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Gamow-Teller studies within the Subtracted Second RPA

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Recent applications of the subtracted second random-phase approximation (SSRPA), based on Skyrme functionals, to the study of Gamow-Teller excitations and beta-decay are presented and discussed. The comparison with the conventional random-phase approximation (RPA) results and experimental data is also discussed. It is found that the amount of Gamow-Teller strength obtained in SSRPA is much lower than the RPA one, and it agrees better with experimental data. The inclusion of two-particle-two-hole configurations is responsible for this quenching, avoiding thus the use of any "ad-hoc" quenching factors normally adopted in this kind of studies. This result may have implications for the computation of nuclear matrix elements in the same framework for neutrinoless double-beta decay.

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1. Introduction

The Random Phase Approximation (RPA) model is a largely used microscopic approach for the description of the nuclear collective excitations. In this approximation, the excited states are build as superpositions of 1 particle–1 hole (1ph) (and 1 hole -1 particle) configurations. This picture is able to provide the global features of Giant Resonances (GR) such as the centroid energy, the total strength and the corresponding energy-weighted sum rules. However, other important properties, as for example the GR's fine structure and the spreading width, generated by the coupling between 1ph configurations with more complex degrees of freedom are by construction beyond the RPA capabilities. The Second RPA (SRPA) model is a natural extension of RPA where a more general description of the nuclear excitations is considered and provides a valuable tool for the prediction of spreading width and fine structure properties, due to the introduction of 2 particle-2 hole (2ph) (and 2 hole -2 particle) configurations. The number of the 2ph configurations, even in the coupled scheme is orders of magnitude larger than the 1ph one. For this reason, the very few applications of the SRPA have been done by adopting very small 2ph model spaces and resorting to strong approximations in the evaluation of the corresponding equations of motion. Only in the last years, large-scale SRPA calculations have been performed, showing merits and limits of this approach. Performing such calculations has allowed one to show some features of the SRPA that could not be seen in previous applications. In particular, the SRPA spectrum is systematically lowered by several MeV with respect to the RPA [1–3], losing then the good description of the centroid energies obtained at RPA level.

This pathalogical behaviour can be traced back to the use of effective interactions that have fitted at mean-field level, like for example the Skyrme and the Gogny ones, and then employed in beyond mean field calculations, introducing possible double counting problems. For this reason, the SRPA model has been recently improved by using the so-called subtraction procedure [4], both for charge-conserving [5, 6] and charge-exchange[7, 8] modes. This procedure cures then the drawbacks and the limitations of the SRPA model, providing thus a robust and stable theoretical tool for a beyond–mean–field description of the excitation spectra of many–body systems.

Charge-exchange excitations and, in particular the Gamow-Teller (GT) resonance provide very crucial information and constraints in the construction of nuclear effective interactions, especially in connection with the different couplings in the spin-isospin channels. Moreover, they are closely linked to electron capture and β decay, which play important roles in nuclear astrophysics. Recent efforts are also motivated in the attempt to investigate and modelize the neutrinoless double-β ($0\nu\beta\beta$) decay. Most of the nuclei for which the $0\nu\beta\beta$ decay might occur are still too complex to be treated from first principles with *ab-initio* methods. More phenomenological approaches are therefore still important, even necessary. However, these theoretical schemes do not correctly describe the available data for GT excitations and β-decay half-lives and one must resort to *ad hoc* "quenching factors" to obtain reasonable results for GT strength. This kind of over-prediction is usually ascribed to missing physics, for example the Δ excitation or complex configurations such as two-particle – two-hole (2p-2h)) excitations or two-body weak currents.

2. Gamow-Teller strength in RPA and SSRPA

Charge-exchange RPA and Subtracted SRPA (SSRPA) calculations are performed here on top of Hartree-Fock ground states, using the Skyrme parametrization SGII [9] in a full self-consistent way, e.g. all the terms of the residual interaction are used consistently both in describing the ground state and the nuclear response. The energy cutoff on the 1ph and 2ph configurations are chosen in such a way that the obtained results are stable, especially regarding the strength function distributions and the associated sum rules. More details on the numerical and formal aspects can be found in Refs [7, 8].

In the case of charge-exchange excitations, two type of transitions are possible, corresponding to a change in the isospin projection $\Delta T_z = +1$ or -1. The corresponding strength functions are evaluated by using the GT one-body transition operators

$$\hat{O}^{\pm} = \sum_{i=1}^{A} \sum_{\mu} \sigma_{\mu}(i) \tau^{\pm}(i), \tag{1}$$

where $\tau^{\pm}$ are the isospin raising (+) and lowering (−) operators, $\tau^{\pm} = t_x \pm i t_y$, σ_μ is the spin operator, and A is the number of nucleons.

The $\hat{O}^{+}$ ($\hat{O}^{-}$) operator generates the GT$^+$ (GT$^-$) strength where a neutron (proton) is added and a proton (neutron) is removed. The non-energy-weighted sum rule, the so called Ikeda sum rule, relating the integrated strengths S of the GT$^-$ and the GT$^+$ spectra to the difference of the number of neutrons N and protons Z reads as

$$S_{GT^-} - S_{GT^+} = 3(N - Z). \tag{2}$$

This sum rule, being model independent, provides an important test for theoretical calculations and can be easily deduced by using the properties of the isospin operators if the condition of completeness of states is fulfilled. Within RPA and extended-RPA models, this sum rule holds if the quasiboson approximation is used. The main effect of the inclusion of 2ph configurations is to push, with respect to the RPA case, a significant amout of the SSRPA strength to high energies. As a consequence, the full value of the Ikeda sum rule is exhausted only by summing up to high excitation energy, much higher than in RPA. In order to illustrate this behaviour, in Fig. 1 we show for ^{48}Ca that the full sum rule is exhausted at $E \sim 12$ MeV in RPA, while in SSRPA one needs to integrate the strength up to ~ 70 MeV to obtain the full value. The inclusion of the 2ph configurations allows for a strongly improved agreement of the integrated strength with the experimental cumulative Ikeda sum in the nucleus ^{48}Ca, as it will be shown in more detail below.

The GT$^-$ spectrum for ^{48}Ca obtained by employing the SGII Skyrme interaction is shown in panel (a) of Fig. 2. In order to make easier the comparison among different results, the theoretical discrete spectra are folded with a Lorentzian function having a width of 1 MeV and compared with experimental data from Ref. [10]. The most demanding operations from a computationally point of view consist in the inversion of the A_{22} matrix, describing the coupling of the 2ph among themselves, needed in the subtraction procedure, and in the diagonalization itself required in solving the SSRPA equations. We compare below different approximation schemes. We indicate with the acronym SSRPAFF, the full calculations where the A_{22} matrix is fully treated in both operations, without

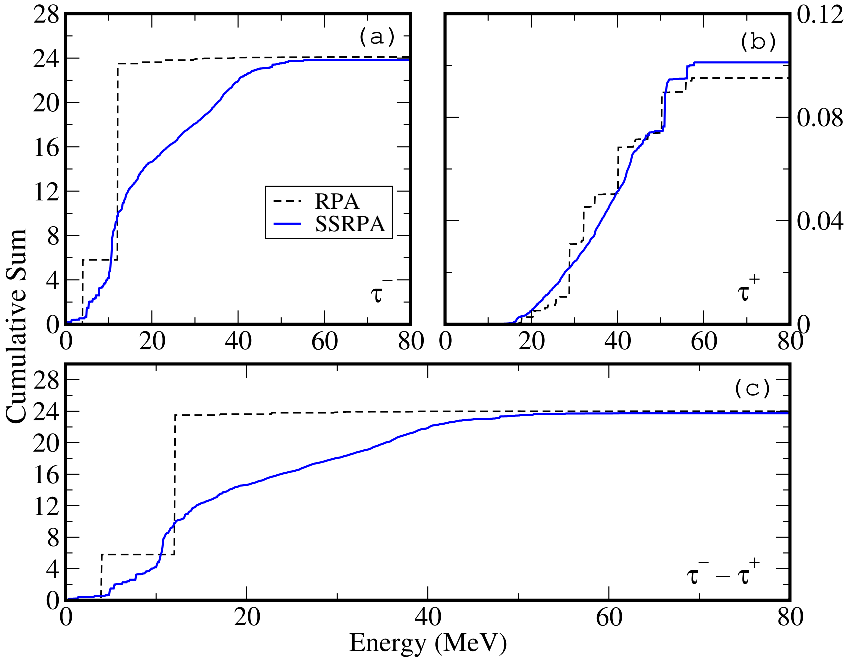

Figure 1: Cumulative sum in the GT^- (panel (a)) and GT^+ (panel (b)) and their difference (panel (c)), for ^{48}Ca, in RPA (black dashed line) and SSRPA (full blue line).

resorting thus to any kind of approximation. We use the acronym SSRPADD to indicate results where the diagonal approximation is used in dealing with the A_{22} matrix, both in the inversion and in the diagonalization procedures. This is of course the most approximated and numerically easy scheme. Finally, as an intermediate approximation, the SSRPADF one indicates a calculation where the diagonal approximation for the matrix A_{22} is adopted only in the subtraction procedure, where this matrix is inverted, while A_{22} is fully treated in the diagonalization.

In Figure 2, we show the results obtained within the three schemes. We can see that the SSRPAFF model leads to a visible improvement of the results compared to RPA. The double diagonal approximation (SSRPADD) provides a worse reproduction of the full spectrum, yet showing a considerable improvement with respect to RPA. The hybrid SSRPADF computation leads to better results, but still different from the SSRPAFF ones. We can see that the SSRPADF approximation reproduces sensibly less well than SSRPAFF the experimental strength. This is especially clear in the displayed cumulative sums shown in the panel (b). Such a result indicates that, in the calculations done for ^{48}Ca, the inversion of the matrix in the subtraction procedure cannot be carried out by adopting a simple diagonal approximation in the matrix and needs a more accurate treatment. It can be noticed that the experimental low-energy peak located at around 4 MeV, which is nicely described in RPA and in SSRPADD, in SSRPAFF is described by a broader distribution. However, as it can be seen in the right panel, the experimental and SSRPAFF cumulative strength are equal at around 5 MeV, suggesting that the strength is only fragmented over many states.

In order to exclude that the observed behaviour is not a peculiar effect of a specific interaction but an intrinsic effect of the SSRPA, we show in Fig. 3 the RPA and SSRPA results obtained

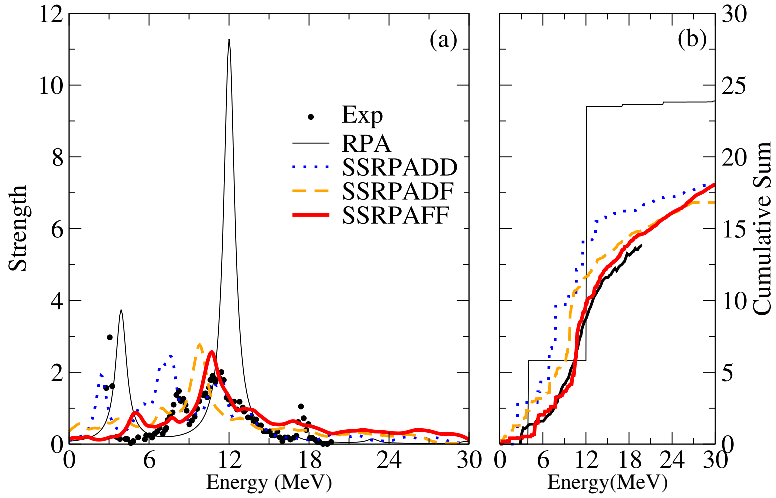

Figure 2: (a) GT$^-$ strength distributions for the nucleus ^{48}Ca obtained with the SGII Skyrme interaction in MeV^{-1} compared with the experimental data [10]. RPA and all SSRPA strengths are obtained by folding with a Lorentzian having a width of 1 MeV. (b) Cumulative sum of the strength up to the excitation energy of 20 MeV. The excitation energy is referred with respect to the mother nucleus. See text for more details.

with three different parameterizations – SGII, SkM* [11], and SkMP [12] — together with their experimental counterparts. Panels (b), (c), and (d) show the cumulative strengths up to 20 MeV, whereas panel (a) displays the percentages of the Ikeda sum rule from strength below 30 MeV. For this last quantity, Ref. [10] has reported both the experimental value and its uncertainty. In all three lower panels the integrated strength in the SSRPA is smaller than in the RPA, especially for SGII and SkM*. With these two parameterizations, the percentages of the Ikeda sum in panel (d) are quite close to the experimental value. The lower panels show that the detailed structure of the experimental spectrum is best reproduced by the parameterization SGII.

The ^{90}Zr(p,n) and ^{90}Zr(n,p) reactions were performed at the Research Center for Nuclear Physics [13] and more recently in Ref. [14]. A consistent study of data coming from both (p,n) and (n,p) channels led in Ref. [14] to the estimation of the experimental quenching for the GT Ikeda sum rule. In particular, the GT$^-$ strength integrated up to 50 MeV was found to be equal to 29.3, very close to the full value of the Ikeda sum rule for this nucleus ($3(N-Z)=30$), while the contribution of the GT$^+$ strength integrated up to the same excitation energy is found to be equal to 2.9. For heavier systems than ^{48}Ca, it has been shown that the SSRPADF scheme is indeed working quite well [8], the SSRPADF result will be thus shown below. In Fig. 4 we show the GT$^-$ strength distributions for ^{90}Zr and ^{132}Sn, panels (a) and (c), respectively and in panels (b) and (d) the corresponding cumulative sums. The SGII interaction is used and both the RPA and SSRPA results are shown. Other RPA strength distributions for ^{90}Zr have been discussed in Ref. [15], obtained by using different Skyrme interactions. However, independently of the employed Skyrme parameterization, the RPA strength integrated up to 25 MeV is in all cases already almost equal to 30, considerably larger than the experimental value. Panel (b) shows that the SSRPA model improves considerably the agreement with the experimental integrated strength, providing a much

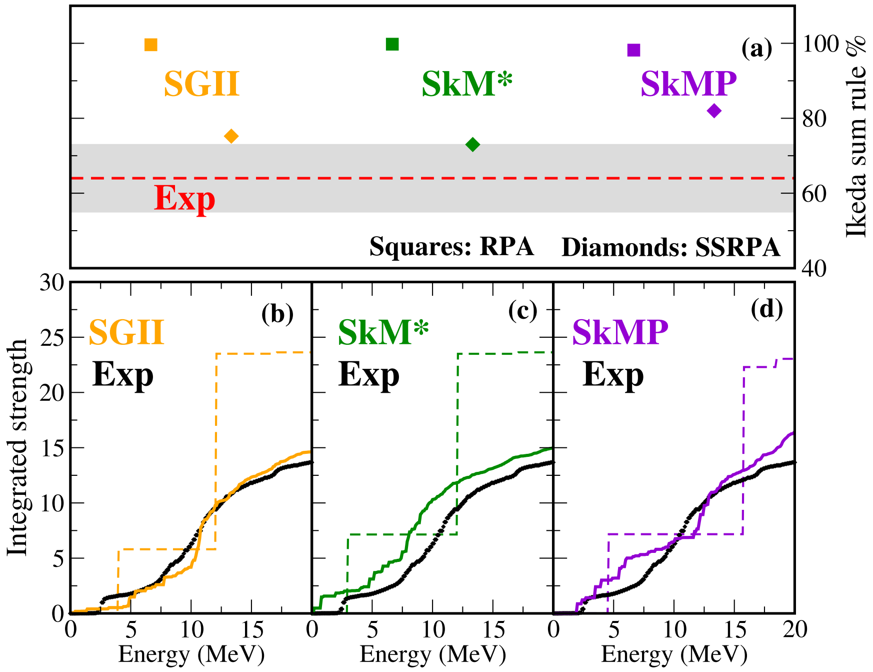

Figure 3: (a) Experimental percentage of the Ikeda sum rule integrated up to 30 MeV extracted from Ref. [10] (red dashed horizontal line), and its associated uncertainty (grey area). RPA and SSRPA percentages obtained with the parameterizations SGII, SkM*, and SkMP are also shown. (b), (c), (d) Cumulative RPA and SSRPA strengths compared with the experimental values in black symbols extracted from Ref. [10]. The solid (dashed) lines correspond to the SSRPA (RPA) results obtained with the parameterizations SGII (b), SkM* (c), and SkMP (d). The excitation energy is referred with respect to the mother nucleus.

lower value than in RPA (22.3) at the excitation energy of 25 MeV. The corresponding experimental value is slightly above 20, as one can see in the figure. The slope of the SSRPA curve nicely follows the experimental one and the slight shift which is visible is generated by the already mentioned small shift (to lower values) in the excitation energies compared to the experiment. For the nucleus ^{132}Sn, we see that, even if prediction at 20 MeV is located above the experimental value (we notice that the overall shift of the SSRPA spectrum to lower energies is also responsible for this) we may observe that we obtain anyway a strong improvement with respect to the RPA. Other important effects, not taken into account explicitly in our approach, such as short-range correlations, non-nucleonic degrees of freedom or the inclusion of more complex configurations, might be needed in order to further improve the agreement with the experimental data for heavier systems. However, one can see that, in general, the SSRPA model provide a very strong and robust improvement with respect to the RPA model in describing the quenching of GT strengths, globally leading to a much better agreement with the experimental measurements.

charge

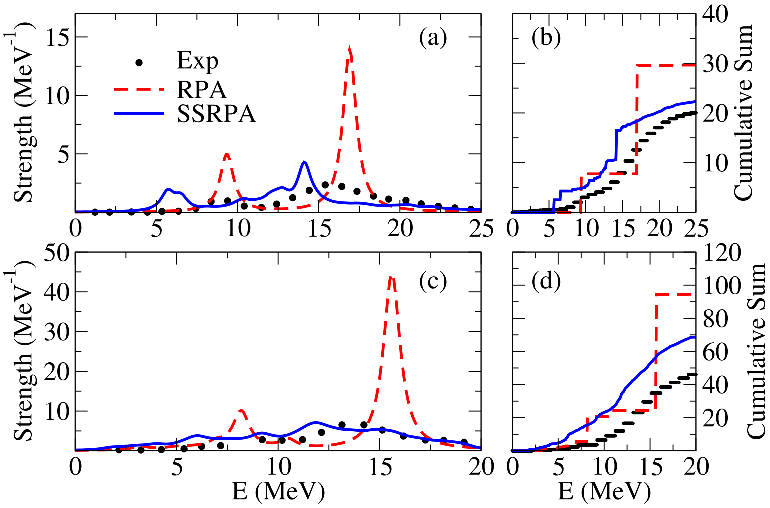

Figure 4: (a) GT$^-$ folded spectra obtained with the Skyrme interaction SGII for the nucleus ^{90}Zr obtained in SSRPA and RPA together with the experimental data taken form Ref. [14] ; (b) Strengths of panel (a) integrated up to 25 MeV; (c) Same as in (a) but for the nucleus ^{132}Sn. The experimental results are extracted from Ref. [16]; (d) Strengths of panel (c) integrated up to 20 MeV. The excitation energy is referred with respect to the mother nucleus.

3. Conclusions

The GT strength distributions in ^{48}Ca, ^{90}Zr and ^{132}Sn have been studied within the SSRPA model and compared with the RPA results. For ^{48}Ca, we have compared the validity of different approximation schemes in treating the 2p2h configurations. This study reveals that, for this nucleus the inversion of the A_{22} matrix in the subtraction procedure must be performed without any type of approximations. If in particular the matrix to invert is approximated as diagonal the quality of the predictions results in a worse agreement with the experimental results, although a considerable improvement with respect to RPA is still obtained. For heavier systems, the diagonal approximation is used only in the inversion of the A_{22} matrix, which has been tested to be a reasonable approximation [8].

The main result is that the total SSRPA strength below 20-30 MeV is much smaller than in RPA and in better agreement with the corresponding experimental values. We show that this result depends very weakly on the employed parameterization and it is an intrinsic effect of the inclusion of the 2p2h configurations. Their density strongly increases with the excitation energy, leading to a high-energy tail in the spectrum.

The capability of describing charge exchange excitations in a efficient and reliable is expected to have a strong impact also in astrophysical studies where GT resonances play a crucial role. It also promises to improve energy density functional-based calculations of the nuclear matrix elements governing $0\nu\beta\beta$ decay, helping in understanding the strong model dependent results observed comparing the prediction of the nulcear matrix elements [17].

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