

Quark mass dependence of doubly heavy tetraquark binding

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The existence of bound doubly heavy tetraquark states was confirmed by the recent LHCb discovery of the doubly charmed T_{cc} , less than 1 MeV below the meson pair threshold. Others states with two heavy (bottom or charm) quarks could also be bound, perhaps more deeply. Here we discuss our previous work, and the improvements in our current, updated analysis of various heavy-heavy-light-light tetraquark candidates, including the light and heavy quark mass dependence of the binding.

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1. Introduction

The existence of strong-interaction-stable doubly heavy tetraquarks, $q_1 q_2 \bar{h} \bar{h}$ (where h are heavy quarks with $m_h \geq m_c$ and q_1, q_2 are the light quark flavours respectively) in the heavy mass limit $m_h \rightarrow \infty$ has long been suspected [1–3]. Phenomenologically, such states have access to a binding contribution from the colour Coulomb interaction between heavy antiquarks in a colour $\bar{3}_c$ configuration, which is proportional to the reduced mass of the heavy antiquark pair. Additional binding contributions are present for $J^P = 1^+$ or $J^P = 0^+$ heavy-heavy-light-light tetraquarks, where the light quarks are either $I = 0$ (ud) or $I = 1/2$ (ls , with $l \in \{u, d\}$). In this case, the light degrees of freedom are in the flavor anti-symmetric, colour $\bar{3}_c$, light-quark spin $J_l = 0$ “good light diquark” configuration, known to be attractive from the observed splittings in the heavy baryon spectrum, and lattice calculations [4].

To what extent this binding survives to physical heavy masses $m_b \geq m_h \geq m_c$ remains an active field of study on the lattice [5–23]. In the case of $ud\bar{c}\bar{c}$, a 2021 LHCb measurement [24, 25] found a bound $J^P = 1^+$ state lying less than 1 MeV below the DD^* threshold. This binding is well below the precision of current lattice calculations, but the finding strongly implies binding in similar channels with heavier heavy quarks, such as $ud\bar{b}\bar{b}$.

These proceedings concern our recent work [26], containing an updated analysis of the $ud\bar{h}\bar{h}$, $ls\bar{h}\bar{h}$, $ud\bar{b}\bar{b}$, and $ls\bar{b}\bar{b}$ $J^P = 1^+$ and $J^P = 0^+$ tetraquark states, in the context of the previous work by the collaboration [8, 12, 27]. We shall begin by briefly summarising the previous work, before moving onto the changes and updates made in the present analysis, and the results.

2. Previous work

The work in [26] is the culmination of previous studies [8, 12, 27], which investigated both the light and heavy mass dependence of the tetraquark binding, as well as a number of different tetraquark channels. Both the previous work and the current update employ PACS-CS $N_f = 2 + 1$ Wilson-Clover ensembles [28], with $a^{-1} = 2.194(10)\text{GeV}$ [29], with Ref. [12] and the current update adding new ensembles supplementing the PACS-CS set. Non-Relativistic QCD (NRQCD) is used to reach heavy masses above m_c .

2.1 Light quark mass dependence

The collaboration’s first steps towards our present calculation were taken in [8]. This focused on the $J^P = 1^+$ $ud\bar{b}\bar{b}$ and $ls\bar{b}\bar{b}$ tetraquarks, with three values for the pion mass in the calculation ($M_\pi \in \{165, 299, 415\} \text{ MeV}$). Two different operators which should overlap with the tetraquark state were used; a meson-meson like operator,

$$M(x) = \bar{b}_a^\alpha(x) \gamma_5^{\alpha\beta} u_a^\beta(x) \bar{b}_b^\kappa(x) \gamma_i^{\kappa\rho} u_b^\rho(x) - u \leftrightarrow d, \quad (1)$$

and a diquark-antidiquark like operator,

$$D(x) = (u_a^\alpha(x))^T (C \gamma_5)^{\alpha\beta} d_b^\beta(x) \bar{b}_a^\kappa(x) (C \gamma_i)^{\kappa\rho} (\bar{b}_b^\rho(x))^T, \quad (2)$$

where C is the charge conjugation operator.

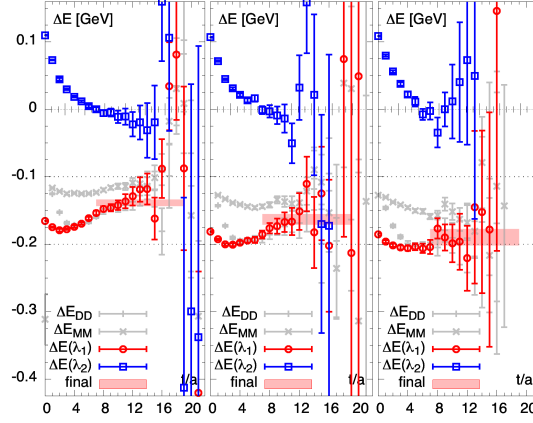


Figure 1: Taken from [8], ΔE (for $ud\bar{b}\bar{b}$) extracted from Eq. (4), from different values of t , for the two Eigenvalues λ_1 (red) and λ_2 (blue). The final value taken for $\Delta E(\lambda_1)$ is shown as a red band.

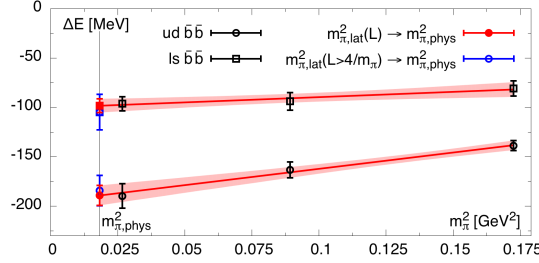


Figure 2: Taken from [8], $ud\bar{b}\bar{b}$ and $ls\bar{b}\bar{b}$ for the three different M_π values, with a linear extrapolation in M_π^2 shown in red. The red and blue points give the extrapolated value at physical pion mass, the blue one excluding the leftmost data point, for which $M_\pi L < 4$.

The two-point correlation function corresponding to each combination of M and D at source and sink, $C_{O_1 O_2}(t)$, is combined with the two-point correlation functions for the threshold pseudoscalar ($C_{PP}(t)$) and vector ($C_{VV}(t)$) meson states to create a matrix for use in a Generalised Eigenvalue Problem (GEVP),

$$F(t) = \frac{1}{C_{PP}(t)C_{VV}(t)} \begin{pmatrix} C_{DD}(t) & C_{DM}(t) \\ C_{MD}(t) & C_{MM}(t) \end{pmatrix}, \quad (3)$$

which has a lowest Eigenvalue given by λ_1 :

$$\begin{aligned} F(t)v &= \lambda(t)F(t_0)v \\ \lambda_1(t) &= Ae^{-\Delta E(t-t_0)}, \end{aligned} \quad (4)$$

which can be fitted to extract the splitting between the tetraquark and threshold state ΔE .

Fig. 1 is taken from [8], and shows the resulting ΔE values for different t choices in red, as well as the final value (red band). These results show plateaus which appear to rise at large t values. Taking the binding results for the three different ensembles used, a linear fit in M_π^2 is extrapolated to the physical pion mass. This fit is shown in Fig. 2, taken from [8]. The fits both appear linear,

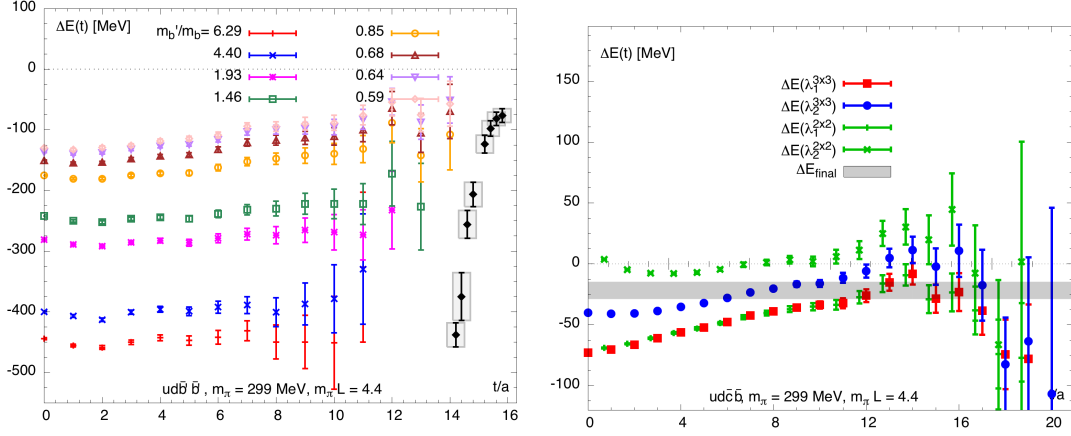


Figure 3: Taken from [27]. Left: ΔE (for $ud\bar{h}\bar{h}$, where the notation $b' \equiv h$ is used) extracted from Eq. (4), from different values of t and choices of m_h , for the Eigenvalue λ_1 . The final value taken for $\Delta E(\lambda_1)$ is shown as a black point. Right: ΔE plateaus for $ud\bar{b}\bar{c}$, showing λ_1 and λ_2 for the 2×2 and 3×3 GEVP cases.

with stronger pion mass dependence visible in the $ud\bar{b}\bar{b}$ case. Physical pion mass binding energies of 189(10) MeV and 98(7) MeV are quoted for the $ud\bar{b}\bar{b}$ and $ls\bar{b}\bar{b}$ tetraquarks respectively. These results suggest that the tetraquarks are deeply bound relative to the thresholds, and whilst no finite volume (FV) analysis is carried out in this paper, the depth of this binding is greater than any expected shift from FV effects. This is further supported by the presence of an excited state close to threshold.

2.2 Heavy quark mass dependence

The next steps were taken in [27]. This work used one ensemble from the previous work with $M_\pi = 299$ MeV, and again focused on the $J^P = 1^+$ tetraquarks. This time, a variable heavy mass was employed, with $am_h \in \{0.9, 1.0, 1.2, 1.6, 3.0, 4.0, 8.0, 10.0\}$, giving $0.1 \leq \frac{m_b}{m_h} \leq 1.8$, and four tetraquark channels were studied: $ud\bar{b}\bar{h}$, $ud\bar{h}\bar{h}$, $ls\bar{b}\bar{h}$ and $ls\bar{h}\bar{h}$. The analysis proceeds as in [8] above, but in the $q_1q_2\bar{b}\bar{h}$ case, a 3×3 matrix is permitted for the GEVP by the exchange $b \leftrightarrow h$ in $D(x)$. Higher order Eigenvalues give access to excited states. The ΔE plateaus in [27] shown in Fig. 3, particularly for $ud\bar{b}\bar{c}$, exhibit similar behaviour to those in [8], with it hard to determine whether or not they are still rising when the noise begins to dominate at large t values.

The heavy mass dependence of the $J^P = 1^+$ binding for $ud\bar{b}\bar{h}$, $ud\bar{h}\bar{h}$, $ls\bar{b}\bar{h}$ and $ls\bar{h}\bar{h}$ is shown in Fig. 4, with $b' \equiv h$. The coloured bands show phenomenologically motivated fit functions, based on the colour Coulomb and ‘good-light’ diquark interactions discussed above. See [27] for the explicit forms and a detailed discussion.

We see the binding increasing with increasing reduced mass, heading off to $-\infty$ for the $\bar{h}\bar{h}$ tetraquarks, and reaching a finite value, accessible to our simulation via a static propagator, in the $\bar{b}\bar{h}$ cases. This is in keeping with our expectations from phenomenology. We also note that as $r \equiv m_b/m_h \equiv m_b/m_{b'}$ increases for $m_h \rightarrow m_c$, the binding becomes much shallower in all cases. The explicit calculation of $ud\bar{b}\bar{c}^1$ reflects this, giving a binding of 38(23) MeV.

¹using a relativistic action for the c

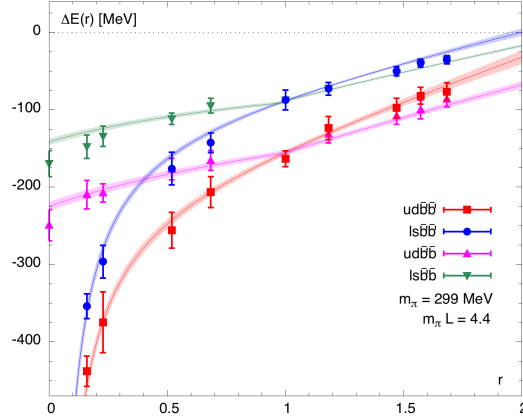


Figure 4: Taken from [27]. The heavy mass dependence of the tetraquark binding energies, plotted against $r = m_b/m_h$. Note that $b' \equiv h$.

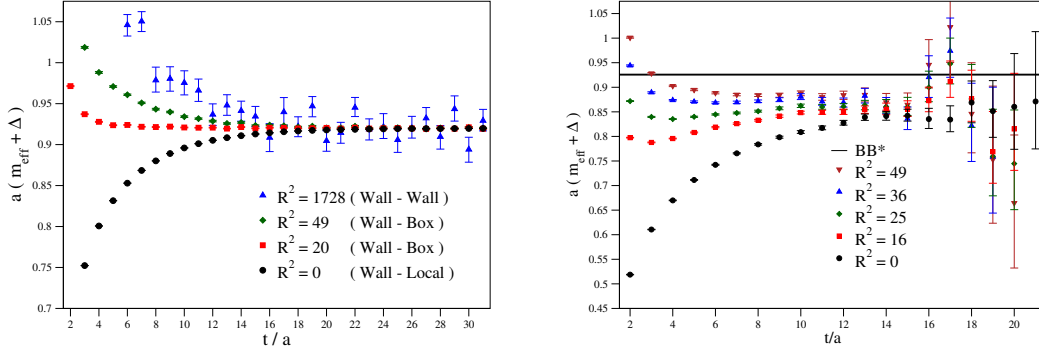


Figure 5: Taken from [12]. The effective masses of the B_c meson (left) and $ud\bar{b}\bar{b}$ tetraquark (right), for a variety of R^2 choices.

2.3 Introduction of box sinks and exploration of other tetraquarks

In the above analyses, propagators were tied together at a single point (a local sink). However, this introduces a bias in the structure of the tetraquark state. The change introduced in [12] was the construction of so called ‘box sinks’, the final part of the story leading to our present analysis. This involves summing the propagators over all N spatial points within a chosen radius, R ,

$$S^{B,R}(t) = \frac{1}{N} \sum_{r^2 \leq R^2} S(x+r, t). \quad (5)$$

In the limit $R^2 = 0$, this is equivalent to a local sink, as before, whilst for large values of R^2 , we approach a wall sink, where all spatial points on the lattice are summed over.

Fig. 5 shows the effect of varying R^2 on the effective mass, both for a B_c meson, and a $ud\bar{b}\bar{b}$ tetraquark. It’s clear that changing the radius of the box sink alters the way that the operator overlaps with excited states, and an optimal choice can improve convergence to the ground state significantly.

In addition, including in our analysis multiple radii with different sized excited state contributions gives us a much better handle on systematics associated with radius choices.

Using the box sink method and one pion mass (192 MeV), [12] studied $J^P = 1^+$ and $J^P = 0^+$ tetraquarks for a large number of flavour combinations: $ud\bar{b}\bar{c}$, $ud\bar{b}\bar{s}$, $ud\bar{s}\bar{c}$, $ud\bar{b}\bar{b}$, $ls\bar{b}\bar{b}$, $sc\bar{b}\bar{b}$ and $ls\bar{b}\bar{c}$ in much the same manner as has been discussed above (see [12] for full details).

The results of this study were a significant reduction in the bindings of the $J^P = 1^+$ $ud\bar{b}\bar{b}$ and $ls\bar{b}\bar{b}$ to around 110 and 40 MeV, respectively. This was attributed to a much better convergence to the plateau than had been achievable in [8, 27] without the box sinks, as well as the use of larger volume ensembles. It was also observed that the $ud\bar{b}\bar{c}$ was not deeply bound, but shallow binding could not be ruled out without a finite volume analysis. No binding was found in other channels.

3. Current, updated analysis

Our latest analysis, [26], takes the box sink method developed in [12], and applies it to the light quark and heavy quark mass dependence studies of [8, 27]. Taking the findings of [12], we focus our study on $J^P = 1^+$ and $J^P = 0^+$ tetraquarks of the form $ud\bar{h}\bar{h}$, $ud\bar{b}\bar{h}$, $ls\bar{h}\bar{h}$ and $ls\bar{b}\bar{h}$.

We use the same set of PACS-CS ensembles as above [29], but include two additional ensembles, at lighter pion mass, which we have generated with a larger spatial volume of $L/a = 48$, and hence a larger minimum $M_\pi L = 3.6$. Details of these ensembles are given in Tab. 1, and further details

Label	V	κ_l	$N_{\text{conf}} \times N_{\text{src}}$	aM_π	$M_\pi L$
E1	$32^3 \times 64$	0.13700	399×4	0.32205(18)	10.3
E2	$32^3 \times 64$	0.13727	400×4	0.26193(19)	8.4
E3	$32^3 \times 64$	0.13754	400×4	0.18960(29)	6.1
E5	$32^3 \times 64$	0.13770	800×4	0.13622(27)	4.4
E7	$48^3 \times 64$	0.13777	94×8	0.08719(47)	4.2
E9	$48^3 \times 64$	0.13779	88×4	0.07536(58)	3.6

Table 1: The ensembles used. Ensemble labels and volume in lattice units, V , are given, as well as κ_l values, the statistics in terms of configurations and sources per configuration, the pion mass in lattice units (aM_π) and the $aM_\pi L$, where L is the spatial extent in lattice units. $a^{-1} = 2.194(10)$ GeV from the Ω mass at the physical point [29].

about their generation can be found in [26].

As well as the expanded range of six M_π values for the light mass dependence part of the study, we employ nine am_h values for our heavy mass dependence study, on the E5 ensemble. On each ensemble, we use four² different box sink radii, to ensure good resolution of the ground state, as discussed in Sec 2.3. As in [12], we use up to 36 different combinations of source and sink operators.

²Two in the case of E7.

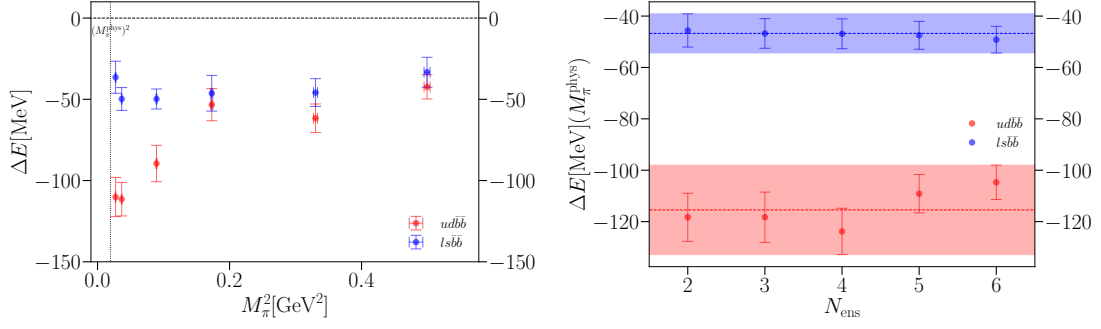


Figure 6: New results from [26]. Left: the updated version of Fig. 2. Right: the result at physical M_π when N_{ens} data points are included in the fit, with the coloured bands giving our final results.

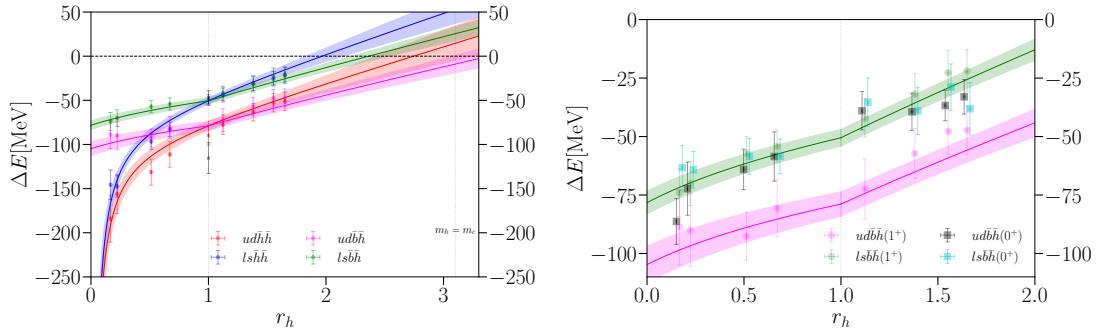


Figure 7: New results from [26]. Left: the updated version of Fig. 4, showing the heavy mass dependence of $J^P = 1^+$ tetraquark binding. Black stars indicating the physical pion mass results from Fig. 6. Right: a comparison of $J^P = 1^+$ and $J^P = 0^+$ bindings for $\bar{b}h$ type tetraquarks.

Instead of using a GEVP to extract the binding energies, we use correlated, multi-exponential fits to both the tetraquark and threshold meson states of the form,

$$C_2^{O_i^{\text{src}} O_j^{\text{snk}}}(t) = \sum_{n=0}^N a_n^{\text{src}i} a_n^{\text{snk}j} (e^{-E_n t} \pm e^{-E_n (T-t)}), \quad (6)$$

where $T = 64$ is the (periodic) time dimension of the lattice.

We obtain correlated values for the ground state energies E_0 . As discussed, our data contains different source and sink combinations, from both the operator choices and box sink radii. These translate into different values for the amplitudes $a_n^{\text{src}/\text{snk}}$, with a common set of energy levels E_n .

These Bayesian fits are performed using the *corrfit*, *lsqfit* and *gvar* python packages [30–32], and we conduct a rigorous stability analysis to ensure that we are fitting accurately. This is discussed in more detail in [26]. These fits offer a different but equivalent approach to the GEVP analysis, which may be beneficial in the case where the excited states in the spectrum are closely spaced.

The results of our light mass dependence study are shown in Fig. 6, which should be compared with the previous analysis Fig. 2. The left panel shows the six pion mass values, with the lightest two coming from our new larger $M_\pi L$ ensembles. As before, it seems $ud\bar{b}\bar{b}$ is much more sensitive

to the light mass than $ls\bar{b}\bar{b}$. We include much larger M_π values than in the previous study, and it's not necessarily valid to extend the linear fit form $A + BM_\pi^2$ to include these large pion masses. For this reason, we perform a fit to increasing numbers of data points, from the leftmost two, up to all six, and plot the resulting M_π^{phys} extrapolation in the right hand pane of Fig. 6. We then take as our final result the band which covers the spread of these values with their respective error bars. Whilst we don't perform a full finite volume analysis, exponential FV effects are included in this fit (see [26]) contributing an increase in binding of about 0.4σ in $ud\bar{b}\bar{b}$, with no meaningful effect on $ls\bar{b}\bar{b}$. These final results are given by,

$$\Delta E_{ud\bar{b}\bar{b}}(M_\pi^{\text{phys}}) = -115(17) \text{ MeV} \quad (7)$$

$$\Delta E_{ls\bar{b}\bar{b}}(M_\pi^{\text{phys}}) = -46.7(7.6) \text{ MeV}, \quad (8)$$

which represent somewhat shallower binding than in our previous analyses.

The variation of the binding with heavy masses dependence is show in Fig. 7, which should be compared with Fig. 4. The left hand pane shows the phenomenologically motivated fit to the $J^P = 1^+$ data, again showing a good fit. On the right, we compare $J^P = 0^+$ and $J^P = 1^+$ data for the $ud\bar{b}\bar{h}$ and $ls\bar{b}\bar{h}$ cases. We see that the $ls\bar{b}\bar{h}$ 0^+ and 1^+ give similar results, whilst the $ud\bar{b}\bar{h}$ case shows a relatively consistent difference of order 20 MeV.

4. Conclusions

The introduction of box sinks, as well as two new, larger lattice volumes, more pion masses and multi-exponential fits have allowed us to build on the previous work in [8, 12, 27]. We conducted studies of the light and heavy mass dependence of the $J^P = 1^+$ $ud\bar{b}\bar{b}$, $ud\bar{b}\bar{h}$, $ls\bar{b}\bar{b}$ and $ls\bar{b}\bar{h}$ tetraquark binding, as well as a comparison with the $J^P = 0^+$ $ud\bar{b}\bar{h}$, and $ls\bar{b}\bar{h}$ case.

We find bindings at physical pion masses which are reduced with respect to our previous studies, and fall in line with a general trend towards shallower bindings in recent analyses [9, 10, 19, 20, 22].

Whilst our heavy mass dependence analysis does not extend down to the charm mass, we can extrapolate by eye to see that, taking into effect the light mass dependence, it is roughly compatible with the confirmed existence of the $1^+ T_{cc}$ just below threshold [24, 25]. A similar extrapolation suggests that the $1^+ ud\bar{b}\bar{c}$ could also be bound, with shallow enough binding to permit an electromagnetic decay. If the $J^P = 0^+$ state is unbound, then the 1^+ state should have a near 100% branching fraction to $B\bar{D}\gamma$, producing highly collimated B - \bar{D} pairs in the lab frame. Even if the $J^P = 0^+$ state is bound, the 1^+ state will decay electromagnetically to a soft photon plus the 0^+ state as well as to $B\bar{D}\gamma$, with a sizeable branching fraction. This $B\bar{D}\gamma$ branch will have the same potentially useful, statistically enhanced signal as it would if 0^+ state were unbound.

References

- [1] J. Ader, J. Richard and P. Taxil, *Do narrow heavy multi - quark states exist?*, *Phys. Rev. D* **25** (1982) 2370.
- [2] L. Heller and J. Tjon, *On the Existence of Stable Dimesons*, *Phys. Rev. D* **35** (1987) 969.

- [3] A.V. Manohar and M.B. Wise, *Exotic $Q\bar{Q}$ anti- q anti- q states in QCD*, *Nucl. Phys. B* **399** (1993) 17 [[hep-ph/9212236](#)].
- [4] A. Francis, P. de Forcrand, R. Lewis and K. Maltman, *Diquark properties from full QCD lattice simulations*, *JHEP* **05** (2022) 062 [[2106.09080](#)].
- [5] P. Bicudo, K. Cichy, A. Peters and M. Wagner, *BB interactions with static bottom quarks from Lattice QCD*, *Phys. Rev. D* **93** (2016) 034501 [[1510.03441](#)].
- [6] P. Bicudo, J. Scheunert and M. Wagner, *Including heavy spin effects in the prediction of a $\bar{b}\bar{b}ud$ tetraquark with lattice QCD potentials*, *Phys. Rev. D* **95** (2017) 034502 [[1612.02758](#)].
- [7] P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer and M. Wagner, *$ud\bar{b}\bar{b}$ tetraquark resonances with lattice QCD potentials and the Born-Oppenheimer approximation*, *Phys. Rev. D* **96** (2017) 054510 [[1704.02383](#)].
- [8] A. Francis, R.J. Hudspith, R. Lewis and K. Maltman, *Lattice Prediction for Deeply Bound Doubly Heavy Tetraquarks*, *Phys. Rev. Lett.* **118** (2017) 142001 [[1607.05214](#)].
- [9] P. Junnarkar, N. Mathur and M. Padmanath, *Study of doubly heavy tetraquarks in Lattice QCD*, *Phys. Rev. D* **99** (2019) 034507 [[1810.12285](#)].
- [10] L. Leskovec, S. Meinel, M. Pflaumer and M. Wagner, *Lattice QCD investigation of a doubly-bottom $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$* , *Phys. Rev. D* **100** (2019) 014503 [[1904.04197](#)].
- [11] P. Mohanta and S. Basak, *Construction of $bb\bar{u}\bar{d}$ tetraquark states on lattice with NRQCD bottom and HISQ up and down quarks*, *Phys. Rev. D* **102** (2020) 094516 [[2008.11146](#)].
- [12] R.J. Hudspith, B. Colquhoun, A. Francis, R. Lewis and K. Maltman, *A lattice investigation of exotic tetraquark channels*, *Phys. Rev. D* **102** (2020) 114506 [[2006.14294](#)].
- [13] M. Pflaumer, L. Leskovec, S. Meinel and M. Wagner, *Investigation of Doubly Heavy Tetraquark Systems using Lattice QCD*, in *Asia-Pacific Symposium for Lattice Field Theory*, 9, 2020 [[2009.10538](#)].
- [14] M. Pflaumer, L. Leskovec, S. Meinel and M. Wagner, *Existence and Non-Existence of Doubly Heavy Tetraquark Bound States*, *PoS LATTICE2021* (2022) 392 [[2108.10704](#)].
- [15] M. Wagner, C. Alexandrou, J. Finkenrath, T. Leontiou, S. Meinel and M. Pflaumer, *Lattice QCD study of antiheavy-antiheavy-light-light tetraquarks based on correlation functions with scattering interpolating operators both at the source and at the sink*, *PoS LATTICE2022* (2023) 270 [[2210.09281](#)].
- [16] M. Pflaumer, C. Alexandrou, J. Finkenrath, T. Leontiou, S. Meinel and M. Wagner, *Antiheavy-antiheavy-light-light four-quark bound states*, *PoS LATTICE2022* (2023) 075 [[2211.00951](#)].

- [17] B. Colquhoun, A. Francis, R.J. Hudspith, R. Lewis and K. Maltman, *Investigating exotic heavy-light tetraquarks with 2+1 flavour lattice QCD*, *PoS LATTICE2021* (2022) 144.
- [18] B. Colquhoun, A. Francis, R. Hudspith, R. Lewis and K. Maltman, *Ruling out some predictions of deeply-bound light-heavy tetraquarks using lattice QCD*, *Rev. Mex. Fis. Suppl.* **3** (2022) 0308044.
- [19] T. Aoki, S. Aoki and T. Inoue, *Lattice study on a tetraquark state T_{bb} in the HAL QCD method*, *Phys. Rev. D* **108** (2023) 054502 [2306.03565].
- [20] R.J. Hudspith and D. Mohler, *Exotic tetraquark states with two b^- quarks and $JP=0+$ and $I+B_s$ states in a nonperturbatively tuned lattice NRQCD setup*, *Phys. Rev. D* **107** (2023) 114510 [2303.17295].
- [21] L. Mueller, P. Bicudo, M. Krstic Marinkovic and M. Wagner, *Antistatic-antistatic-light-light potentials from lattice QCD*, 12, 2023 [2312.17060].
- [22] C. Alexandrou, J. Finkenrath, T. Leontiou, S. Meinel, M. Pflaumer and M. Wagner, *$\bar{b}\bar{b}ud$ and $\bar{b}\bar{b}us$ tetraquarks from lattice QCD using symmetric correlation matrices with both local and scattering interpolating operators*, 2404.03588.
- [23] S. Meinel, M. Pflaumer and M. Wagner, *Search for b^-b^-us and b^-c^-ud tetraquark bound states using lattice QCD*, *Phys. Rev. D* **106** (2022) 034507 [2205.13982].
- [24] LHCb collaboration, *Observation of an exotic narrow doubly charmed tetraquark*, *Nature Phys.* **18** (2022) 751 [2109.01038].
- [25] LHCb collaboration, *Study of the doubly charmed tetraquark T_{cc}^+* , *Nature Commun.* **13** (2022) 3351 [2109.01056].
- [26] B. Colquhoun, A. Francis, R.J. Hudspith, R. Lewis, K. Maltman and W.G. Parrott, *Improved analysis of strong-interaction-stable doubly-bottom tetraquarks on the lattice*, 2407.08816.
- [27] A. Francis, R.J. Hudspith, R. Lewis and K. Maltman, *Evidence for charm-bottom tetraquarks and the mass dependence of heavy-light tetraquark states from lattice QCD*, *Phys. Rev. D* **99** (2019) 054505 [1810.10550].
- [28] PACS-CS collaboration, *Physical Point Simulation in 2+1 Flavor Lattice QCD*, *Phys. Rev. D* **81** (2010) 074503 [0911.2561].
- [29] PACS-CS collaboration, *Charm quark system at the physical point of 2+1 flavor lattice QCD*, *Phys. Rev. D* **84** (2011) 074505 [1104.4600].
- [30] P. Lepage, *gplepage/corrfitter: corrfitter version 8.2*, Nov., 2021. 10.5281/zenodo.5733391.
- [31] P. Lepage and C. Gohlke, *gplepage/lqfit: lqfit version 13.0.1*, May, 2023. 10.5281/zenodo.7931361.
- [32] P. Lepage, C. Gohlke and D. Hackett, *gplepage/gvar: gvar version 11.11.11*, June, 2023. 10.5281/zenodo.8025535.