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QCD sum rule approach to Okamoto-Nolen-Schiffer anomaly

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A novel approach is proposed to link the charge symmetry breaking (CSB) nuclear interaction and the low-energy constants in quantum chromodynamics (QCD) for the first time by matching the CSB effect in nuclear matter. Resulting CSB interaction is applied to study the Okamoto-Nolen-Schiffer anomaly, still lacking of a satisfactory microscopic understanding, on the energy differences of mirror nuclei by taking ^{17}F-^{17}O, ^{15}O-^{15}N, ^{41}Sc-^{41}Ca, and ^{39}Ca-^{39}K as typical examples. The QCD-based CSB interactions are found to resolve the anomaly successfully within theoretical uncertainties.

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1. Introduction

An anomaly in the energy differences of mirror nuclei and isobaric analogue states was found more than 50 years ago and is called the Okamoto-Nolen-Schiffer (ONS) anomaly [1, 2]. This anomaly has been standing many years, but not yet well understood from a microscopic point of view. It was first reported by Okamoto for the case of ^{3}He-^{3}H system, and Nolen and Schiffer made a systematic study from light to heavy nuclei within the framework of the independent-particle model. They found that the theoretical values of the energy difference underestimate the experimental values by 3–9 %. In Table 1, the Coulomb energy differences between mirror nuclei with the mass $A = 16 \pm 1$ and $A = 40 \pm 1$ are tabulated. The Coulomb energies are calculated by Hartree-Fock (HF) approximation with a energy density functional (EDF) SGII. The Coulomb exchange energy is calculated in the exact integration of HF wave functions. As is clearly seen in Table 1, the calculated Coulomb energy is always smaller than the experimental value by 150–400 keV.

2. ONS anomaly and its physical implications

To cure this anomaly, many extra corrections have been discussed such as the finite proton size, the center of mass effect, the Thomas-Ehrman effect, the isospin impurity, the electromagnetic spin-orbit interaction, the proton-neutron mass difference in the kinetic energy, the core polarization effect, and the vacuum polarization. These corrections altogether explain only about 1 % of the discrepancy [3], but are found not enough to solve the problem. Some possible solutions are further listed as follows:

1. 10–20 % smaller proton radii of valence particles than those of cores to increase the Coulomb energies of valence particles. This prescription will certainly fill the gaps between calculated and experimental energy differences of mirror nuclei. However, empirically, the trend is opposite: the proton radii of valence particles are always larger than those of cores.

2. For the CSB interaction, some meson exchange interactions of isospin breaking pairs, such as ρ-ω, π-η and δ-σ meson exchange interactions, mediated by photons.

3. For charge independence breaking interaction (CIB), the difference between charged pion and neutral pion exchange interactions. The mass difference between two kinds of pions will create CIB interactions in the pion exchange potentials.

4. Introduce phenomenological CSB and CIB interactions such as the Skyrme type. The strengths can be determined by optimizing systematically empirical binding energy differences between mirror nuclei.

Among them, most plausible source to fill the gap is the charge symmetry breaking (CSB) nuclear interaction [1, 4–7]. This idea is originated from the difference of scattering lengths between *nn* and *pp* scatterings. The experimental scattering lengths are extracted from the nucleon-nucleon

Table 1: Differences of HF Coulomb direct and exchange energies between mirror nuclei with masses $A = 16 \pm 1$ and 40 ± 1. The SGII EDF is adopted for HF calculations. The unit is MeV.

Nuclei	^{17}F-^{17}O	^{15}O-^{15}N	^{41}Sc-^{41}Ca	^{39}Ca-^{39}K
Orbital	$1d_{5/2}$	$(1p_{1/2})^{-1}$	$1f_{7/2}$	$(1d_{3/2})^{-1}$
$\Delta E_{\rm D}$ (Coulomb)	3.596	3.272	7.133	6.717
$\Delta E_{\rm E}$ (Coulomb)	−0.203	0.026	−0.267	0.260
Sum	3.392	3.298	6.866	6.977
Expt. [16]	3.543	3.537	7.278	7.307

scattering data to be

$$\begin{aligned} a_{pp} &= -17.3 \pm 0.4\,\text{fm}, \\ a_{nn} &= -18.7 \pm 0.6\,\text{fm}, \\ a_{pn} &= -23.7 \pm 0.2\,\text{fm}. \end{aligned} \tag{1}$$

The difference between a_{pp} and a_{nn} shows the existence of the CSB force, while the difference between a_{pn} and $\left(a_{pp} + a_{nn}\right)/2$ suggests the CIB force of NN interaction.

Recently, phenomenological CSB interactions (often taken to be a Skyrme-type contact interaction) have been introduced to systematically calculate the isospin symmetry breaking (ISB) effect on top of the Coulomb interaction; they provide successful results for describing the isobaric analogue states, the mass differences of iso-doublet and iso-triplet nuclei, and also the double-beta decays [8–14]. However, both the magnitude and the sign of the parameters in phenomenological CSB interactions have not been well determined. Meanwhile, microscopic calculations of observables sensitive to isospin symmetry breaking terms in the nuclear Hamiltonian have also become available [15] although CSB effects have not been isolated in detail.

The aim of this contribution is to provide a QCD-based understanding of CSB by making a quantitative link between the Skyrme-type CSB interactions [10] and the CSB effect due to the u-d quark mass difference and the associated spontaneous symmetry breaking (SSB) in QCD [17–19]. To this end, we adopt a QCD sum rule approach, which can provide a microscopic model to discuss the CSB effect based on the idea of partial restoration of SSB in the nuclear medium.

To study QCD dynamics, Lattice QCD (LQCD) is the most fundamental approach, but demands extremely high computer power and is only feasible by usage of top-level super computer. To avoid such high computational burden, the chiral effective theory and the relativistic mean field model were proposed and applied for nuclear many-body problems successfully. Analogous theories in nuclear physics and condensed matter physics are Hartree-Fock theory and BCS model for superconductor. QCD sum rule approach has been known an effective and quantitative model to derive the baryon masses and hadron masses in term of the SSB due to the quark-antiquark $q\bar{q}$ chiral condensation in vacuum. This model can be extended to apply for the study of the nuclear medium effect on SSB, which is intimately related with CSB effect of the nucleon-nucleon interaction as will be discussed in the following section.

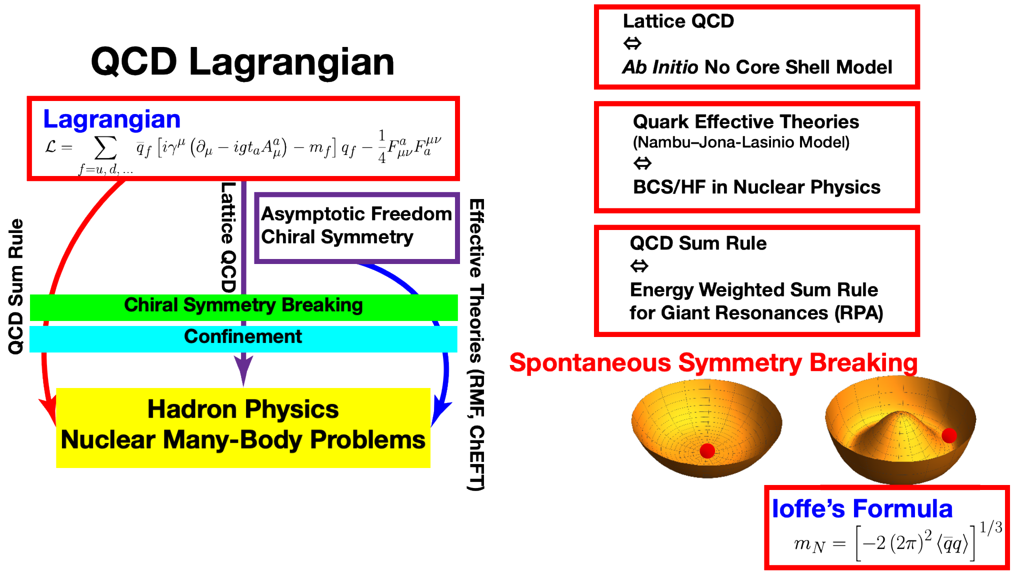

Figure 1: Various QCD approaches for hadron dynamics and quantum many-body problems. See the text for details.

3. CSB EDF from the QCD sum rule and its application

Let us start with the binding-energy difference between the neutron and the proton $\Delta_{np}(\rho)$ in infinite nuclear matter ($N = Z$) with the baryon density ρ, as defined by a difference of the momentum independent part of the Lorentz-scalar self-energies. In the leading order of the u-d quark mass difference and the quantum electrodynamics (QED) effect, an approximate formula has been obtained from the QCD sum rules (QSR) [18];

$$\Delta_{np}(\rho) \simeq C_1 G(\rho) - C_2, \tag{2a}$$

$$G(\rho) = \left(\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0} \right)^{1/3}. \tag{2b}$$

Here, $\langle \bar{q}q \rangle$ and $\langle \bar{q}q \rangle_0$ are, respectively, the isospin averaged in-medium and in-vacuum chiral condensate. The coefficient C_1 is proportional to the u-d quark mass difference δm [1], through the isospin-breaking constant $\gamma \equiv \left\langle \bar{d}d \right\rangle_0 / \langle \bar{u}u \rangle_0 - 1$ as $C_1 = -a\gamma$ with a positive numerical constant a determined by the Borel QSR method [18]. On the other hand, C_2 is a constant originating both from δm and the QED effect. Equation (2) is valid at low density $\rho < \rho_0 = 0.17\,\text{fm}^{-3}$ where the dimension-3 chiral condensate gives a dominant contribution in the operator product expansion in QSR. In the following, we take $C_1 = 5.24^{+2.48}_{-1.21}$ MeV. Since the C_2-term is density independent, it is cancelled out in the following analysis.

Equation (2) implies that $\Delta_{np}(\rho)$ tends to decrease in the nuclear medium associated with the partial restoration of chiral symmetry $G(\rho) < 1$. The in-medium chiral condensate in the leading

[1] The renormalization group invariant mass difference reads $\delta m \equiv m_d - m_u \simeq 3.6\,\text{MeV}$ [20].

order with the Fermi-motion correction has a universal form [21, 22]

$$\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0} \simeq 1 + k_1 \frac{\rho}{\rho_0} + k_2 \left(\frac{\rho}{\rho_0}\right)^{5/3} \quad \text{with} \quad k_1 = -\frac{\sigma_{\pi N}\rho_0}{f_\pi^2 m_\pi^2} < 0, \quad k_2 = -k_1 \frac{3k_{\mathrm{F0}}^2}{10 m_N^2} > 0, \tag{3}$$

where $\sigma_{\pi N}$ is the π-N sigma term, m_π (m_N) is the pion (nucleon) mass, and f_π is the pion decay constant. The Fermi-momentum of the symmetric nuclear matter at saturation is denoted by $k_{\mathrm{F0}} = \left(3\pi^2\rho_0/2\right)^{1/3} = 268\,\mathrm{MeV}$. Systematic calculations using the in-medium chiral perturbation theory shows that the full chiral corrections up to next-to-next-to leading order over Eq. (3) is numerically small for $\rho < \rho_0$ [23].

We decompose the mass difference between mirror nuclei $\Delta E = E\,(Z+1, N) - E\,(Z, N+1)$ into the Coulomb HF contribution ΔE_{C} and the ONS anomaly δ_{ONS} as

$$\Delta E = \Delta E_{\mathrm{C}} + \delta_{\mathrm{ONS}}. \tag{4}$$

On the basis of Eq. (2), the CSB effect to δ_{ONS} from the partial restoration of chiral symmetry in the uniform and symmetric ($N = Z$) nuclear matter δ_{chiral} can be estimated as [18]

$$\delta_{\mathrm{chiral}} \equiv \Delta_{np}\,(0) - \Delta_{np}\,(\rho) = C_1\,[1 - G\,(\rho)]\,. \tag{5}$$

Let us make an alternative evaluation of δ_{ONS} in Eq. (4) on the basis of the CSB interaction of EDF. First of all, the general form of $E\,(Z, N)$ for uniform nuclear matter up to the second order of $\beta = (N-Z)\,/A$ reads [14]

$$\frac{E}{A} \simeq \varepsilon_0\,(\rho) + \varepsilon_1\,(\rho)\,\beta + \varepsilon_2\,(\rho)\,\beta^2. \tag{6}$$

In particular, for $N = Z$, we find the $\Delta E|_{N=Z} = -2\varepsilon_1\,(\rho)$ where the effect of ε_0 and ε_2 disappears. Note that ε_1 is a genuinely CSB-type term coming only from the CSB EDF. We take the Skyrme-type CSB interaction [10] to evaluate its contribution to $\varepsilon_1\,(\rho)$ as

$$\begin{aligned} V_{\mathrm{CSB}}\,(\boldsymbol{r}) = \Big[& s_0\,(1 + y_0 P_\sigma)\,\delta\,(\boldsymbol{r}) + \frac{s_1}{2}\,(1 + y_1 P_\sigma)\left(\boldsymbol{k}^{\dagger 2}\delta\,(\boldsymbol{r}) + \delta\,(\boldsymbol{r})\,\boldsymbol{k}^2\right) \\ & + s_2\,(1 + y_2 P_\sigma)\,\boldsymbol{k}^\dagger \cdot \delta\,(\boldsymbol{r})\,\boldsymbol{k}\Big]\,\frac{\tau_{1z} + \tau_{2z}}{4}, \end{aligned} \tag{7}$$

where $\tau_{iz} = +1$ (-1) for neutrons (protons) is the z-direction of isospin operator of nucleon i, $\boldsymbol{k} = (\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2)\,/2i$, $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$, and $P_\sigma = (1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\,/2$ is the spin-exchange operator. In Eq. (7), s_0 and y_0 are the strength parameters of the contact CSB and its spin-exchange interactions, while s_1 (s_2) and y_1 (y_2) are the parameters of the momentum dependent s-wave (p-wave) CSB and its spin-exchange interactions, respectively. Equation (7) gives contributions to $\varepsilon_1\,(\rho)$ and hence δ_{ONS} as [14]

$$\delta_{\mathrm{Skyrme}} = -\frac{\tilde{s}_0}{4}\rho - \frac{1}{10}\left(\frac{3\pi^2}{2}\right)^{2/3}(\tilde{s}_1 + 3\tilde{s}_2)\,\rho^{5/3}, \tag{8}$$

where we have defined the effective coupling strengths,

$$\tilde{s}_0 \equiv s_0\,(1 - y_0)\,, \quad \tilde{s}_1 \equiv s_1\,(1 - y_1)\,, \quad \tilde{s}_2 \equiv s_2\,(1 + y_2)\,. \tag{9}$$

Note that the Thomas-Fermi approximation is adopted to evaluate the kinetic energy terms in Eq. (6).

Table 2: Parameters of the Skyrme-type CSB interactions constrained from the low-energy constants in QCD. To evaluate the CSB effect in finite nuclei where $\tilde{s}_1$ and $\tilde{s}_2$ contribute independently, two characteristic parameter sets (Case I and Case II) are introduced.

$\tilde{s}_0$ (MeV fm^3)	$-15.5^{+8.8}_{-12.5}$	
$\tilde{s}_1 + 3\tilde{s}_2$ (MeV fm^5)	$0.52^{+0.42}_{-0.29}$	
	Case I	Case II
$\tilde{s}_0$ (MeV fm^3)	$-15.5^{+8.8}_{-12.5}$	$-15.5^{+8.8}_{-12.5}$
$\tilde{s}_1$ (MeV fm^5)	$0.52^{+0.42}_{-0.29}$	—
$\tilde{s}_2$ (MeV fm^5)	—	$0.18^{+0.14}_{-0.10}$

There have been attempts to extract $\tilde{s}_{0,\,1,\,2}$ empirically by using various experimental data; the energy of isobaric analogue states (IAS) [8], the mass differences of isobar and isotriplet nuclei [11]. The value of $\tilde{s}_0$ estimated from IAS in ^{208}Pb is $\tilde{s}_0 = -52.6 \pm 1.4\,\text{MeV fm}^3$, while the mass differences of mirror nuclei lead to two estimates; $\left(\tilde{s}_0, \tilde{s}_{1,\,2}\right) = \left(-29.2 \pm 1.2\,\text{MeV fm}^3, 0\right)$ and $(\tilde{s}_0, \tilde{s}_1, \tilde{s}_2) = \left(44 \pm 8\,\text{MeV fm}^3, -56 \pm 16\,\text{MeV fm}^5, -31.2 \pm 3.2\,\text{MeV fm}^5\right)$. Since the contributions of $\tilde{s}_0$ and $\tilde{s}_{1,\,2}$ tend to cancel each other in physical observables, it is rather difficult to determine the magnitude and the sign of each term only from the existing experimental data.

On the other hand, our approach is to constrain $\tilde{s}_{0,\,1,\,2}$ from the low-energy constants in QCD, $C_1 = -a\gamma$ and $\sigma_{\pi N}$, by matching $\delta_{\text{Skyrme}}(\rho)$ in Eq. (8) and $\delta_{\text{chiral}}(\rho)$ expanded up to $O\left(\rho^{5/3}\right)$ at low densities. Then, we obtain

$$\tilde{s}_0 = -\frac{4}{3}\frac{C_1\sigma_{\pi N}}{f_\pi^2 m_\pi^2}, \qquad \tilde{s}_1 + 3\tilde{s}_2 = \frac{1}{m_N^2}\frac{C_1\sigma_{\pi N}}{f_\pi^2 m_\pi^2}. \tag{10}$$

The values $\tilde{s}_0$ and $\tilde{s}_1 + 3\tilde{s}_2$ determined are summarized in Table 2.

Let us now turn to the comparison of the theoretical values with our CSB interaction with the experimental mass difference of the mirror nuclei. The direct and exchange contributions of the Coulomb interaction (ΔE_{D} and ΔE_{E} with $\Delta E_{\text{C}} = \Delta E_{\text{D}} + \Delta E_{\text{E}}$) are obtained with the exact treatment of the exchange term. The sum of extra contributions (denoted Extra in Fig. 2) including the finite-size effect of nucleon, the center-of-mass effect on nuclear density, the Thomas-Ehrman effect δ^1_{NN}, the isospin impurity δ^2_{NN}, the electromagnetic spin-orbit interaction, the core polarization effect of the last nucleon, the proton and neutron mass difference in the kinetic energy, and the vacuum polarization. Each contribution varies from -150 keV to 150 keV, while the net result is at most 100 keV due to a strong cancellation. As can be seen in Fig. 2, the Extra contributions are too small to fill the difference between $\Delta E_{\text{Expt.}} - \Delta E_{\text{C}}$, but the CSB contributions constrained by the low-energy constants in QCD fill the gap quite successfully. We adopt two EDFs (SGII and SAMi) in the ΔE_{C} calculations, and found the difference is very small. Thus, the EDF model dependence is concluded to be a minor effect.

4. Summary

In summary, we determined the EDF parameters of Skyrme-type CSB interactions, not only the contact term ($\tilde{s}_0$) but also the momentum-dependent terms ($\tilde{s}_{1,\,2}$), by utilizing the low-energy

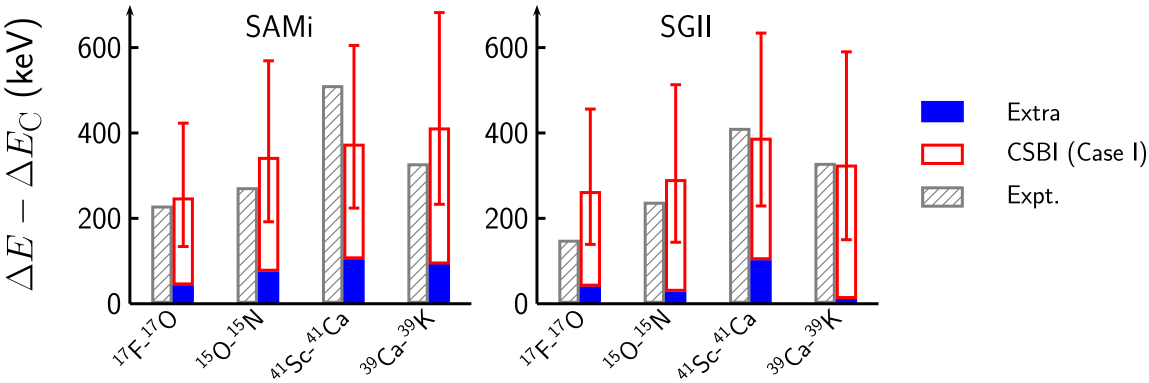

Figure 2: Comparisons of the experimental ONS anomaly $\Delta E_{\mathrm{Expt.}} - \Delta E_{\mathrm{C}}$ (grey hatched bars) and the corresponding theoretical estimates in two EDFs (SGII and SAMi). The contribution from the QCD-based CSB interaction (CSBI) in Case I and the extra contributions are indicated by the red bars with error bars and the blue bars, respectively.

constants in QCD and the density dependence of chiral condensation of $\bar{q}q$ pair in the nuclear medium. This is the first attempt to derive the coupling strengths of CSB EDF from the QCD-based approach. The resulting CSB interaction is applied to resolve the ONS anomaly: the numerical results for the mirror nuclei ($A = 16 \pm 1$ and 40 ± 1 with the isosymmetric core $N = Z = A/2$) with the two Skyrme EDFs (SGII and SAMi) show good agreement with experimental data both in sign and magnitude within the theoretical uncertainty. We will further develop a microscopic approach to derive CIB interaction from the mass difference between charged pion and neutral pion by the density matrix expansion method, and apply it for the RPA calculations of IAS in $N > Z$ nuclei in the next step of the project.

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