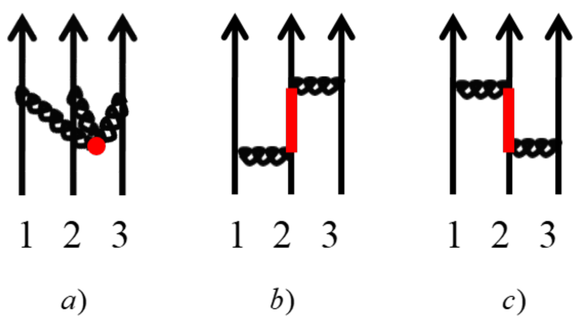

Figure 1: Quark three-body interaction from a) three-gluon interaction and b), c) two-gluon exchange with excited quark intermediate state.

2. Three-quark potentials

Let us now consider the three-quark potentials. In Ref.[9], we derived the form of the three-quark potentials using the diagrams in Fig.1 as follows.

$$\begin{aligned}
L_{123}^{C-C} &= \frac{4}{3}\left(\frac{\lambda_2^c\lambda_3^c}{m_1}+\frac{\lambda_1^c\lambda_3^c}{m_2}+\frac{\lambda_1^c\lambda_2^c}{m_3}\right)+2d^{abc}(\lambda_1^a\lambda_2^b\lambda_3^c)\left(\frac{1}{m_1}+\frac{1}{m_2}+\frac{1}{m_3}\right),\\
L_{123}^{S-S} &= \frac{1}{m_1m_2m_3}\left[\frac{4}{3}\left(\frac{(\sigma_2\cdot\sigma_3)(\lambda_2^c\lambda_3^c)}{m_1^2}+\frac{(\sigma_1\cdot\sigma_3)(\lambda_1^c\lambda_3^c)}{m_2^2}+\frac{(\sigma_1\cdot\sigma_2)(\lambda_1^c\lambda_2^c)}{m_3^2}\right)\right.\\
&\quad +2d^{abc}(\lambda_1^a\lambda_2^b\lambda_3^c)\left(\frac{\sigma_2\cdot\sigma_3}{m_1^2}+\frac{\sigma_1\cdot\sigma_3}{m_2^2}+\frac{\sigma_1\cdot\sigma_2}{m_3^2}\right)\\
&\quad \left.-2\epsilon_{ijk}\sigma_1^i\sigma_2^j\sigma_3^k f^{abc}\lambda_1^a\lambda_2^b\lambda_3^c\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{1}{m_3^2}\right)\right],\\
L_{123}^{C-S} &= \frac{4}{3}\left[\frac{\lambda_2^c\lambda_3^c}{m_1}\left(\frac{\sigma_1\cdot\sigma_3}{m_1m_3}+\frac{\sigma_1\cdot\sigma_2}{m_1m_2}\right)+\frac{\lambda_1^c\lambda_3^c}{m_2}\left(\frac{\sigma_2\cdot\sigma_3}{m_2m_3}+\frac{\sigma_1\cdot\sigma_2}{m_1m_2}\right)+\frac{\lambda_1^c\lambda_2^c}{m_3}\left(\frac{\sigma_2\cdot\sigma_3}{m_2m_3}+\frac{\sigma_1\cdot\sigma_3}{m_1m_3}\right)\right]\\
&\quad +2d_{abc}(\lambda_1^a\lambda_2^b\lambda_3^b)\left[\frac{1}{m_1}\left(\frac{\sigma_1\cdot\sigma_3}{m_1m_3}+\frac{\sigma_1\cdot\sigma_2}{m_1m_2}\right)+\frac{1}{m_2}\left(\frac{\sigma_2\cdot\sigma_3}{m_2m_3}+\frac{\sigma_1\cdot\sigma_2}{m_1m_2}\right)\right.\\
&\quad \left.+\frac{1}{m_3}\left(\frac{\sigma_2\cdot\sigma_3}{m_2m_3}+\frac{\sigma_1\cdot\sigma_3}{m_1m_3}\right)\right]. \qquad (1)
\end{aligned}$$

In this work, we investigate the three-quark interactions using the following Hamiltonian.

$$H = \sum_{i<j<k}^{n}\left(AL_{ijk}^{C-C}+BL_{ijk}^{S-S}+CL_{ijk}^{C-S}\right). \qquad (2)$$

Here, the superscripts C and S show the two-body color-color and color-spin interactions, respectively, used at the vertices. The values for the three-body parameters are $A = -367.522\,\text{MeV}^2$, $B = -2.85156\times10^{11}\,\text{MeV}^6$, $C = -7.68351\times10^6\,\text{MeV}^4$. Using these three-quark potentials, we can obtain much better fitting results with the same parameter set for both meson and baryon spectrums [9]. Additionally, as one can see, there are spin-dependent contributions to the quark three-body forces that have not been considered before.

The three-quark color interaction factor can be calculated using permutation matrices when all the particles are quarks. For the diagonal component of a multiquark state, it is given as follows[11]:.

$$\sum_{i<j<k} d^{abc}\lambda_i^a\lambda_j^b\lambda_k^c = \frac{4}{3}C^{(3)}-\frac{10}{3}C^{(2)}+\frac{80}{27}n, \qquad (3)$$

$$\sum_{i<j<k} f^{abc}\lambda_i^a\lambda_j^b\lambda_k^c = 0, \qquad (4)$$

where $C^{(3)} = \frac{1}{18}(p-q)(2p+q+3)(p+2q+3)$ and $C^{(2)} = \frac{1}{3}(p^2+q^2+pq+3p+3q)$ are the cubic and quadratic Casimir invariants of SU(3), and n is the total number of quarks. However, it should be noted that this formula is only valid for multiquark system. When there are antiquarks, then we should use the commutation and anticommutation relations to calculate the three-quark color matrices.

Table 1: Three-quark potentials for T_{cc}. The unit is MeV.

Color	$\sum_{i<j<k} AL_{ijk}^{C-C}$	$\sum_{i<j<k} BL_{ijk}^{S-S}$	$\sum_{i<j<k} CL_{ijk}^{C-S}$
$\mathbf{3}_{12}\bar{\mathbf{3}}_{34}$	-4.8424	0.0319	20.9444
$\bar{\mathbf{6}}_{12}\mathbf{6}_{34}$	31.4754	0.0907	8.6177

Table 2: Three-quark potentials for $\chi_{c1}(3872)$. The unit is MeV.

Color	$\sum_{i<j<k} AL_{ijk}^{C-C}$	$\sum_{i<j<k} BL_{ijk}^{S-S}$	$\sum_{i<j<k} CL_{ijk}^{C-S}$
$\mathbf{1}_{13}\mathbf{1}_{24}$	19.3695	0.0427	-1.3654
$\mathbf{8}_{13}\mathbf{8}_{24}$	7.2636	-0.2793	4.2077

3. Results

3.1 Tetraquarks: $T_{cc}, \chi_{c1}(3872)$

Assuming the quantum numbers are $I(J^P) = 0(1^+)$ for T_{cc} and $I^G(J^{PC}) = 0^+(1^{++})$ for $\chi_{c1}(3872)$, the two independent color-spin bases for the former and latter can be conveniently chosen to be $(|\mathbf{3}_{12}\bar{\mathbf{3}}_{34}\rangle, |\bar{\mathbf{6}}_{12}\mathbf{6}_{34}\rangle)$ and $(|\mathbf{1}_{13}\mathbf{1}_{24}\rangle, |\mathbf{8}_{13}\mathbf{8}_{24}\rangle)$, respectively. Here, the subscripts $1, 2, 3, 4$ represent $\bar{q}, \bar{q}, c, c$ for T_{cc} and $\bar{c}, \bar{q}, c, q$ for $\chi_{c1}(3872)$, and the numbers in bold are the color multiplets of the quark pairs.

Solving for the ground state of $\chi_{c1}(3872)$ with two-body interactions only, one finds it to be a well-separated meson-meson state dominated by the $|\mathbf{1}_{13}\mathbf{1}_{24}\rangle$ color-spin state. For the T_{cc}, the ground state is dominated by the $|\mathbf{3}_{12}\bar{\mathbf{3}}_{34}\rangle$ color state with the energy being slightly above or below the threshold, depending on the form of the r-dependence of the color-spin potential[12, 13]. Using these ground states, we represent the contribution from the three-quark potential for T_{cc} and $\chi_{c1}(3872)$ tetraquarks in Table 1 and 2.

For T_{cc}, the largest contribution comes from the term proportional to d^{abc} in L^{C-S}, while for $\chi_{c1}(3872)$, it is from the first term of L^{C-C} given in Eq. (1). For both cases, one finds that the sums of the quark three-body interactions are repulsive of more than 10 MeV, which places the masses of compact configurations far above the thresholds and pushes the quarks into separate meson states.

3.2 Dibaryons: H, $N\Omega$

In this work, we also calculate the three-quark potentials for $H(uuddss, I = 0, S = 0)$ and $N\Omega(uudsss, I = \frac{1}{2}, S = \frac{3}{2})$ which are most promising candidates for compact dibaryon states. There are two flavor states for H dibaryon, which are F_1 and F_{27}. Similarly, for $N\Omega$, there are two flavor states which are F_8 and F_{27}. However, in terms of two-body color-spin interaction, only F_1 for H and F_8 for $N\Omega$ show attractive interaction[14], so we expect that these flavors states are dominant states for H and $N\Omega$. However, as we can see in the Table. 3 and 4, both flavor states show repulsive three-quark interactions which are mostly from L^{C-S}. On the one hand, the other flavor states for both H and $N\Omega$ show attractive three-quark interactions. Therefore, if there is compact dibaryon state for H and/or $N\Omega$, then it is likely a mixture of two flavor states.

Table 3: Three-quark potentials for H. The unit is MeV.

Flavor	$\sum_{i<j<k} AL_{ijk}^{C-C}$	$\sum_{i<j<k} BL_{ijk}^{S-S}$	$\sum_{i<j<k} CL_{ijk}^{C-S}$
F_1	-21.0008	17.6644	34.0685
F_{27}	-21.0008	24.1615	-20.3534

Table 4: Three-quark potentials for $N\Omega$. The unit is MeV.

Flavor	$\sum_{i<j<k} AL_{ijk}^{C-C}$	$\sum_{i<j<k} BL_{ijk}^{S-S}$	$\sum_{i<j<k} CL_{ijk}^{C-S}$
F_8	-19.056	1.3458	27.8001
F_{27}	-19.056	5.3551	-6.2815

4. Summary

In this work, we calculated the three-quark potentials for tetraquarks and dibaryons. We have further demonstrated that such quark three-body interactions provide a non-trivial amount of repulsion in a compact configuration for both the T_{cc} and $\chi_{c1}(3872)$, strongly suggesting that these will fall apart and become molecular-type meson systems bound by pion exchange type of interaction. For dibaryons, similar to the tetraquark case, the dominant flavor states also show repulsive interaction. Since such quark three-body interactions are crucial in the study of compact multiquark configurations with quark and anti-quark numbers larger than three, we need to investigate its effect more deeply in the future.

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