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Nuclear Matter properties from the ladder resummation method

J. M. Alarcón[a,*]

Universidad de Alcalá, Grupo de Física Nuclear, Partículas y Astrofísica, Departamento de Física y Matemáticas, 28805 Alcalá de Henares (Madrid), Spain

E-mail: jmanuel.alarcon@uah.es

We present a new method to compute the energy per particle in nuclear matter at low densities. It relates the energy per particle in nuclear matter at low densities directly to the nucleon-nucleon scattering amplitude in the vacuum. The results are independent of any regulator and can be used for many purposes, including the calculation of the equation of state of nuclear matter at low densities. In particular, the results for pure neutron matter can be used to constrain the equation of state of neutron stars at low densities, which is important for the determination of the equation of state of neutron stars from first principles.

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[*]Speaker

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PoS(QNP2024)172

1. Introduction

The energy per particle in nuclear matter provides information about the equation of state, which has important astrophysical applications. Example of these applications are the determination of properties of neutron stars like mass-radius relation or tidal deformabilities, and capture of dark matter in neutron stars. Theoretical nuclear models provide results for the equation of state that can be tested in astrophysical observations. This can be used to improve the different models. In the case of neutron stars, the range of densities in its interior covers a wide range, and allow us to explore nuclear matter under extreme conditions. There are many different theoretical approaches to calculate the equation of state of nuclear matter. We can sort them into two different groups: The first category is the *Ab initio* method, and examples of these methods are the Quantum Monte Carlo and Effective Field Theory (EFT) approaches. The other category is the Phenomenological methods, like the Mean-field models.

In this proceeding I present the results based on the method developed in [1, 2]. In this approach we wanted to provide a calculation based on EFT at finite densities that incorporates by construction in-medium effects like particle-particle and hole-hole propagation or in-medium counting. Also, we avoided the use of any regulator in the calculations, so the approach doesn't have any systematics associated to the functional form of the regulator or fine tuning of the cutoff scale. And finally, we aimed to connect the in-medium calculations to vacuum calculations, so the calculation is parameter-free.

2. Formalism

The starting point is the idea of [3] where a method to resum the in-medium ladder diagrams to all orders with a contact interaction (S-wave scattering length) was proposed. That work was extended to include the effective range in S-wave and the scattering volume in P-wave, but the results show some off-shell effects when reaching the unitary limit [4]. We observe that an expansion of the generating functional of in-medium Green function obtained in [5] provides the diagrams to be resumed in the ladder approximation. In [1] we perform the sum of the Hartree and Fock diagrams, and obtained a contribution due to the interaction of

$$\mathcal{E}_V = \frac{i}{2}\mathrm{Tr}\left(\sum_{d=1}^{\infty}\frac{(t_m L_d)^d}{d}\right) = -\frac{i}{2}\mathrm{Tr}\log\left[I - t_m L_d\right] \tag{1}$$

where

$$L_m(p,\vec{a}) = -m_N\int\frac{d^3k}{(2\pi)^3}\left[\theta(k_F-|\vec{a}+\vec{k}|)+\theta(k_F-|\vec{a}-\vec{k}|)\right]\frac{|\vec{a}+\vec{k},\vec{a}-\vec{k}\rangle\langle\vec{a}+\vec{k},\vec{a}-\vec{k}|}{\vec{k}^2-\vec{p}^2-i\varepsilon} \tag{2}$$

$$L_d(p,\vec{a}) = i\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\theta(k_F-|\vec{k}_1|)\theta(k_F-|\vec{k}_2|)(2\pi)^4\delta(k_1+k_2-2a)|\vec{k}_1,\vec{k}_2\rangle\langle\vec{k}_1,\vec{k}_2| \tag{3}$$

$$t_m = t_V \sum_{m=0}^{\infty} (L_m t_V)^m = (t_V^{-1} - L_m)^{-1} \tag{4}$$

being m_N the nucleon mass, $\vec{a} = (\vec{k}_1 + \vec{k}_2)/2$ and k_F the Fermi momentum. In the same work we found that the *in-medium t-matrix* t_m satisfies the following integral equation in partial waves

$$t_m(p', p) = V(p', p) + \frac{m_N}{(2\pi)^2} \int_0^{\infty} \frac{k^2 dk}{k^2 - p^2 - i\varepsilon} V(p', k)[Y^* Y] t_m(k, p) \tag{5}$$

This integral equation can be solved exactly if we consider a contact (momentum dependent interaction). This kind of interaction is valid up to $k_F \sim \frac{M_\pi}{2}$. Vacuum integrals (integrals with no Fermi sea insertions) are regularized through a cutoff (Λ) which in the end of the calculation is sent to infinity. We solve the integral equation for t_m with an arbitrary potential based on momentum-dependent contact terms and write the low-energy constants (LECs) in terms of the effective range parameters. Finally, we send the cutoff to infinity and observe that the in-medium on-shell t-matrix is related to the vacuum on-shell t-matrix in the following way

$$\tilde{t}_m = \left(\tilde{t}_V^{-1} - L_m\right)^{-1} \tag{6}$$

where the tilde means that the external momenta are taken on the mass shell.

3. Results for nuclear matter

This method has been used to study the unitary limit [1] and systems of cold atoms interacting in P-wave [6]. To make calculations for nuclear matter, we use the Nijmegen phase shifts to reconstruct the on-shell vacuum t-matrix that we need to compute the in-medium t-matrix on the mass shell.

The results of the calculations are shown in Fig. 1. There is shown the energy per particle of symmetric nuclear matter (SNM) and pure neutron matter (PNM). Both results are compared to different theoretical calculations. In Fig. 2 we show the convergence of the calculation by including different partial waves to compute the in-medium t-matrix. Finally, in Fig. 3 we show the results for the pressure as a function of the Fermi momentum.

We estimate the uncertainty of the calculation associated to the use of a momentum-dependent contact interaction by multiplying the off-shell momentum dependence in the loop integral L_m times the gaussian regulator

$$\exp\left[-\frac{(k - M_\pi/2)^2}{\lambda^2}\right], \qquad \lambda > M_\pi. \tag{7}$$

Additionally, we also explore the uncertainty of our calculation by considering a density-dependent nucleon mass of the form

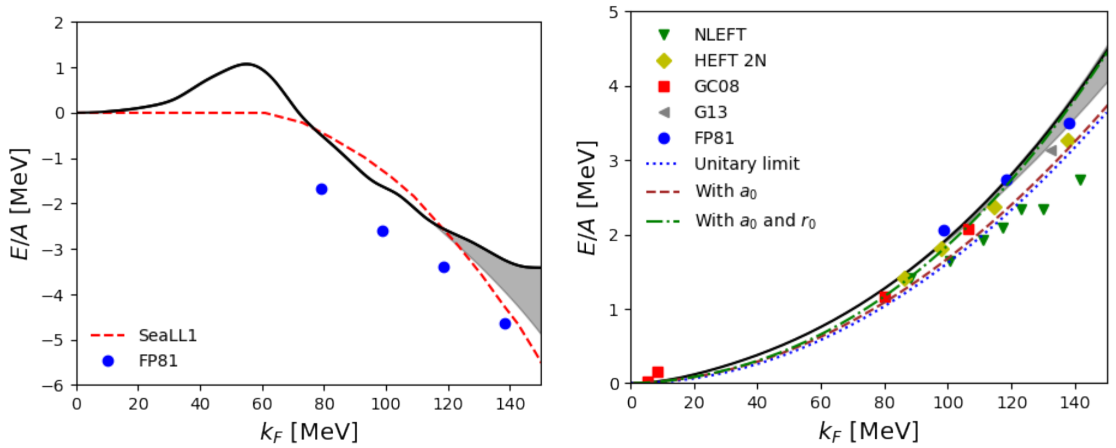

Figure 1: Energy per particle of symmetric nuclear matter (left) and pure neutron matter (right).

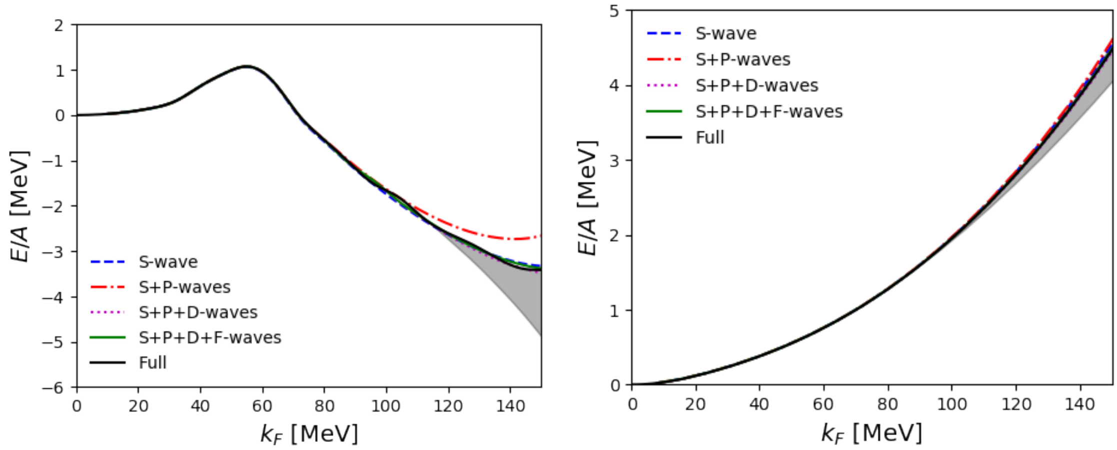

Figure 2: Convergence of the calculation of the energy per particle in symmetric nuclear matter (left) and pure neutron matter (right).

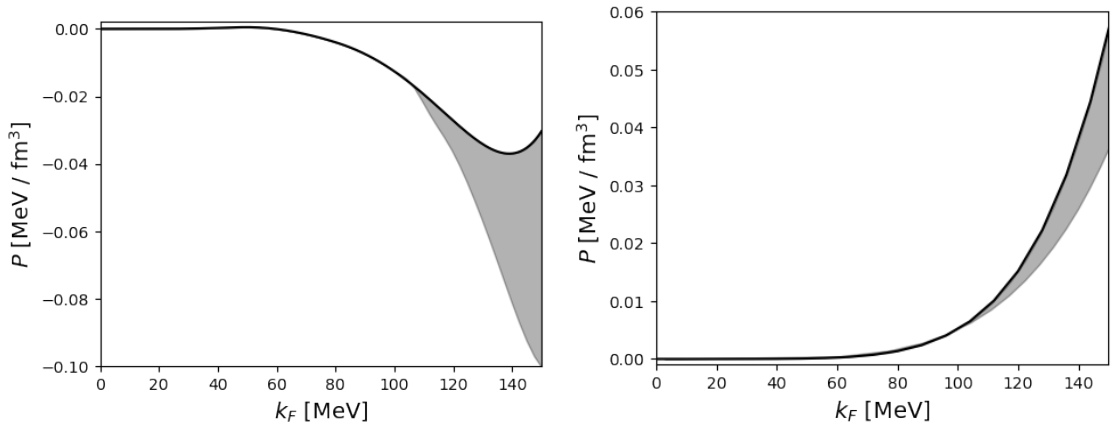

Figure 3: Pressure as a function of the Fermi momentum (k_F) in symmetric nuclear matter (left) and pure neutron matter (right).

$$m_N(n) = \frac{m_N}{1 + \frac{n}{n_s}\left(\frac{m_N}{m_N^*} - 1\right)}, \qquad m_N^* \in [0.7 m_N, 0.9 m_N] \tag{8}$$

Both types of uncertainty are reflected through the grey bands in Figs.1-3

We use these results to study the properties of nuclear matter at saturation density. With that aim, we consider the parametrization of the symmetry energy $S(n) = [E/A(n)]_{PNM} - [E/A(n)]_{SNM}$ given by [7]

$$S(n) = C_s \left(\frac{n}{n_s}\right)^{\gamma_s} \tag{9}$$

where $n_s = 0.16$ fm^{-3} is the saturation density. We fit this parametrization to our results at low densities to get the parameters C_s and γ_s

$$C_s = 34.77(15)\text{ MeV}, \tag{10}$$

$$\gamma_s = 0.667(3), \tag{11}$$

from where we get the symmetry energy at saturation S_0 and the slope L, defined as

$$S_0 = S(n_s) \tag{12}$$

$$L = 3n_s \left.\frac{dS(n)}{dn}\right|_{n_s} \tag{13}$$

$$31.10 \leq S_0 \leq 36.58\text{ MeV} \tag{14}$$

$$57.82 \leq L \leq 78.29\text{ MeV} \tag{15}$$

4. Summary and Conclusions

We develop a method to compute the energy per particle in nuclear matter at low densities. With this method, we related the energy per particle in nuclear matter at low densities directly to the in-vacuum nucleon-nucleon scattering amplitude. The method provides results independent of any regulator and can be used to study the unitary limit in nuclear matter, cold atoms systems and the equation of state of nuclear matter. In particular, the results for pure neutron matter can be used to constrain the equation of state of neutron stars at low densities [8]. These constrains turn out to be important for the determination of the equation of stat of neutron stars from first principles [9].

PoS(QNP2024)172

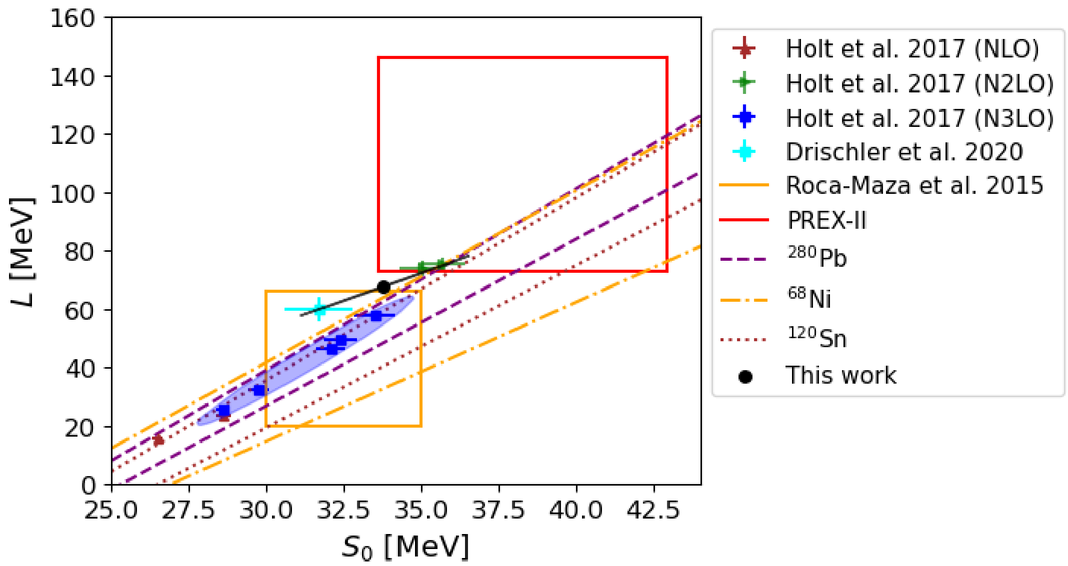

Figure 4: Values obtained for S_0 and L using our results for E/A in symmetric nuclear matter and pure neutron matter. They are compared to theoretical and experimental determinations.

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