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Consistent description of mean-field instabilities and clustering phenomena within a unified dynamical approach

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Nuclear matter at subsaturation densities is expected to be inhomogeneous, owing to the existence of many-body correlations, which constitutes an essential feature for the construction of a reliable equation of state. A first emergent phenomenon related to this aspect is the fragmentation process, experimentally observed in heavy-ion collisions at intermediate energies as the result of mechanical (spinodal) instabilities driven by the mean-field, in connection to the occurrence of a liquid-gas phase transition. On the other hand, at even smaller densities, owing to residual few-body correlations, also the formation of light clusters as deuterons, or particularly strongly bound α particles which dissolve with increasing density due to the Pauli principle, is considered well established.

A consistent description of light clusters at low densities and the formation of heavy fragments through spinodal instabilities within the same theoretical approach is however still missing nowadays. In this work, we propose then a novel approach to include light clusters degrees of freedom within a non-relativistic kinetic theory based on energy density functionals, providing a unified dynamical framework to account at once for both phenomena, when out of equilibrium processes, as they occur in nuclear reactions, are considered. Implications for general aspects of reactions dynamics and in the widest scope of astrophysical applications are envisaged and discussed.

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Heavy-ion reactions serve as a powerful probe for transient states of nuclear matter (NM) under conditions far from saturation [1–3]. Such reactions offer valuable insights into the NM equation of state (EOS), a concept closely linked to various astrophysical phenomena [4, 5]. Specifically, during the expansion following initial compression in central heavy-ion collisions (HICs) at Fermi or intermediate energies (with beam energies $E/A \approx$ 30–300 MeV/nucleon), it becomes possible to investigate regions of low density and moderate temperature within the NM phase diagram [6]. In these conditions, large-scale correlations can arise from volume (spinodal) instabilities related to the liquid-gas phase transition, thus contributing to multifragmentation processes [7]. Additionally, at low densities, few-body correlations driven by short-range nucleon-nucleon interactions become prominent, facilitating the formation of light clusters such as deuterons and α particles [8, 9].

Energy density functional (EDF) models provide an effective framework to describe these processes from a thermodynamical perspective, incorporating clusters as explicit degrees of freedom (DOF) and characterizing dilute NM as a mixture of nucleons and nuclei [8, 10]. However, for analyzing out-of-equilibrium processes in dynamical scenarios, transport approaches are generally required. Yet, developing a transport model that consistently addresses both light clusters from few-body correlations and volume instabilities leading to heavier fragments remains a significant challenge for many-body theory [11, 12].

Further complexity arises as light clusters are expected to dissolve at higher densities (beyond the so-called Mott density) due to in-medium effects, primarily arising from Pauli blocking [13]. Beyond this threshold, bound clusters can only exist if their momentum exceeds a critical value, known as the Mott momentum [13]. Additionally, few-body correlations may persist beyond the Mott density in the form of continuum correlations, though these are often excluded from EDF-based models due to the difficulty in representing NM at high densities [9].

In this study, we introduce a novel approach to solve nuclear dynamics in the heterogeneous sub-saturation regime using a linear response analysis. Our aim is to build a unified theoretical framework that encompasses various fragment formation mechanisms in HICs by accounting for the interplay between nucleonic and light cluster DOF, while incorporating in-medium effects. Specifically, we examine how light clusters, which primarily emerge in the compression phase, influence the development of spinodal instabilities during the expansion phase, leading to fragmentation dynamics. The structure of this paper is as follows: Section 1 reviews the theoretical formalism initially proposed in Ref. [14]; Section 2 presents the main findings; Section 3 concludes with future research directions.

1. Theoretical formalism

The theoretical formalism was developed in Ref. [14], and readers seeking a detailed explanation are directed to that source. Here, we provide a summary of the primary formulas and concepts. Let us consider a system of nucleons—neutrons (n) and protons (p)—and one light cluster species (d), all in thermodynamic equilibrium at temperature T. The density ρ_j for each species is defined by

$$\rho_j = g_j \int_{\Lambda_j} \frac{d\mathbf{p}}{(2\pi\hbar)^3} f_j, \qquad j = n, p, d \tag{1}$$

where g_j denotes the spin degeneracy, f_j represents the phase-space distribution function, and Λ_j is the Mott momentum, which may vary with density and temperature and is introduced for clusters to incorporate in-medium effects ($\Lambda_q = 0$ for $q = n, p$) [12–14]. The total baryon density is given by $\rho_b = \sum_j \rho_j A_j$, where A_j and ρ_j ($j = n, p, d$) denote the mass numbers and densities.

Introducing a small perturbation δf_j to the initial distribution functions f_j, we use a linear response analysis to derive the collisionless (Vlasov) form of the Boltzmann equation [15]:

$$\partial_t(\delta f_j) + \nabla_{\mathbf{r}}(\delta f_j) \cdot \nabla_{\mathbf{p}}\varepsilon_j - \nabla_{\mathbf{p}} f_j \cdot \nabla_{\mathbf{r}}(\delta\varepsilon_j) = 0, \tag{2}$$

where the single-particle energy $\varepsilon_j = \frac{(2\pi\hbar)^3}{g_j}\frac{\delta\mathcal{E}}{\delta f_j(\mathbf{p})}$ (and its corresponding variation $\delta\varepsilon_j$) are derived from the energy density functional (EDF) $\mathcal{E}$, with the potential component $\mathcal{U}$ provided by a momentum-independent Skyrme-like interaction [14].

Using Eq. (1), the density fluctuation is expressed as:

$$\delta\rho_j(\mathbf{r}, t) = g_j \int_{\Lambda_j} \frac{d\mathbf{p}}{(2\pi\hbar)^3}\delta f_j - \delta_{jd}\sum_l \Phi_\lambda^{dl}\delta\rho_l, \tag{3}$$

where δ_{jd} is the Kronecker delta, with the second term on the r.h.s. capturing the variation in local density of light clusters due to shifts in in-medium effects controlled by the density-dependent cut-off. To better understand the impact of in-medium effects, we also consider a scenario in which this cut-off is fixed during density fluctuation propagation ($\Phi_\lambda^{dj} = 0$).

Equation (2) supports plane-wave solutions in which δf_j oscillates with a frequency ω and wave vector $\mathbf{k}$, represented as $\delta f_j \sim \sum_{\mathbf{k}} \delta f_j^{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$. Applying the Landau procedure [16], we obtain:

$$\delta\rho_j = -\chi_j \sum_l \tilde{F}_{0\lambda}^{jl}\delta\rho_l - \delta_{jd}\sum_l \Phi_\lambda^{dl}\delta\rho_l, \tag{4}$$

where $\chi_j = \chi_j(\omega, \mathbf{k})$ is the Lindhard function incorporating the cut-off, and $\tilde{F}_{0\lambda}^{jl}$ are the modified Landau parameters [14]. Solving the homogeneous system defined in Eq. (4) yields the dispersion relation that links frequency ω to wave number k. The spinodal region, where ω becomes imaginary and density fluctuations grow over time, leading to the system fragmenting, is identified by solving for $\omega = 0$ [7, 17, 18].

2. Results

We present results for the simplest scenario of symmetric nuclear matter (SNM) with deuterons. The left panel of Fig. 1 illustrates the spinodal region in the (ρ_b, T) plane. Specifically, the red area shows the case with the in-medium effects fully incorporated into the dynamics, compared to the cyan area, which omits the density dependence of the cut-off. Including light clusters as explicit DOF significantly affects the extent of the spinodal region. When in-medium effects are included, the spinodal boundary of the composite system closely matches that of pure nucleonic matter (grey line). In contrast, ignoring in-medium effects ($\Phi_\lambda^{dj} = 0$) would broaden the instability region. Notably, a separated instability region below 0.002 fm^{-3} and a metastable region at higher densities emerge at low temperatures in the full calculations, consistent with recent findings from Ref. [20].

Within the spinodal region, a purely imaginary ω signifies the growth rate of unstable modes. This rate is plotted against wave number k in the right panels of Fig. 1, for the same cases as in

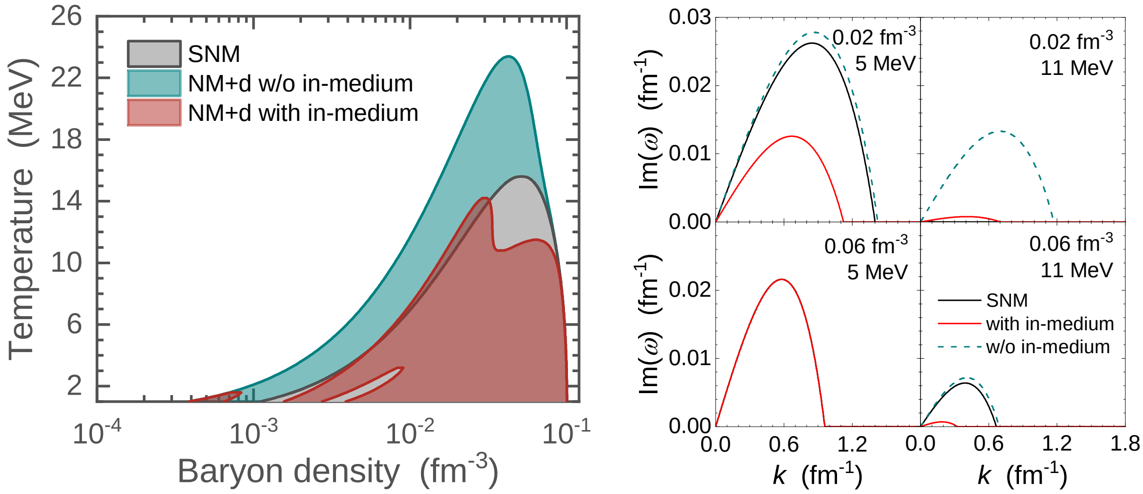

Figure 1: (Left panel) Spinodal region in the (ρ_b, T) plane for: 1) pure nucleonic matter (SNM, grey); 2) NM with deuterons, including in-medium effects along the dynamics (red); 3) NM with deuterons, neglecting in-medium effects along the dynamics ($\Phi_\lambda^{dj} = 0$, cyan). (Right panels) Growth rate of the instability, Im(ω), as a function of the wave-number k, for the same cases as in the left panels, at two density and two temperature values. Readapted from Refs. [14, 19].

the left panel, across various density and temperature values. The growth rate has a maximum, indicating that density fluctuations grow preferentially at a specific k. However, in-medium effects tend to reduce and shift this maximum to lower k-values, slowing the instability growth rate and eventually leading to different dominant fragmentation modes. Conversely, if in-medium effects are disregarded, the opposite trend appears. The impact of in-medium effects becomes less significant at densities beyond $\rho_0/3$, at least up to moderate temperatures ($T \lesssim 8$ MeV).

Further insight into the direction of the unstable modes in terms of density fluctuations is provided by the $(\delta\rho_S/\delta\rho_d)$ ratio, where $\rho_S = \rho_n + \rho_p$ represents the total isoscalar nucleonic density. In the left panels of Fig. 2, the relative ratio $\Delta = (\delta\rho_S/\delta\rho_d)/(\rho_S/\rho_d)$ is plotted as a function of ρ_b within the spinodal region for the two scenarios of in-medium effects discussed. Positive or negative values indicate that nucleon and deuteron fluctuations move in or out of phase, respectively. As also depicted in the sketch of Fig. 2, when in-medium effects are omitted, light clusters move in phase with the nucleons, enhancing instabilities and potentially supporting the formation of larger fragments. Conversely, with in-medium effects included, deuterons move out of phase respect to nucleons, migrating toward lower-density regions as nucleon density fluctuations increase and fragments form. Local in-medium effects thus generate a “distillation” mechanism, causing deuterons to eventually separate, potentially raising their yield in MF-based simulations of HICs at intermediate energies, as recent experimental data suggest [21].

3. Summary, outlooks and conclusions

In conclusion, we have introduced a new framework based on a linearized Vlasov approach to study the dynamics of dilute nuclear systems comprising nucleons and light clusters, incorporating in-medium Mott effects responsible for cluster dissolution.

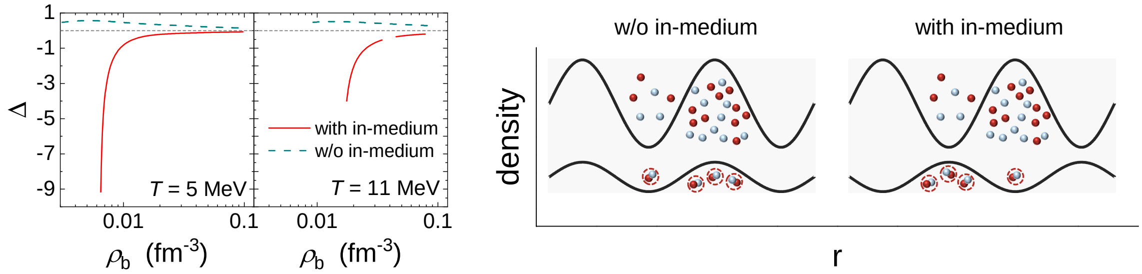

Figure 2: (Left panels) The relative ratio Δ (see text) as a function of the total baryon density ρ_b for NM with deuterons, neglecting (cyan) or including (red) in-medium effects in the dynamics, for two temperature values. Lines are drawn only for the density values lying inside the spinodal region. The sketch on the right illustrates the spatial fluctuations in nucleon density (top) and deuteron density (bottom), with in-medium effects either neglected or included throughout the propagation. Readapted from Refs. [14, 19].

Our findings reveal that light clusters, especially when in-medium effects are included, have a significant impact on the unstable modes that drive multifragmentation. In the absence of in-medium effects, clusters move in tandem with nucleons, facilitating fragment formation. By contrast, in-medium effects activate a “distillation” mechanism, leading clusters to migrate to lower-density regions, which slows down instability growth and modifies the prevailing fragmentation patterns.

Achieving a complete and unified understanding of the mechanisms behind fragment formation will require advancing beyond the current quasi-analytical approach toward fully numerical simulations. Nonetheless, this work opens a promising pathway for exploring out-of-equilibrium processes in heavy-ion collisions and other astrophysical scenarios.

References

[1] H. Zheng, S. Burrello, M. Colonna and V. Baran, *Pre-equilibrium effects in charge-asymmetric low-energy reactions*, *Phys. Lett. B* **769** (2017) 424.

[2] H. Zheng, S. Burrello, M. Colonna, D. Lacroix and G. Scamps, *Connecting the nuclear equation of state to the interplay between fusion and quasifission processes in low-energy nuclear reactions*, *Phys. Rev. C* **98** (2018) 024622.

[3] A. Sorensen, K. Agarwal, K.W. Brown, Z. Chajęcki, P. Danielewicz, C. Drischler et al., *Dense nuclear matter equation of state from heavy-ion collisions*, *Prog. Part. Nucl. Phys.* **134** (2024) 104080.

[4] S. Huth et al., *Constraining Neutron-Star Matter with Microscopic and Macroscopic Collisions*, *Nature* **606** (2022) 276.

[5] C.Y. Tsang, M.B. Tsang, W.G. Lynch, R. Kumar and C.J. Horowitz, *Determination of the equation of state from nuclear experiments and neutron star observations*, *Nat. Astron.* **8** (2024) 328.

[6] M. Colonna, *Collision dynamics at medium and relativistic energies*, *Prog. Part. Nucl. Phys.* **113** (2020) 103775.

[7] P. Chomaz, M. Colonna and J. Randrup, *Nuclear spinodal fragmentation*, *Physics Reports* **389** (2004) 263.

[8] S. Typel, G. Röpke, T. Klähn, D. Blaschke and H.H. Wolter, *Composition and thermodynamics of nuclear matter with light clusters*, *Phys. Rev. C* **81** (2010) 015803.

[9] S. Burrello and S. Typel, *Embedding short-range correlations in relativistic density functionals through quasi-deuterons*, *Eur. Phys. J. A* **58** (2022) 120.

[10] S. Burrello, F. Gulminelli, F. Aymard, M. Colonna and A.R. Raduta, *Heat capacity of the neutron star inner crust within an extended nuclear statistical equilibrium model*, *Phys. Rev. C* **92** (2015) 055804.

[11] A. Ono, *Dynamics of clusters and fragments in heavy-ion collisions*, *Prog. Part. Nucl. Phys.* **105** (2019) 139.

[12] R. Wang, Y.-G. Ma, L.-W. Chen, C.M. Ko, K.-J. Sun and Z. Zhang, *Kinetic approach of light-nuclei production in intermediate-energy heavy-ion collisions*, *Phys. Rev. C* **108** (2023) L031601.

[13] G. Röpke, *Nuclear matter equation of state including two-, three-, and four-nucleon correlations*, *Phys. Rev. C* **92** (2015) 054001.

[14] R. Wang, S. Burrello, M. Colonna and F. Matera, *Dynamics of dilute nuclear matter with light clusters and in-medium effects*, *Phys. Rev. C* **110** (2024) L031601.

[15] S. Burrello, M. Colonna, G. Colò, D. Lacroix, X. Roca-Maza, G. Scamps et al., *Interplay between low-lying isoscalar and isovector dipole modes: A comparative analysis between semiclassical and quantum approaches*, *Phys. Rev. C* **99** (2019) 054314.

[16] L. Landau, *On the theory of the fermi liquid*, *Sov. Phys. JETP* **8** (1959) 70.

[17] S. Burrello, M. Colonna and F. Matera, *Pairing effects on spinodal decomposition of asymmetric nuclear matter*, *Phys. Rev. C* **89** (2014) 057604.

[18] S. Burrello, M. Colonna and F. Matera, *Pairing effects on neutrino transport in low-density stellar matter*, *Phys. Rev. C* **94** (2016) 012801.

[19] S. Burrello, M. Colonna, F. Matera and R. Wang, *Dilute nuclear matter with light clusters and in-medium effects: Towards a unified dynamical framework*, *EPJ Web Conf.* **311** (2024) 00005.

[20] G. Röpke, D. Voskresensky, I. Kryukov and D. Blaschke, *Fermi liquid, clustering, and structure factor in dilute warm nuclear matter*, *Nucl. Phys. A* **970** (2018) 224.

[21] B. Borderie, N. Le Neindre, M. Rivet, P. Désesquelles, E. Bonnet, R. Bougault et al., *Phase transition dynamics for hot nuclei*, *Phys. Lett. B* **782** (2018) 291.