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Low-energy K^+N scattering revisited and in-medium strange quark condensate

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The chiral ward identity connects the in-medium quark condensate to the soft limit value of a correlation function of the pseudoscalar fields evaluated in nuclear medium. For the strange quark condensate, one considers the correlation function of the pseudoscalar fields with strangeness. The correlation function describes in-medium propagation of kaons and it is obtained phenomenologically by kaon-nucleon scattering in the low density approximation. We describe the kaon-nucleon scattering amplitude in chiral perturbation theory and its low energy constants are determined by existent K^+N scattering data. Performing analytic continuation of the scattering amplitude obtained by chiral perturbation theory, we can take the soft limit of the scattering amplitude. With this amplitude, we evaluate the in-medium strange quark condensate based on hadron phenomenology.

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1. Introduction

The Nambu-Goldstone bosons are special hadrons that appear as a consequence of dynamical breaking of chiral symmetry. Their dynamics in low energies are strongly constrained by the dynamical symmetry breaking and described well by chiral effective theories such as chiral perturbation theory. Kaons are considered to be the Nambu-Goldstone bosons, and low energy KN scatterings are expected to be described by chiral perturbation theory, because KN is the lowest channel in the strangeness $S = +1$ channel and there are no resonances coupling strongly with the KN channel. The Nambu-Goldstone bosons are sensitive to the change of the vacuum structure, and thus they can be a probe of the in-medium change of the quark condensates. Actually, the up and down quark condensates in nuclear medium are studied with pion-nucleus systems, such as pionic atoms and low energy pion nucleus scattering [1–5]. It is found that the magnitude of the u and d quark condensates may be reduced by about 40% at the nuclear saturation density [5, 6]. This is known as partial restoration of chiral symmetry in nuclear medium. For a systematic study of partial restoration of chiral symmetry, it is interesting to see how the strange quark condensate behaves in nuclear matter. In addition, it was recently suggested in Ref. [7] that there could be present a broad resonance in the $S = +1$ channel with isospin $I = 0$ around $W = 1650$ MeV. In these contexts it is very important to revisit the K^+N scattering in the aspect of chiral symmetry [8]. This report is based on our recent work [9].

2. Correlation function approach

To formulate the quark condensate in a nuclear medium, let us consider the following correlation function [5, 9–12]:

$$\Pi_{5\mu}(x;\rho) = \langle\Omega|\mathrm{T}[A_\mu(x)P^\dagger(0)]|\Omega\rangle, \tag{1}$$

where $A_\mu(x)$ is the axial vector current, $P(x)$ is the pseudoscalar field and $|\Omega\rangle$ is the ground state of the symmetric nuclear medium specified by the nuclear density ρ. We consider the K^+ channel to discuss the strange quark condensate and, thus, the axial vector current and the pseudoscalar field are given by the u and $\bar{s}$ fields as

$$A_\mu = \frac{1}{\sqrt{2}}\bar{s}\gamma_\mu\gamma_5 u, \qquad P = \sqrt{2}\bar{s}i\gamma_5 u, \tag{2}$$

respectively. These operators observe chiral algebra; the axial vector current $A_\mu(x)$ is one of the Noether currents associated with the SU(3) chiral symmetry and the pseudoscalar current $P(x)$ is transformed under the chiral transformation generated by $Q_5 = \int d^3x A_0(x)$ as

$$[Q_5, P^\dagger(x)] = -iS(x), \tag{3}$$

where the scalar field is defined by $S = \bar{u}u + \bar{s}s$. We have also the PCAC relation connecting the divergence of the axial current with the pseudoscalar field as

$$\partial^\mu A_\mu(x) = \frac{m + m_s}{2} P(x), \tag{4}$$

with the light quark mass $m = m_u = m_d$ in the isospin limit and the strange quark mass m_s.

The divergence of the operator product in Eq. (1) can be calculated as

$$\partial^\mu \mathrm{T}[A_\mu(x)P^\dagger(0)] = \mathrm{T}[\partial^\mu A_\mu(x)P^\dagger(0)] + \delta(x_0)\left[A_0(x), P^\dagger(0)\right], \tag{5}$$

which is known as Ward identity. Using this identity, we obtain a relation of correlation functions in the momentum space as

$$-iq^\mu \Pi_{5\mu}(q;\rho) = \frac{m+m_s}{2}\Pi(q;\rho) + \int d^3x e^{-\boldsymbol{p}\cdot\boldsymbol{x}} \left\langle \Omega \middle| \left[A_0(x), P^\dagger(0)\right] \middle| \Omega \right\rangle, \tag{6}$$

where the correlation function $\Pi(q)$ is defined as the Fourier transform of the correlation function for the pseudoscalar fields, $\Pi(x,\rho)$, as

$$\Pi(x,\rho) = \langle \Omega | T[P(x)P^\dagger(0)] | \Omega \rangle. \tag{7}$$

Taking the soft limit $q_\mu \to 0$ of Eq. (6) with Eq. (3), we obtain

$$\left[-iq^\mu \Pi_{5\mu}(q;\rho)\right]_{q=0} = \frac{m+m_s}{2}\,\Pi(q;\rho)|_{q=0} - i\langle \Omega | \bar{u}u + \bar{s}s | \Omega \rangle. \tag{8}$$

Because there are no zero modes accompanied with the spontaneous breaking of chiral symmetry for the finite quark masses, the left hand side of Eq. (8) vanishes at the soft limit. Then we find

$$\langle \bar{u}u + \bar{s}s \rangle^* \equiv \langle \Omega | \bar{u}u + \bar{s}s | \Omega \rangle = -i\frac{m+m_s}{2}\,\Pi(q;\rho)|_{q=0}. \tag{9}$$

For the chiral limit, the first term in the right hand side of Eq. (8) goes to zero, while the left hand side gives finite contributions because the correlation function must have a massless pole of the Nambu-Goldstone boson that cancels to q_μ. Because the relation (9) is obtained based on the operator relation (5), the corresponding relation in vacuum is also satisfied:

$$\langle \bar{u}u + \bar{s}s \rangle_0 \equiv \langle 0 | \bar{u}u + \bar{s}s | 0 \rangle = -i\frac{m+m_s}{2}\,\Pi(q;0)|_{q=0}. \tag{10}$$

The ratio of Eqs. (9) to (10) provide us the in-medium modification of the quark condensate. Thus, for the evaluation of the in-medium change of the quark condensate, we need to calculate the correlation function $\Pi(q;\rho)$ in the nuclear medium and take its soft limit.

According to low density expansion [13, 14], the nuclear matrix element can be expanded as

$$\langle \Omega | T[P(x)P^\dagger(0)] | \Omega \rangle = \langle 0 | T[P(x)P^\dagger(0)] | 0 \rangle + \rho \langle N | T[P(x)P^\dagger(0)] | N \rangle + O(\rho^{n>1}), \tag{11}$$

where $|N\rangle$ is a nucleon state. With the reduction formula, the nucleon matrix element can be written by using the isospin averaged kaon-nucleon scattering amplitude $T_{KN}(q)$ [15] as

$$\int d^4x e^{iq\cdot x} \langle N | T[P(x)P^\dagger(0)] | N \rangle = \frac{i}{q^2 - M_K^2}\frac{G_K^2}{q^2 - M_K^2}\left(-\frac{T_{KN}(q)}{2M_N}\right), \tag{12}$$

where G_K is the coupling of $P(x)$ to the kaon state in vacuum, $G_K = \langle 0 | P | K^+ \rangle$, and q is the forward kaon momentum. Finally we obtain the in-medium change of the quark condensate in terms of phenomenological quantities of the K^+N scattering amplitude $T_{KN}(q)$ in the linear density approximation as

$$\frac{\langle \bar{u}u + \bar{s}s \rangle^*}{\langle \bar{u}u + \bar{s}s \rangle_0} = 1 + \frac{\rho}{M_K^2}\frac{T_{KN}(q)|_{q=0}}{2M_N}, \tag{13}$$

where we have to take unphysical soft limit $q_\mu \to 0$ for the kaon momentum in the KN scattering amplitude. This can be done if the KN scattering amplitude is obtained as an analytic function. Chiral perturbation theory is one of the theoretical tools to obtain analytic forms of the scattering amplitudes.

3. Scattering amplitudes obtained by chiral perturbation theory

As discussed in the previous section, the low-density behavior of the in-medium quark condensate with strange quarks can be expressed by using the KN scattering amplitude at the soft limit. To access the soft limit of the scattering amplitude we need to have its analytic expression. Here we perform such analytic continuation to the soft limit by using the scattering amplitude obtained in chiral perturbation theory. We calculate the KN scattering amplitudes with isospin $I = 0$ and $I = 1$ up to the next leading orders of the chiral expansion. We also add some terms of the next-to-next-to leading order that contain the strange quark mass in order to improve extrapolation of the strange quark sector. In the calculation we do not impose the on-shell conditions for the initial and final kaons because the soft limit is not on the mass shell. The KN scattering amplitudes in the present calculation have 12 independent low-energy constants (LECs). By taking the soft limit of the isospin-averaged KN scattering amplitudes we obtain

$$\frac{\langle \bar{u}u + \bar{s}s\rangle^*}{\langle \bar{u}u + \bar{s}s\rangle_0} = 1 + \frac{\rho}{2M_N M_K^2}\frac{3T_{KN}^{I=1}(q)\big|_{q=0} + T_{KN}^{I=0}(q)\big|_{q=0}}{4} = 1 + \frac{3b^{I=1} + b^{I=0}}{F_K^2}\rho. \tag{14}$$

Among the twelve LECs two of them are relevant to the quark condensate.

As detailed in Ref. [9], we fix the model parameters by chi-square fitting with using existing experimental data up to $P_{\rm lab}$ = 800 MeV/c, where inelastic contributions, such as pion production, become significant; K^+p elastic differential cross sections ($P_{\rm lab}$ = 145 to 726 MeV/c), charge-exchange $K^+n \to K^0p$ differential cross sections ($P_{\rm lab}$ = 434 to 780 MeV/c), and $I = 1$ and $I = 0$ total cross sections. We did not use the differential cross section of K^+n elastic scattering due to large ambiguities. When we compare the differential cross sections of the K^+p elastic scattering between the theoretical calculation and the experimental data, we add Coulomb correction in the theoretical scattering amplitudes. To investigate theoretical uncertainties, we try four options for the chi square fitting; choice of the data of the $I = 0$ total cross section and inclusion of resonance contributions. In FITs 1 and 2, we use different data sets of the $I = 0$ total cross section because these two data sets look inconsistent with each other. In FITs 3 and 4, we include contribution of a possible broad resonance with strangeness $S = +1$ in the P_{01} and P_{03} channel, respectively. The broad resonance has been suggested in Ref. [7] by the analysis of the chiral unitary approach for the KN scattering. There are found two candidates of a $S = +1$ resonance; one with 1617 MeV mass, 305 MeV width and $J^P = \frac{1}{2}^+$, and the other with 1678 MeV mass, 463 MeV width and $J^P = \frac{3}{2}^+$.

4. Result

The K^+p elastic scattering cross sections are reproduced well thanks to a wide momentum range of accurate experimental data, while the scattering cross sections with $I = 0$ are not satisfactorily reproduced. Comparison of the calculated cross sections with the experimental data are shown in Figs. 1 to 6 in Ref. [9]. After fitting the low energy constants, we find the value of the slope parameter for the in-medium reduction of the quark condensate in Eq. (14) as summarized in Table 1. This result shows that the quark condensate may be reduced by 10 to 20% at the saturation density in FITs 2, 4 and 3', while other two FITs would suggest somewhat unrealistic behavior. (See Fig. 7 in Ref. [9].) This is because the KN scattering amplitude with $I = 0$ is not theoretically

Table 1: Values of the slope parameter $(3b^{I=1} + b^{I=0})$ in Eq. (14) in unit of GeV^{-1} [9]. FIT 3′ is a second best solution of FIT3.

$[\mathrm{GeV}^{-1}]$	FIT 1	FIT 2	FIT 3	FIT 4	FIT 3′
$3b^{I=1} + b^{I=0}$	−6.87	−1.86	2.02	−0.96	−1.98

under control in chiral perturbation theory, in particular in low energies. The K^+n scattering cross sections are experimentally extracted by the kaon-deuteron scattering. One should be careful of the Fermi motion correction and the final state interaction in the three body system.

5. Conclusion

We have investigated the in-medium quark condensate which can be evaluated by the correlation function of the pseudoscalar field in the soft limit. It connects to the change of the quark condensate in nuclear medium to the low energy scattering amplitude and one can obtain the reduction of the quark condensates in nuclear medium based on hadronic phenomenology. We have described the K^+N scattering amplitudes by using chiral perturbation theory, which enables us to perform analytic continuation of the scattering amplitudes to the soft limit. We have found that the chiral perturbation theory calculation perfectly (nicely) reproduces the K^+p elastic scattering amplitude up to $P_{\mathrm{lab}} = 500$ (800) MeV/c, while the K^+n scattering amplitude with $I = 0$ is not reproduced well. Thus, the $I = 0$ amplitude is not yet under control theoretically and there are still ambiguities in low energy amplitudes to extrapolate to the soft limit.

For prospects, because the K^+n scattering cross sections are observed in K^+d reactions, it is very important to perform theoretical calculation of the K^+d reaction and direct comparison with experiments [16]. This is possible by following machinery developed in Ref. [17] for K^-d reaction. Alternatively, to observe $I = 0$ KN scattering amplitudes a K^0 beam may be available in the K-long facility [18]. To separate out the strange quark condensate form Eq. (13), we need to consider the flavor SU(3) breaking effects both in-vacuum and in-medium calculations. Further calculations beyond the linear density are extremely interesting. For this purpose the correlation function must be calculated directly based on in-medium chiral perturbation theory.

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