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Gravitational-wave imprints of nonconvex dynamics in binary neutron star mergers

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Explaining gravitational-wave (GW) observations of binary neutron star (BNS) mergers requires an understanding of matter beyond nuclear saturation density. Our current knowledge of the properties of high-density matter relies on electromagnetic and GW observations, nuclear physics experiments, and general relativistic numerical simulations. Using a phenomenological nonconvex equation of state (EoS) we conduct a suite of numerical-relativity simulations of BNS mergers and identify observable imprints on the GW spectra of the remnant. Nonconvex regions may be associated with first order phase transitions from nuclear/hadronic matter to deconfined quark matter, present in some realistic EoS from nuclear physics. The dynamics triggered by our nonconvex EoS induces a significant shift in the peak frequency of the dominant oscillation mode of the post-merger remnant (of order $\Delta f \gtrsim 380$ Hz) with respect to that of binaries with convex (or regular) dynamics.

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1. Introduction

Gravitational waves (GWs) from binary neutron star (BNS) mergers encode key information about the nature of matter above nuclear saturation density ($n_0 = 0.15 \pm 0.01\,\mathrm{fm}^{-3}$). Following the merger, most of the gravitational-wave energy is emitted at frequencies above 2 kHz. This high-frequency emission may be detectable with next-generation observatories such as the Einstein Telescope and Cosmic Explorer [1, 2]. The possible appearance of phase transitions from nuclear hadronic matter to quark-gluon plasma or to matter phases containing exotic particles (e.g. hyperons) in BNS mergers may impact the stability, dynamics, and final fate of the remnant, and thus the associated GW signal [3]. In particular, the dominant GW frequency at merger f_{peak} exhibits a significant deviation from a quasi-universal relation (EoS-independent) with the tidal deformability Λ, an effect that could be observationally identified.

Physical processes, such as phase transitions to "exotic" states of matter, affect the monotonic increase of the speed of sound with density. The speed of sound is closely related to the so-called *fundamental derivative* $\mathcal{G}$ and hence to the *convexity* of the EoS. Namely, the convexity of a thermodynamical system is determined by the sign of the $\mathcal{G}$ on the $p - V$ plane [4], a quantity directly linked to the derivative of the speed of sound. Here p is the pressure and $V = \rho_0^{-1}$ is the specific volume, with ρ_0 the rest-mass density. If the fundamental derivative is negative then the EoS is non-convex allowing for the appearance of expansive shock waves and compressive rarefactions, while if $\mathcal{G} > 0$ the EoS is convex leading to compressive shocks and compressive rarefaction waves. Note that, at the phase transition the fundamental derivative is discontinuous. Studies of non-convexity in relativistic fluids relevant for the topic summarized in this paper are discussed in detail in [5–8].

The effects of non-convex thermodynamics in BNS mergers are incorporated using the same phenomenological EoS employed in [5, 6], i.e. a $\Gamma-$law EoS allowing for shock heating. Following [5] we assume that the adiabatic index Γ is not constant but depends on the rest-mass density (see below). This allows us to mimic some key features of tabulated, nuclear-matter EoS such as the non-monotonic dependence of the speed of sound (or the adiabatic index) with the rest-mass density, allowing the appearance of non-convex dynamics. We find that the use of a non-convex EoS does influence the post-merger dynamics in a significant way, in particular we observe the frequency of the peak first visible in the GW spectra right after merger, f_{peak}. Depending on the parameters of our EoS, deviations from a $\Lambda - f_{\mathrm{peak}}$ quasi-universal relation can be as large as $\Delta f_{\mathrm{peak}} \geq 380\,\mathrm{Hz}$ with respect to that of binaries with pure convex evolution. Such frequency shifts are reminiscent of the results reported by [3], where they were attributed as due to a strong first-order phase transitions from nuclear/hadronic matter to deconfined quark matter. We argue that the explanation for the observed frequency shift lies in the presence of anomalous, non-convex dynamics in the binary remnant. Our explanation does not exclude the interpretation of the shifts as being due to a first-order phase transition, as the dynamics is indeed non-convex in that case. The explanation based on a first-order phase transition can be seen as a particular manifestation of a more general cause, namely the possible non-monotone behavior of the EoS of NSs above nuclear saturation density. For details on the execution of the simulations the reader may refer to [8].

2. Non-convex thermodynamics and EoS

The convexity properties of Newtonian hydrodynamical flows is determined by the EoS through the concept of the fundamental derivative defined as

$$\mathcal{G} \equiv -\frac{1}{2} V \frac{\left.\frac{\partial^2 p}{\partial V^2}\right|_s}{\left.\frac{\partial p}{\partial V}\right|_s} = 1 + \left.\frac{\partial \ln c_s}{\partial \ln \rho_0}\right|_s \ , \tag{1}$$

with s being the specific entropy. This quantity measures the convexity of the isentropes on the $p - V$ plane. When $\mathcal{G} > 0$ the system is convex and its dynamics involves expansive rarefaction waves and compressive shocks. This is the usual regime in which many astrophysical scenarios develop. By contrast, when $\mathcal{G} < 0$ the system is non-convex and its (anomalous) dynamics involves compressive rarefaction waves and expansive shocks.

A generalized fundamental derivative for *relativistic* fluids was found in [9] and is given by

$$\mathcal{G}_R = \mathcal{G} - \frac{3}{2} c^2_{s(R)} \, , \tag{2}$$

where $c_{s(R)}$ is the relativistic speed of sound that can be related to c_s through $c_s^2 = h\, c^2_{s(R)}$, where $h = 1 + \epsilon + p/\rho_0$ is the specific enthalpy and ϵ is the specific internal energy density.

The effects of a non-convex EoS on the stability and dynamics of isolated neutron stars were probed in [6] employing a phenomenological Gaussian Γ-law (GGL) EoS $p = (\Gamma - 1)\, \rho_0\, \epsilon$. Here Γ is an effective thermal index that, to mimic its dependency on the nucleon effective mass for densities above half nuclear saturation, is given by [5]

$$\Gamma = \Gamma_{\rm th} + (\Gamma_1 - \Gamma_{\rm th})\, \exp\left[-\frac{(\rho_0 - \rho_1)^2}{\Sigma^2}\right] \, , \tag{3}$$

where $\Gamma_{\rm th}$, Γ_1, ρ_1, and Σ are free constant parameters. The results reported by [6] indicate that a non-convex dynamics can accelerate the onset of the collapse of a neutron star to a black hole with respect to that of a convex dynamics. Non-convexity also leaves an imprint on the GW signal, amplifying the amplitude of the GWs emitted by the collapsing star. The maximum amplitude is about twice as large as in the convex case. These imprints are large enough to be detectable by third-generation, ground-based detectors.

3. Results

We now proceed to summarize the results of our simulations, which are fully described in [8]. For comparison purposes, we also consider convex models, which are evolved using a constant value of the thermal index Γ, achieved by setting $\Gamma_{\rm th} = \Gamma_1$ in Eq. (3). During inspiral the dynamics for convex and non-convex EoS is roughly similar. We observe that a non-convex dynamics tends to reduce the neutron star compactness compared to those with a convex dynamics. The binary companion induces tidal forces that stretch the star out along the line connecting the centroids of the two stars. This effect triggers expansive shock waves in the bulk of a non-convex star pushing out its outer layers, which in turn accelerates the merger. We note that this behavior depends on the EoS used to construct the initial data. This effect appears strong in soft EoS binaries since they tend to have smaller values of $\mathcal{G}_R$, the merger happens up to $\gtrsim 5$ ms earlier than their convex counterpart. On the other hand, stiff EoSs are less affected, reporting roughly the same time of merger ($\Delta t_{\rm mer} \lesssim 1$ ms) than their constant $\Gamma = 1.8$ EoS counterparts.

Following merger, a massive remnant forms with two dense cores rotating about each other. Depending on the compactness of the progenitors, the two cores collide during the first $\lesssim 5$ ms after merger giving birth to a highly differentially rotating star wrapped in a Keplerian-like cloud of low density matter. The remnant settles down shortly after that. During early post-merger the bulk of the GGL-EoS-remnant undergoes a transient period where non-convex relativistic regions ($\mathcal{G}_R < 0$) expand and contract continuously. We observe that depending on the softness of the EoS these regions, as expected, may spread covering a large part of the bulk of the remnant or be confined around its central core. Once the remnant settles down we observe that, in all cases, the non-convex regions remain bounded within $\rho_0 \gtrsim 10^{13.7}\,\mathrm{g\,cm^{-3}}$. We note that, for all EoSs, the remnants resulting from non-convex evolutions are in general more extended than those of the convex ones, which implies that the presence of a non-convex dynamics tends to increase the pressure support in the remnant. Therefore, one may conclude that non-convex dynamics hardens the EoS of the remnant. Consistent with this, the angular velocity profiles shown for some of our models indicate that non-convex remnants tend to rotate more rapidly at the core than convex ones. Notice that this effect has also been observed in remnants undergoing quark-hadron phase transitions [10].

3.1 Gravitational wave spectra

The GW spectra are computed assuming a distance to the source of 50 Mpc, optimal orientation and sky localization, and using a fixed time window from 20 ms before merger to 5 ms past merger. During inspiral, the existence of $\mathcal{G}_R < 0$ regions slightly modifies the maximum GW frequency which reaches a maximum value below 2.2 kHz depending on the initial-data EoS. Following merger, the GW amplitude and frequency strongly depend on both the softness of the initial-data EoS and of the existence of non-convex regions. Our simulations reveal that the frequency $f_{\rm peak}$ of the dominant oscillation mode of the remnant for binaries with non-convex dynamics is always smaller than that of their convex dynamics counterparts. In the most extreme cases analyzed, a frequency shift of $\Delta f_{\rm peak} \sim 300$ Hz was found for the stiff DD2 EoS evolved with the GGL EoS with $\Gamma_1 = 3.0$ and a corresponding shift of $\Delta f_{\rm peak} \sim 1$ kHz was found for the soft APR4 EoS with $\Gamma_1 = 3.5$. The frequency shifts and "outliers" in the $f_{\rm peak} - \Lambda$ quasi-universal relation found by [3] may be attributed to the existence of non-convex dynamics in the post-merger remnant discussed here. Fig. 1 displays the dominant frequency as a function of the tidal deformability Λ for all evolutions. As expected, when the BNS merger is simulated using a convex EoS (green circles) $f_{\rm peak}$ is correlated with Λ in a quasi-universal (EoS-insensitive) manner according to

$$f_{\rm peak}^{\rm const\,\Gamma} = \left[(4.52 \pm 0.79) \times 10^{-7}\,\Lambda^2 - (1.80 \pm 0.18) \times 10^{-3}\,\Lambda + (3.99 \pm 0.08)\right]\ \mathrm{kHz}\,. \tag{4}$$

However, when the system is subject to non-convex dynamics the peak frequency is more than $1-\sigma$ away from the above quasi-universal relation (yellow and blue points in Fig. 1). We notice that a $1-\sigma$ deviation in our previous fit corresponds to ~ 64 Hz. This significant shift in frequency can only be attributed in our study to the presence of a non-convex dynamics and not to the presence

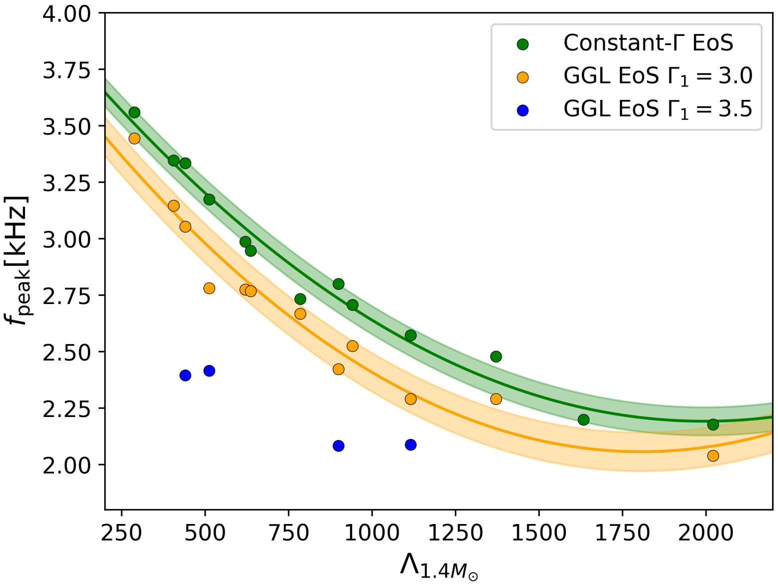

Figure 1: GW peak frequency of the post-merger remnant $f_{\rm peak}$ as a function of the tidal deformability Λ. Each circle corresponds to one initial-data EoS, the corresponding colors indicating the EoS used in the evolutions (see legend). Solid curves display the quasi-universal relations given by Eqs. (4) and (5). Colored regions represent the standard deviation of the corresponding fit.

of a physical process, such as a phase transition in the binary remnant, as our GGL EoS does not account for such an effect, and neither to spurious artifacts induced by numerical interpolations of the tabulated EoS. Notice that, in contrast with the results reported in [3] where the effects of the first-order phase transition *increase* the peak frequency of the dominant mode, the appearance of a non-convex dynamics *decreases* this frequency. This discrepancy is simply due to the effective stiffening of the EoS remnant associated with our specific choice of the free parameters of our phenomenological GGL EoS (see Eq. (3)). However, the *magnitude* of the frequency shifts is, in both cases, similar. We expect that a survey of parameters of the GGL EoS, in particular accounting for a softening of the EoS remnant, may not only reproduce the magnitude of the shifts but also reconcile the direction of the shift of the peak frequency with the values reported by [3].

Interestingly, we find that the outliers to the fitting formula (4) corresponding to binaries evolved with the GGL EoS setting $\Gamma_1 = 3.0$ satisfy their own quasi-universal relation according to

$$f^{\rm GGL}_{\rm peak} \quad = \quad \left[(5.4 \pm 1.1) \times 10^{-7}\, \Lambda^2 - (1.95 \pm 0.25) \times 10^{-3}\, \Lambda + (3.82 \pm 0.12)\right] \,{\rm kHz}\,, \tag{5}$$

with a deviation of $\Delta f_{\rm peak} \lesssim 380\,{\rm Hz}$ with respect to binaries subject to convex dynamics. This finding suggests that the appearance of non-convex regions might induce a displacement of the quasi-universal relation on the $f_{\rm peak} - \Lambda$ plane for non-convex EoS. We note that the effects of a non-convex dynamics on the post-merger GWs can be enhanced by fine-tuning the free parameters of the thermal index in Eq. (3). In particular, by setting $\rho_1 = 9.1 \times 10^{14}\,{\rm g\,cm^{-3}}$, $\Sigma = 0.35\,\rho_1$, and $\Gamma_1 = 3.5$ we obtain the largest shift in $f_{\rm peak}$, namely $\sim 1\,{\rm kHz}$ and the blue markers in Fig. 1).

To further corroborate that the frequency shifts observed in our non-convex evolutions are due to the "anomalous" dynamics and not to the specific parameters of the GGL EoS of our fiducial model, we also simulate BNS mergers for the initial-data EoS DD2 with values of the GGL EoS parameters so that the evolution remains *convex* throughout (see [8] for details). In all cases we find that $10\,\mathrm{Hz} \lesssim \Delta\mathrm{f}_{\mathrm{peak}} \lesssim 50\,\mathrm{Hz}$, i.e. less than $1-\sigma$ away from the quasi-universal relation of Eq. (4) found for convex EoS. This suggests that the significant shifts in the peak frequency reported in this paper should be induced by the non-convex dynamics.

4. Discussion

In this paper we have performed numerical-relativity simulations of BNS mergers subject to non-convex dynamics, allowing for the appearance of expansive shock waves and compressive rarefactions. To this aim we have used a phenomenological non-convex EoS proposed in [5] and also used in [6]. The latter work showed that the appearance of non-convex dynamics during the gravitational collapse of uniformly rotating NS leaves a distinctive imprint on the GW signal, and served as a motivation for this study. Further motivation was gathered by our attempt to provide an explanation to the loss of the $\Lambda - f_{\mathrm{peak}}$ quasi-universal relation found by [3] for EoS admitting a strong first-order phase transition. We have surveyed a number of BNS initial configurations modeled with a piecewise-polytropic representation of different (cold) nuclear EoS. Those have been subsequently evolved with a $\Gamma-$law EoS to allow for shock heating, considering two different possibilities for the adiabatic index Γ, either a constant value, which induces a convex (regular) dynamics, or a variable index depending on the rest-mass density, which induces a non-convex (anomalous) dynamics.

By comparing the two types of dynamics – convex vs non-convex – we have identified observable differences in the GW spectra of the remnant. In particular, we have found that non-convexity induces a significant shift in the $\Lambda - f_{\mathrm{peak}}$ quasi-universal relation, of order $\Delta f_{\mathrm{peak}} \gtrsim 380\,\mathrm{Hz}$, with respect to that of binaries with convex dynamics. These values are comparable in magnitude to those reported by [3], attributed however to a first-order phase transition from nuclear/hadronic matter to deconfined quark matter depending on the density jump. As showed in [6], many of the microphysical EoSs display a sensitive reduction of the relativistic fundamental derivative as the baryon number density grows above 1 fm^{-3}. In that regime, even spurious small-scale oscillations may drive the relativistic fundamental derivative towards negative values, triggering non-convex dynamics. This artificial behavior could be attenuated using a finer number of data points.

We finally argue that the ultimate origin of the frequency shift is to be found in the presence of anomalous, non-convex dynamics in the binary remnant typical of strong phase transitions. The phenomenological GGL EoS employed in the simulations reported in this work can only be regarded as a toy model. Nevertheless, it has served the purpose of highlighting the potential relevance the development of non-convex dynamics may have on important observables in BNS mergers such as the GW emission. We stress that our GGL EoS does not account for phase transitions and is not affected by potential spurious artifacts induced by the numerical access to a tabulated EoS as all derivatives can be computed analytically. A natural extension of this work will be to revisit these simulations using actual microphysical EoS allowing for such non-convex dynamics. Those could also be carried out in combination with the analytic model for modelling phase transitions in tabulated EoS recently reported by [7].

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