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Hadronic vacuum polarization contribution to the muon g-2 on Euclidean windows from tau data

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We computed for the first time the τ data-driven Euclidean windows for the hadronic vacuum polarization contribution to the muon g-2. We showed that τ-based results agree with the available lattice window evaluations and with the full result. On the intermediate window, where all lattice evaluations are rather precise and agree, τ-based results are compatible with them. This is particularly interesting, given that the disagreement of the e^+e^- data-driven result with the lattice values in this window is the main cause for their discrepancy, affecting the interpretation of the a_μ measurement in terms of possible new physics.

PoS(QNP2024)216

10th International Conference on Quarks and Nuclear Physics (QNP2024)
8-12 July, 2024
Barcelona, Spain

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1. Introduction: a_μ and HVP

The uncertainty of the SM prediction for the anomalous magnetic moment of the muon, a_μ, is dominated by the hadronic contributions which, according to the White Paper [1] (to be updated by the end of this year), are at the level of 0.37(0.15) ppm in the Hadronic Vacuum Polarization, HVP (Hadronic Light-by-light) pieces, respectively. This uncertainty of 44×10^{-11} is slightly more than twice that of the experimental average, coming from the BNL [2] and FNAL [3, 4] measurements, that is 0.19 ppm, close to the FNAL goal of 0.14 ppm. The E34 experiment at J-PARC aims to perform ultra-precise measurements of a_μ and the muon electric dipole moment using a different method, which will have a completely independent systematic uncertainty [5]. Taken at face value, the difference between the White Paper results and the a_μ average measurement would be slightly larger than 5σ, the usual reference to claim (indirect) new physics discovery.

The SM uncertainty is thus dominated by the HVP contribution, which has been traditionally computed via a dispersion relation in terms of experimental data [6]

$$\begin{aligned} a_\mu^{\mathrm{HVP,LO}} &= \frac{\alpha^2}{3\pi^3}\int_{m_\pi^2}^{\infty} ds\frac{K(s)}{s}R(s)\,,\\ R(s) &= \frac{\sigma^0(e^+e^-\longrightarrow \mathrm{hadrons}(\gamma))}{\sigma_{pt}}\,, \qquad \sigma_{pt}=\frac{4\pi\alpha^2}{3s}\,, \end{aligned} \tag{1}$$

where $K(s)$ is a QED kernel function concentrated at low energies [7]. Alternatively, it can also be obtained using the spectral function of $\tau^-\longrightarrow\nu_\tau$ hadron decays, which can be related to the isovector component of the required e^+e^- cross-section through an isospin rotation. Given the current puzzle with e^+e^- data (that we will summarize shortly), we insist it is a good strategy to keep using both. Particularly, tau data can be very useful for the $\pi\pi$ contribution, which amounts to $\sim 73\%$ of the whole $a_\mu^{\mathrm{HVP,LO}}$ and to $\sim 58\%$ of its uncertainty at low energies [8]. For this final state ($\sqrt{s}$ is the $\pi\pi$ invariant mass),

$$\begin{aligned} \sigma^0_{\pi^+\pi^-(\gamma)}(s) &= \frac{\pi\alpha^2\beta^3_{\pi^+\pi^-}(s)}{3s}|F_V(s)|^2\\ &= \frac{K_\sigma(s)}{K_\Gamma(s)}\frac{d\Gamma_{\pi\pi[\gamma]}}{ds}\frac{R_{\mathrm{IB}}(s)}{S_{\mathrm{EW}}}\,, \end{aligned} \tag{2}$$

where the first factor of the last member includes kinematical functions, the second one is the tau spectra, and short-distance electroweak radiative corrections are encoded in S_{EW}. Isospin-breaking (IB) enters $R_{\mathrm{IB}}(s)$, which can be written as

$$R_{\mathrm{IB}}(s) = \frac{FSR(s)}{G_{\mathrm{EM}}(s)}\frac{\beta^3_{\pi^+\pi^-}(s)}{\beta^3_{\pi^0\pi^-}(s)}\left|\frac{F_V(s)}{f_+(s)}\right|^2\,, \tag{3}$$

where $G_{\mathrm{EM}}(s)$ is the long-distance electromagnetic radiative corrections factor [9, 10], that is challenging, as well as the ratio of the neutral-to-charged pion form factors. The different results obtained either way can be interpreted as new physics (with Wilson coefficients ϵ_i) affecting tau decays [11, 12]

$$\frac{a_\mu^\tau - a_\mu^{e^+e^-}}{2a_\mu^{e^+e^-}} = \epsilon_L^{d\tau}-\epsilon_L^{de}+\epsilon_R^{d\tau}-\epsilon_R^{de}+1.7\epsilon_T^{d\tau}\,, \tag{4}$$

whose implications have also been studied in refs. [13, 14]. The very precise BMW coll. evaluation [15] also challenged the e^+e^- data-driven a_μ^{HVP}. Their result would imply only a 1.7σ deviation with respect to the world average. Furthermore, the recent CMD-3 measurement [16, 17] of the e^+e^- cross-section conflicts severely with previous data, particularly with KLOE's [18] (not so much with the other very accurate measurement, from BaBar [19]). CMD-3 alone would imply an a_μ SM prediction in agreement with the measurement within one sigma, as advocated by the recent mixed lattice–data driven evaluation of ref. [20].

2. Long-distance radiative corrections

The G_{EM} factor was originally studied by Cirigliano *et al.* in refs. [9, 10], where it was computed in Resonance Chiral Theory ($R\chi T$) [21, 22] [1], including those operators that saturate the chiral low-energy constants up to $O(p^4)$ [30–32]. A recalculation of this factor was performed by Flores-Báez *et al.* [33] using a vector meson dominance (VMD) model. Both results agreed but for the contribution due to the diagrams with a ρ-ω-π vertex in VMD (which appears at $O(p^6)$ in $R\chi T$). In ref. [34] we extended the $R\chi T$ computation including operators contributing up to $O(p^6)$ in the chiral expansion and confirmed the important rôle played by the odd-intrinsic parity sector contributions to the $G_{\text{EM}}(s)$. We also considered either the short-distance constraints on the $R\chi T$ operators rendering well-behaved two-point correlators [21, 22] or extending this to three point-functions [36–38]. Our results agree with other tau-based determinations [41–48], with our IB contributions to $a_\mu^{\text{HVP,LO}}|_{\pi\pi}$ in the range $[-20.52, -6.96] \times 10^{-10}$ at 68% confidence level (we are more precise in the recent [49]).

3. a_μ^{HVP} evaluations

The tension between the different sets of e^+e^- data can be appreciated in Fig. 1, showing the $\pi\pi$ contribution to $a_\mu^{\text{HVP, LO}}$ around the ρ peak, found using either $\sigma(e^+e^- \to \text{hadrons})$ (top part of the plot, the average in yellow excludes the CMD-3 point, to emphasize its impact) or the $\tau^- \to \pi^-\pi^0\nu_\tau$ spectrum (bottom of the figure, with mean in green which agrees with CMD-3). Clearly, tau data yields a larger value, by $\sim 10 \times 10^{-10}$.

In ref. [50] we applied these results to the window quantities introduced in Ref. [51], and used in [52]. We recall that the different contributions of these windows, short-distance (SD), intermediate (int) and long-distance (LD), to a_μ^{HVP} scale as $\sim 1 : 10 : 25$, respectively, so that the relative accuracy needed varies substantially between them.

Our most important results for the three different window contributions to a_μ^{HVP} are represented in Fig. 2, where the different τ measurements [53–56] agree remarkably. In the SD and int windows, e^+e^- (from Ref. [52]) and τ data-based results are at variance.

Our τ-based $\pi\pi$ contribution to $a_\mu^{\text{HVP, LO}}$ is complemented with that from the other modes, to confront it directly with the full evaluations. We considered two approaches, as detailed in Ref. [50],

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[1] $R\chi T$ has been used to successfully evaluate other contributions to a_μ [23–29].

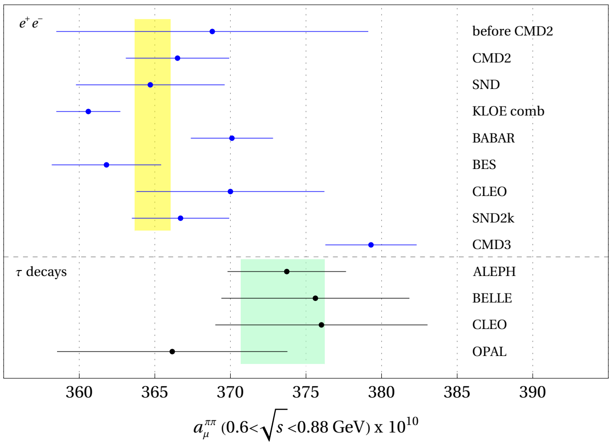

Figure 1: The $\pi\pi(\gamma)$ contribution to $a_\mu^{\text{HVP, LO}}$ around the ρ peak, obtained from the $e^+e^- \to \pi^+\pi^-(\gamma)$ cross section (top) and di-pion τ decays (bottom).

and the difference between them gave the associated error to this procedure. Thus, we obtained the results displayed in Fig. 3. A clear tendency of τ-based evaluations agreeing with the lattice outcomes [57–60] is exhibited, whereas the e^+e^- ones differ clearly with both (our results were corroborated by ref. [61]). This discrepancy is almost entirely due to the light-quark connected contribution, dominated by the $\pi\pi$ channel (it is $\sim 81\%$) [62, 63].

4. Conclusions

There is a global effort in improving the evaluation of the hadronic contributions to a_μ. Specifically, dedicated studies to improve the HVP part –which dominates the SM uncertainty– from lattice, dispersion relations, data-driven methods, improved e^+e^- data and Monte Carlo generators for the low-energy hadron cross-section [64] are being undertaken.

Through the years, the tau-data driven computation has always been approximately $[2, 2.5]\sigma$ away from the experimental average, while the tension with e^+e^- data was systematically larger than three sigmas. The most recent lattice QCD results by the Mainz/CLS, ETMC, RBC/UKQCD Colls. agree remarkably with BMW in the intermediate window. It is then of utmost importance than another lattice computation reaches a comparable accuracy to BMW in the long-distance window.

We showed that tau based results are compatible with the lattice evaluations in the intermediate window, while the e^+e^- data are in tension with both. This puzzle deserves further scrutiny.

Acknowledgments

We gladly acknowledge the QNP24 organizing committees. We thank financial support through the Ministerio de Ciencia e Innovación under Grant No. PID2020–112965 GB-I00, by the Departament de Recerca i Universitats from Generalitat de Catalunya to the Grup de Recerca “Grup de Física Teòrica UAB/IFAE” (Codi: 2021 SGR 00649) and partial Conahcyt support, from project CB2023-2024-3226.

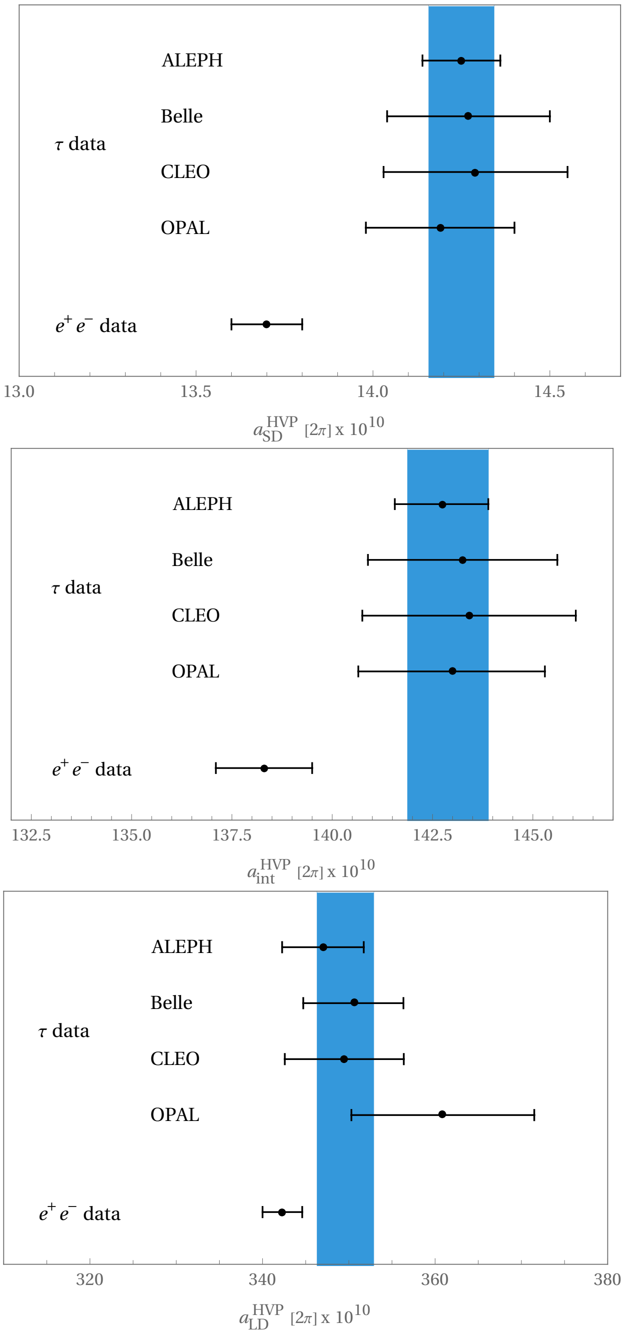

Figure 2: Window quantities (*SD* top, *int* medium, and *LD* bottom) for the 2π contribution below 1.0 GeV to a_μ^{HVP}, corresponding to our reference results. The τ data mean is shown in blue, with the e^+e^- result from [52].

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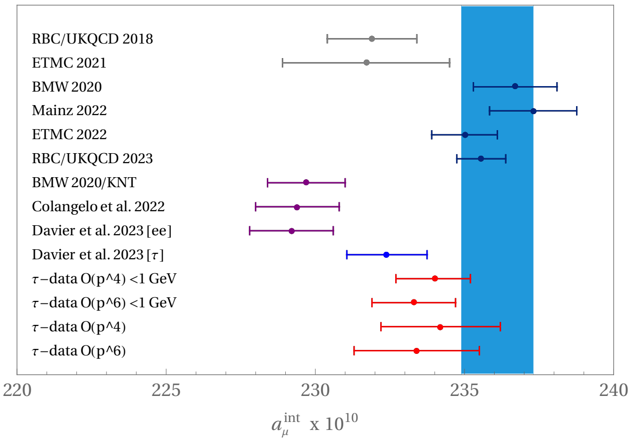

Figure 3: Comparison of the total intermediate window contribution to $a_\mu^{\text{HVP, LO}}$ according to lattice QCD, e^+e^- and τ data-driven evaluations. The blue band is the weighted average of the lattice results excluding those superseded, RBC/UKQCD 2018 [51] and ETMC 2021 [57] (by refs. [58] and [60], respectively).

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