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Study of the pion-mass dependence of ρ-meson properties in lattice QCD

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We collect spectra extracted in the $I = \ell = 1$ $\pi\pi$ sector provided by various lattice QCD collaborations and study the m_π dependence of ρ-meson properties using Hamiltonian Effective Field Theory (HEFT). In this unified analysis, the coupling constant and cutoff mass, characterizing the $\rho - \pi\pi$ vertex, are both found to be weakly dependent on m_π, while the mass of the bare ρ, associated with a simple quark-model state, shows a linear dependence on m_π^2. Both the lattice results and experimental data can be described well. Drawing on HEFT's ability to describe the pion mass dependence of resonances in a single formalism, we map the dependence of the phase shift as a function of m_π, and expose interesting discrepancies in contemporary lattice QCD results.

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1. introduction

Experimentally, the ρ meson is identified as a broad peak around $\sqrt{s} = 770$ MeV in the invariant mass distribution of the isovector P-wave of $\pi\pi$ scattering. It is often identified as a confined $q\bar{q}$ state, consistent with the constituent quark model. This picture is supported by several theoretical arguments, such as the the large-N_c limit of QCD Nevertheless, the sizable decay width, $\Gamma_{\rho\to\pi\pi} \approx 140$ MeV, signifies the ρ meson's strong coupling to the $\pi\pi$ channel. In other words, the observed peak structure results from the interaction between a $q\bar{q}$ state, referred to as the bare ρ, and the $\pi\pi$ continuum at the hadronic level. Consequently, a comprehensive study of the ρ meson necessitates an exploration of the $\pi\pi$ scattering sector. In the past decade, multiple LQCD groups have provided energy levels for the P-wave $\pi\pi$ sector [1–14]. However, there has been little work collating spectra from various collaborations, particularly for $N_f = 2 + 1$, and performing a consistent unified analysis. That is the aim of this work.

The Lüscher formula is the most practical way to relate lattice calculations to the elastic scattering phase shifts of two spinless particles. Therefore, when dealing with a system containing only one $\pi\pi$ channel, it is sufficient to utilize the standard Lüscher formula to relate the finite volume spectrum to the phase shifts. However, in our present study, we also aim to incorporate the $\omega\pi$ channel, in order to assess its impact. While it is not an open channel, it does generate the leading non-analytic behaviour of the ρ mass as a function of m_π. While the Lüscher formalism can certainly include the additional $\omega\pi$ channel, we note that there are other approaches which provide computational convenience with very little overhead in incorporating several two-particle channels.

Alternatively, Hamiltonian effective field theory (HEFT) also incorporates the Lüscher formalism and establishes a connection between the scattering process in infinite volume and the finite volume spectrum of the system. Here, we consistently analyze the spectra provided by several different LQCD collaborations using the HEFT framework, drawing on results extracted in the rest frame, moving frame and elongated frames. Motivated by the physical picture mentioned, the Hamiltonian employed here is studied within a framework that involves a bare ρ, as well as $\pi\pi$ and $\omega\pi$ channels. We obtain the bare ρ mass in various regularization schemes from the lattice energy levels and investigate its dependence on m_π.

2. formalism

2.1 Hamiltonian Model

The Hamiltonian in the center of mass frame of the interacting system is divided into two parts as follows,

$$H = H_0 + H_I \,, \tag{1}$$

where H_0 is the non-interacting part, and H_I is the interaction part. In this work, we include a bare ρ meson, which can be identified as a $q\bar{q}$ state, as well as two coupled channels, $\pi\pi$ and $\pi\omega$. In the infinite volume, characterized by SO(3) symmetry, it is most convenient to express the interaction in the JLS basis. Because of the definite J^P quantum number of the bare ρ meson, it is sufficient to focus on the Hamiltonian in the subspace spanned by $|\alpha = \pi\pi; k^*, J = 1, M, \ell = 1, S = 0\rangle$ and

$|\alpha = \omega\pi; k^*, J = 1, M, \ell = 1, S = 1\rangle$. The JLS indices will be suppressed hereafter. The free energy part of the Hamiltonian in this subspace, H_0, is given by,

$$H_0 = \sum_M m_\rho^B \, |\rho_B, M\rangle \, \langle\rho_B, M| + \sum_{\alpha,M} \int k^{*2} \, dk^* \left(E_{\alpha_1}(k^*) + E_{\alpha_2}(k^*)\right) |\alpha; k^*, M\rangle \, \langle\alpha; k^*, M| \,, \quad (2)$$

where $|\,\rho_B,\ M\rangle$ indicates the bare ρ state with z-component of spin, M, $E_{\alpha_i}(k) = \sqrt{k^2 + m_{\alpha_i}^2}$ with $\alpha_i = \pi$ or ω, m_{α_i} is for the mass of the particle in the α channel and m_ρ^B is the mass of the bare single-particle basis state. The interacting part, H_I, is given within the model by,

$$H_I = \sum_{\alpha,M} \int k^{*2} \, dk^* \, V_\alpha(k^*) \, |\rho_B,\ M\rangle \, \langle\alpha; k^*,\ M| + \text{h.c.} \,, \quad (3)$$

where the interaction term V_α is independent of M and given by

$$V_{\pi\pi}(k^*) = \frac{g_{\rho\pi\pi}}{2\pi\sqrt{3}} \frac{k^*}{\sqrt{m_\rho^{\rm B}} \, E_\pi(k^*)} \, u_{\pi\pi}(k^*) \,, \; V_{\omega\pi}(k^*) = \frac{g_{\omega\rho\pi}}{2\pi\sqrt{6}} \frac{k^* \sqrt{m_\rho^{\rm B}}}{\sqrt{E_\pi(k^*) \, E_\omega(k^*)}} \, u_{\omega\pi}(k^*) \quad (4)$$

where $u_{\pi\pi}$ and $u_{\omega\pi}$ are the form factors parameterizing the internal structure of hadrons and ensuring the convergence of loop integrals. The usual dipole form factors are used,

$$u_{\pi\pi}(k) = \left(\frac{\Lambda_{\rho\pi\pi}^2}{k^2 + \Lambda_{\rho\pi\pi}^2}\right)^2 \,, \; u_{\omega\pi}(k) = \left(\frac{\Lambda_{\omega\rho\pi}^2 - \mu_\pi^2}{k^2 + \Lambda_{\omega\rho\pi}^2}\right)^2 \quad (5)$$

where $\mu_\pi = 138.5$MeV. The scattering T-matrix, defined by $S_{fi} = \delta_{fi} - 2\pi i \, \delta^4(p_f - p_i) \, T_{fi}$, can be obtained from the partial wave Lippmann-Schwinger equation. In the present case $T_{\pi\pi,\,\pi\pi}(p, q; E)$ can be obtained analytically

$$T_{\pi\pi,\,\pi\pi}(p, q; E) = V_{\pi\pi}^*(p) \, G(E) \, V_{\pi\pi}(q) \,, \quad (6)$$

where $G(E)$ is the full propagator of the ρ meson defined by $G(E)^{-1} = E - m_\rho^{\rm B} - \Sigma(E)$ with the self-energy $\Sigma(E) = \Sigma_{\pi\pi}(E) + \Sigma_{\omega\pi}(E)$ and

$$\Sigma_{\pi\pi}(E) = \int q^2 \, dq \, \frac{|V_{\pi\pi}(q)|^2}{E - 2E_\pi(q) + i\varepsilon} \,, \; \Sigma_{\omega\pi}(E) = \int q^2 \, dq \, \frac{|V_{\omega\pi}(q)|^2}{E - E_\pi(q) - E_\omega(q) + i\varepsilon} \quad (7)$$

The partial-wave phase shift, $\delta(E)$, for the P-wave $\pi\pi \to \pi\pi$ elastic scattering is then given by,

$$\delta(E) = \arctan\left[\frac{\text{Im}\,\Sigma_{\pi\pi}(E)}{E - m_\rho^{\rm B} - \text{Re}\,\Sigma(E)}\right] \; (\text{mod } \pi) \,, \quad (8)$$

where $\bar{p} = \sqrt{E^2/4 - m_\pi^2}$ is the on-shell momentum. The pole position of the ρ-resonance is located in the lower half plane of the unphysical Riemann sheet of the $\pi\pi$-channel but the first Riemann sheet of the $\omega\pi$-channel and determined by solving the equation $E - m_\rho^{\rm B} - \Sigma(E) = 0$.

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2.2 The Hamiltonian in the finite volume

To obtain the FVH in terms of the states with discrete momentum, one needs to make the following substitutions in Eqs.(2) and (3).

$$|\rho_B,\, M\rangle \to |\rho_B,\, M\rangle_L \ ,\ |\alpha; k^*,\, M\rangle \to \sqrt{\frac{V}{(2\pi)^3}}\,|\alpha; e_{\boldsymbol{n}},\, M\rangle \ ,\ \int d^3\boldsymbol{k}^* \to \frac{(2\pi)^3}{V}\sum_{\boldsymbol{n}\in\mathbb{Z}^3} \tag{9}$$

where $V = \eta L^3$ is the volume of the box, with elongation factor η, and $e_{\boldsymbol{n}}$ denotes a degenerate shell of the non-interacting Hamiltonian in the rest frame, because those states with the same $e_{\boldsymbol{n}}$ share the same $|\boldsymbol{k}^*(\boldsymbol{n})|$. For example, in the rest frame of a cubic box, $\boldsymbol{k}^* = \frac{2\pi}{L}\boldsymbol{n}$ and hence $e_{\boldsymbol{n}} = \boldsymbol{n}^2$. The finite volume basis vector, $|\alpha; e_{\boldsymbol{n}},\, M\rangle$, as the counterpart of infinite volume JLS state, is given by

$$|\alpha; e_{\boldsymbol{n}}, M\rangle = A_\alpha \sum_{m\sigma\sigma_1\sigma_2} C_{\ell S}(JM; m\sigma)\, C_{s_1 s_2}(S\sigma;\sigma_1\sigma_2) \sum_{\boldsymbol{n}\in\{\hat{e}_{\boldsymbol{n}}\}} Y_{\ell m}(\hat{\boldsymbol{k}}^*(\boldsymbol{n}))\, |\boldsymbol{k}^*(\boldsymbol{n}), \sigma_1\sigma_2\rangle\ , \tag{10}$$

where $A_\alpha = \frac{1}{\sqrt{2}}$ for $\pi\pi$ and 1 for $\omega\pi$. Note that these states are not orthonormalized due to the $O(3)$ symmetry breaking. Thus it is necessary to construct an orthogonal basis $|\alpha; e_n, \Gamma, a\rangle$ furnishing an irreducible representation Γ of G. Such states take linear combinations of the basis states $|\alpha; e_{\boldsymbol{n}}, M\rangle$ with reduction coefficients $C_{\Gamma,G}$: $|\alpha; e_n, \Gamma, a\rangle := \sqrt{\frac{1}{Z_\Gamma(e_{\boldsymbol{n}})}}\,\left[C_{\Gamma,G}\right]_{M,a} |\alpha; e_{\boldsymbol{n}}, M\rangle$ and $|\rho_B,\, \Gamma,\, a\rangle = \left[C_{\Gamma,G}\right]_{M,a} |\rho_B,\, M\rangle_L$ where $Z_\Gamma(e_{\boldsymbol{n}})$ is the normalization factor. With these well-defined orthogonal basis states, the FVH in the rest frame, $H^{\text{fin}} = \sum_\Gamma \left(H^{\text{fin}}_{0,\Gamma} + H^{\text{fin}}_{I,\Gamma}\right)$, for a given irreducible representation Γ, is now given by

$$H^{\text{fin}}_{0,\Gamma} = m^{\text{B}}_\rho\, |\rho_{\text{B}},\, \Gamma\rangle\, \langle\rho_{\text{B}},\, \Gamma| + \sum_{\alpha; e_{\boldsymbol{n}}} |\alpha; e_{\boldsymbol{n}}, \Gamma\rangle\, \langle\alpha; e_{\boldsymbol{n}}, \Gamma|\, \left(E_{\alpha_1}(|\boldsymbol{k}^*(\boldsymbol{n})|) + E_{\alpha_2}(|\boldsymbol{k}^*(\boldsymbol{n})|)\right)\ , \tag{11}$$

$$H^{\text{fin}}_{I,\Gamma} = \sum_{\alpha, e_{\boldsymbol{n}}} V^{\text{fin}}_{\alpha,\Gamma}(|\boldsymbol{k}^*(\boldsymbol{n})|)\, |\rho_{\text{B}},\, \Gamma\rangle\, \langle\alpha; e_{\boldsymbol{n}}, \Gamma| + \text{h.c.}\,. \tag{12}$$

where $V^{\text{fin}}_{\alpha,\Gamma}(e_{\boldsymbol{n}}) = \sqrt{\frac{(2\pi)^3}{V}} J_\alpha(e_n)\sqrt{Z_\Gamma(e_n)} V_\alpha(|\boldsymbol{k}^*(\boldsymbol{n})|)$. J_α is the Jacobian for a moving system.

2.3 fitting formulas

We minimize the χ^2 defined as $\chi^2 = (E^{\text{H}} - E^{\text{lat}}_{\text{cm}})^{\text{T}}\mathbb{C}^{-1}(E^{\text{H}} - E^{\text{lat}}_{\text{cm}})$, where $\mathbb{C}$ denotes the covariance matrix of the lattice spectrum. It should be noted that $E^{\text{lat}}_{\text{cm}}$ is the spectrum that has been transformed into the rest frame. If a certain energy level, E^{lat}_n, is extracted from the composite operator with $\boldsymbol{P} \neq 0$, it needs to be converted to $E^{\text{lat}}_{\text{cm},n}$ through $E^{\text{lat}}_{\text{cm},n} = \sqrt{(E^{\text{lat}}_n)^2 - \boldsymbol{P}^2}$.

2.4 formulas for extrapolation

In the framework of Chiral Perturbation Theory, the mass of the ρ is a function of m_π of the form $m^{\text{p}}_\rho = c'_0 + c'_1\, m^2_\pi + c'_2\, m^3_\pi + c'_3\, m^4_\pi \ln\left(\frac{m^2_\pi}{m^2_\rho}\right) + O(m^4_\pi)$ where m^{p}_ρ is related to the pole mass of the ρ, corresponding to the real part of the pole of the T-matrix in the complex plane. As discussed in Refs. [15, 16], the quark mass insertion at tree level only contributes to the m^2_π term up to $O(m^4_\pi)$, with the other two terms arising from pion-loop self energies. The m^3_π term comes from the $\omega\pi$ loop while both $\pi\pi$ and $\omega\pi$ loops contribute to the log term. Within the present framework, $m_\rho^{\rm p} = m_\rho^{\rm B} + {\rm Re}\,\Sigma(m_\rho^{\rm p} - i\Gamma/2)$, so the bare mass $m_\rho^{\rm B}$ is a quadratic function of m_π at the leading order,

$$m_\rho^{\rm B}(m_\pi) = c_0 + c_1\, m_\pi^2\,. \tag{13}$$

This equation will be used to study the extracted $m_\rho^{\rm B}$ as function of m_π in our analysis.

3. numerical results and discussion

The finite volume spectra for the $I = \ell = 1$ $\pi\pi$ sector with dynamical fermions at various pion masses have been provided by several LQCD collaborations over the past decade, including PACS-CS (2011, $N_f = 2+1$) [2], HSC (2013, $N_f = 2+1$) [5], HSC (2015, $N_f = 2+1$) [4], Guo *et al.* (2016, $N_f = 2$) [10], MILC(2016, $N_f = 2+1$) [11], C. Alexandrou *et al.* (2017, $N_f = 2+1$) [7], J. Bulava *et al.* (2019, $N_f = 2+1$) [12], and ETMC (2020, $N_f = 2+1(+1)$) [6]. These lattice data are be fitted in the present work. We will adopt three fitting schemes.

We first adopt scheme A, wherein the interaction $V_{\omega\pi}$ is turned off, i.e., $g_{\omega\rho\pi} \equiv 0$, while $m_\rho^{\rm B}$, $g_{\rho\pi\pi}$, and $\Lambda_{\rho\pi\pi}$ are treated as free fitting parameters. It is found that both $g_{\rho\pi\pi}$ and $\Lambda_{\omega\rho\pi}$ show a very weak dependence on m_π, while $m_\rho^{\rm B}$ is strongly dependent on m_π. In scheme B both $g_{\rho\pi\pi}$ and $\Lambda_{\rho\pi\pi}$ are fixed at $g_{\rho\pi\pi} = 7.07\,(6.75)$ and $\Lambda_{\rho\pi\pi} = 890\,(950)$ MeV for $N_f = 2+1$ ($N_f = 2$). $m_\rho^{\rm B}$ is allowed to vary. In this scheme we find that the m_ρ^B shows a messy distribution, which is unexpected at the beginning. The contribution of the $\omega\pi$ channel is examined in scheme C. In this case $V_{\omega\pi}$ is switched on, however, the two additional parameters, $g_{\omega\rho\pi}$ and $\Lambda_{\omega\rho\pi}$ are taken to be independent of m_π, namely, $g_{\omega\rho\pi} = 18/\text{GeV}$ and $\Lambda_{\omega\rho\pi} = 900$ MeV. $g_{\rho\pi\pi}$ and $\Lambda_{\rho\pi\pi}$ are slightly shifted to be 7.4(7.07) and 900(980) MeV for $N_f = 2+1(N_f = 2)$.[1] The values are determined by the width $\Gamma_{\omega\to 3\pi}$. It still only remains $m_\rho^{\rm B}$ to vary in the fitting. As an illustration the fitting results for $m_\pi = 200$ MeV in three schemes is shown in Fig.1. We state that the lattice data for all these collaborations can be well fitted in our framework.

However, even after including the effect of the $\omega\pi$ coupled channel, the m_π-dependence of $m_\rho^{\rm B}$ is still scattered. It becomes apparent that the bare masses extracted from different lattice groups do not permit a consistent interpretation, which indicates the presence of intrinsic systematic differences between the lattice spectra provided by different collaborations. Such discrepancies lead us to consider the following issues that may influence the lattice results presented. These include: different residual lattice artifacts due to the different gauge and fermion actions considered, varied scale-setting schemes employed by different collaborations and different methods used to extract the finite volume spectra. In the absence of these systematic discrepancies, the results provided by different collaborations are expected to be consistent with each other after finite volume and lattice spacing artifacts are taken into account.

4. extrapolation in m_π

With the fitting results, we now investigate the m_π-dependence of the properties of the ρ meson and extrapolate them into the physical region. In scheme A, our investigations reveal that both $g_{\rho\pi\pi}$

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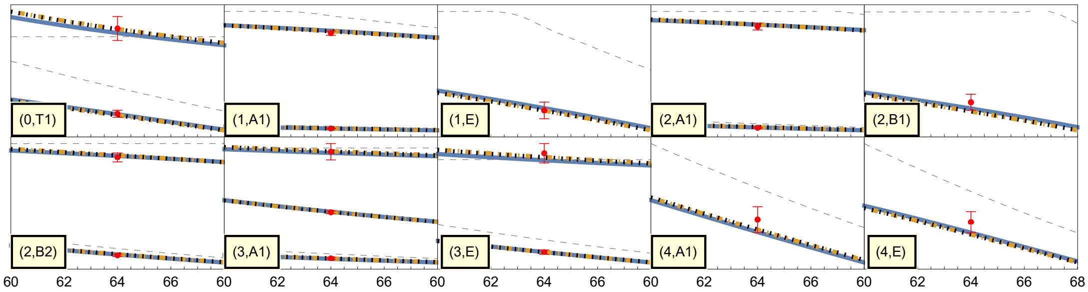

Figure 1: Spectra with $m_\pi = 200$ MeV along with that calculated by HEFT using the fitting results for schemes A, B and C. The x-axis represents the spatial extent L in units of lattice spacing a, while the y-axis indicates the energy level. Text within the yellow box $(\boldsymbol{n}^2, \Gamma)$ signifies spectrum extraction using operators in representation Γ and with total momentum $\boldsymbol{P}^2 = (\frac{2\pi}{L})^2\boldsymbol{n}^2$. Red points indicate the lattice spectrum provided by collaborations. Blue, orange dot-dashed and black dotted lines indicate the spectrum as the function of L calculated by HEFT in schemes A, B and C, respectively. The dashed gray lines indicate the non-interacting energy levels $2E_\pi(\boldsymbol{k}^*)$ and $m_\rho^{\rm B}$. The turning points in the non-interacting energy levels are associated with energy crossings.

and $\Lambda_{\rho\pi\pi}$ display little variation as m_π varies. Thus, we could concentrate on the m_π-dependence of the bare ρ mass, $m_\rho^{\rm B}$, using Eq. (13). It is possible to extrapolate the fitting results of $m_\rho^{\rm B}$ in schemes B and C, but not in scheme A, since there the values of $m_\rho^{\rm B}$ are correlated with $g_{\rho\pi\pi}$ and $\Lambda_{\rho\pi\pi}$.

In principle, it is natural to consider putting all the values of $m_\rho^{\rm B}(m_\pi)$ together and performing a global fit to make full use of the lattice data. However, it is hard to extract useful information, since the data show large inconsistent variations. The possible reasons have been discussed earlier. This suggests that we should make the extrapolation of the data to the physical point collaboration by collaboration. Because for each group, there are only a limited number of values of m_π and the lattice spacing does not change a lot, the lattice spacing effect can be absorbed in c_0 to perform the extrapolations. Furthermore, we have two free parameters in Eq. (13), thus only the data of collaborations having no less than two m_π points are analyzed. The fitting and extrapolation results are shown in Fig. 2 and Table 1.

As shown, there are five collaborations having no less than two different m_π points. For each collaboration, the points show a good linear relation between $m_\rho^{\rm B}$ and m_π^2, whether the $\omega\pi$ loop is included or not. The only notable exception is one point with a large uncertainty from ETMC. With c_0 and c_1 determined, we can obtain the bare mass of the ρ at the physical pion mass. Subsequently, we can get $m_\rho^{\rm p}$ by solving $E - m_\rho^{\rm B} - \Sigma(E) = 0$. The results are listed in the last column of Table 1. For MILC, HSC and Bulava *et al.*, even though their values of c_0 and c_1 are quite different, the extrapolated $m_\rho^{\rm p}$ all agree with the experimental value. However, for ETMC and Guo *et al.*, they are about 30 MeV higher and 50 MeV lower compared to the experimental value, respectively. The relatively high $m_\rho^{\rm p}$ obtained by ETMC is not so surprising, since in their previous work a higher value of m_ρ compared to the others was also reported. The lower $m_\rho^{\rm p}$ extracted from Guo *et al.* also agrees with their own result, which possibly results from using $N_f = 2$.

It is obvious that it makes little sense to talk about $m_\rho^{\rm B}$ solely , as it strongly depends on how the hadronic loops are estimated. It is the slope c_1 that contains more useful, less model dependent,

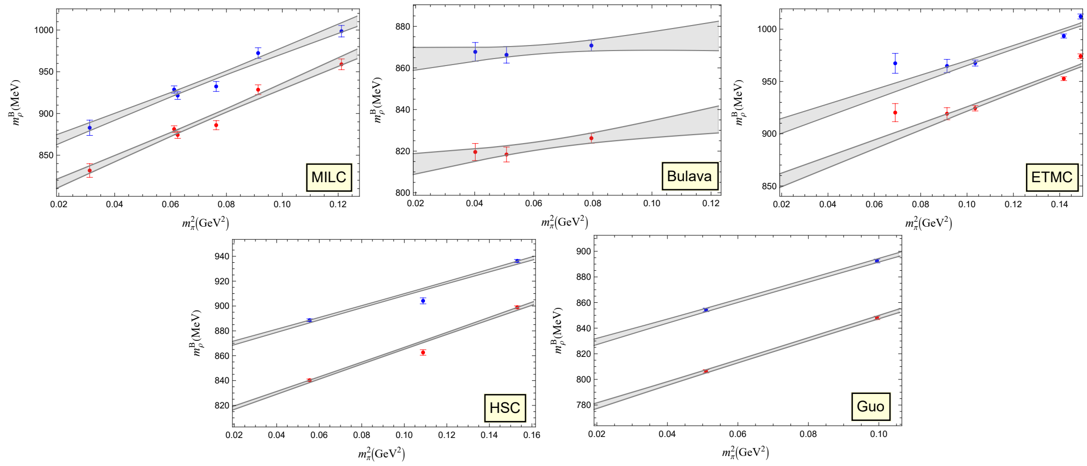

Figure 2: m_π-dependence and extrapolation of $m_\rho^{\rm B}$ for each collaboration. Red and blue points indicate the fitting results of $m_\rho^{\rm B}$ in schemes B and C, respectively. Gray bands represent the quadratic function $m_\rho^{\rm B} = c_0 + c_1\, m_\pi^2$ with uncertainty, where c_0 and c_1 for both schemes are given in Table 1.

Table 1: Extrapolation results are summarized. For each collaboration, the results of B and C are given in the first and second row, respectively. The second and third column present the coefficient c_0 and c_1. The fourth column presents the extrapolated $m_\rho^{\rm B}$ at the physical pion mass. The fifth column presents the pole mass.

COLLAB.	c_0(MeV)	c_1(GeV^{-1})	$m_\rho^{\rm B}(\mu_\pi)$	$m_\rho^{\rm p}(\mu_\pi)$
Bulava	809.8(7.0)	0.21(0.11)	814.0(5.0)	765.0(6.0)
	862.3(7.6)	0.11(0.12)	864.0(6.0)	765.0(6.0)
MILC	788.0(7.3)	1.45(0.10)	816.0(6.0)	768.0(6.0)
	843.3(7.95)	1.32(0.11)	869.0(6.0)	769.0(6.0)
HSC	806.7(1.71)	0.60(0.02)	818.2(1.4)	770.6(1.7)
	861.3(1.9)	0.49(0.02)	870.7(1.6)	771.3(1.7)
ETMC	838.9(7.7)	0.85(0.06)	855.0(7.0)	814.0(8.0)
	892.5(8.3)	0.75(0.06)	907.0(7.0)	809.0(7.0)
Guo	762.2(2.9)	0.86(0.04)	778.8(2.3)	719.3(2.6)
	813.8(3.2)	0.79(0.04)	829.0(2.5)	719.0(2.6)

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physical information concerning the structure of the ρ meson, which can only be extracted from the LQCD data at unphysical m_π. Furthermore, in principle, on the theoretical side the slope c_1 can be calculated at the quark-level in various models. Thus, with the help of c_1 the relevant models could be distinguished.

5. Summary

In this work, we collected finite volume spectra for the $I = \ell = 1$ $\pi\pi$ sector provided by LQCD collaborations over the past decade. These spectra were fit in a consistent manner within the framework of Hamiltonian Effective field theory. We employed three schemes to fit the energy levels obtained at various pion masses. Through scheme A, we found that $g_{\rho\pi\pi}$ and $\Lambda_{\rho\pi\pi}$ exhibit a weak dependence on m_π, while m_ρ^{B} shows a messy distribution. In the following scheme B, we set these two parameters as constant in order to obtain the m_π-dependence of the bare ρ basis state, $m_\rho^{\mathrm{B}}(m_\pi)$. In scheme C, where the $\omega\rho\pi$ vertex was included with additional constraints from experimental data, we again extracted $m_\rho^{\mathrm{B}}(m_\pi)$. However, in scheme B,C, m_ρ^{B} is still scattered. Finally, we used the linear relation between m_ρ^{B} and m_π^2 to perform an extrapolation. Because the relationship between m_ρ^{B} and m_π^2 was highly dependent on the LQCD group whose data we used, we were unable to fit all m_ρ^{B} simultaneously, and resorted to extrapolating collaboration by collaboration. Based upon the extrapolations of data from the five LQCD groups, it was found that for each collaboration, $m_\rho^{\mathrm{B}}(m_\pi)$ could be described well by Eq.(13). When extrapolated to physical pion mass μ_π, the m_ρ^{p} for MILC, Bulava *et al.* and HSC agree with the experimental measurements. However, from the current LQCD data, the extracted value of c_1 is dependent on the lattice collaboration whose data is used. This indicates the important discrepancies in the results of today's lattice QCD calculation among different collaborations.

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References

[1] C. B. Lang, Daniel Mohler, Sasa Prelovsek, Matija Vidmar, *Coupled channel analysis of the rho meson decay in lattice QCD*, Phys. Rev. D **84** (2011) 054503 [1105.5636], Phys.Rev.D **89** (2014) 5 059903 (erratum).

[2] S. Aoki *et al.*, *ρ Meson Decay in 2+1 Flavor Lattice QCD*, Phys. Rev. D **84** (2011) 094505 [1106.5365].

[3] Gunnar S. Bali *et al.*, *ρ and K^* resonances on the lattice at nearly physical quark masses and* $N_f = 2$, Phys. Rev. D **93** (2016) 054509 [1512.08678].

[4] David J. Wilson *et al.*, *Coupled $\pi\pi, K\bar{K}$ scattering in P-wave and the ρ resonance from lattice QCD*, Phys. Rev. D **92** (2015) 094502 [1507.02599].

[5] Jozef J. Dudek *et al.*, *Energy dependence of the ρ resonance in $\pi\pi$ elastic scattering from lattice QCD*, Phys. Rev. D **87** (2013) 034505 [1212.0830], Phys.Rev.D **90** (2014) 099902 (erratum).

[6] Markus Werner *et al.*, *Hadron-Hadron Interactions from $N_f = 2+1+1$ Lattice QCD: The ρ-resonance*, Eur. Phys. J. A **56** (2020) 61 [1907.01237].

[7] Constantia Alexandrou *et al.*, *P-wave $\pi\pi$ scattering and the ρ resonance from lattice QCD*, Phys. Rev. D **96** (2017) 034525 [1704.05439].

[8] Craig Pelissier and Andrei Alexandru, *Resonance parameters of the rho-meson from asymmetrical lattices*, Phys. Rev. D **87** (2013) 014503 [1211.0092].

[9] Xu Feng, Karl Jansen, and Dru B. Renner, *Resonance Parameters of the rho-Meson from Lattice QCD*, Phys. Rev. D **83** (2011) 094505 [1011.5288].

[10] Dehua Guo *et al.*, *Rho resonance parameters from lattice QCD*, Phys. Rev. D **94** (2016) 034501 [1605.03993].

[11] Ziwen Fu and Lingyun Wang, *Studying the ρ resonance parameters with staggered fermions*, Phys. Rev. D **94** (2016) 034505 [1608.07478].

[12] Christian Andersen *et al.*, *The $I = 1$ pion-pion scattering amplitude and timelike pion form factor from $N_f = 2+1$ lattice QCD*, Nucl. Phys. B **939** (2019) 145 [1808.05007].

[13] Matthias Fischer *et al.*, *The ρ-resonance from $N_f = 2$ lattice QCD including the physical pion mass*, Phys. Lett. B **819** (2021) 136449 [2006.13805].

[14] Arkaitz Rodas, Jozef J. Dudek, and Robert G. Edwards, *The quark mass dependence of $\pi\pi$ scattering in isospin 0, 1 and 2 from lattice QCD*, Phys. Rev. D 108 (2023) 034513 [2303.10701].

[15] Derek Bruce Leinweber, Anthony William Thomas, Kazuo Tsushima, and Stewart Victor Wright, *Chiral behavior of the rho meson in lattice QCD*, Phys. Rev. D **64** (2001) 094502 [0104013].

[16] Peter C. Bruns and Ulf-G. Meissner, *Infrared regularization for spin-1 fields*, Eur. Phys. J. C **40** (2005) 97 [0411223].