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NJ/ψ and $N\eta_c$ interactions from lattice QCD

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The interaction between nucleon and charmonia (J/ψ and η_c) is expected to deepen our understanding of various aspects in nonperturbative QCD ranging from the origin of nucleon mass to J/ψ mass modification in nuclear medium and properties of hidden-charm pentaquark states. Here, we present the low-energy NJ/ψ and $N\eta_c$ interactions based on $(2+1)$ flavor lattice QCD simulations with nearly physical pion mass $m_\pi = 146$ MeV. The interactions, extracted from the spacetime correlations of the nucleon and charmonium system by using the HAL QCD method, are found to be attractive in all distances and manifest a characteristic long-range tail consistent with the two-pion exchange interaction. The resulting scattering lengths are around 0.3 fm, 0.4 fm and 0.2 fm for NJ/ψ with spin $3/2$, with spin $1/2$, and $N\eta_c$, respectively. Our results are orders of magnitude larger than those from the photoproduction experiments assuming the vector meson dominance.

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1. Introduction

The low-energy interaction between a nucleon (N) and a charmonium (J/ψ and η_c) has deep connections with fundamental questions in QCD, such as how hadrons gain/lose mass in vacuum/medium, what kind of multiquark hadrons can exit.

Firstly, the trace anomaly contribution to the nucleon mass is related to the forward scattering amplitude of N and J/ψ. This is because the amplitude involves the nucleon matrix element of gluon field $\langle N|GG|N\rangle$, as shown in Ref. [1]. In addition, the J/ψ mass modification in nuclear medium can be obtained from the N-J/ψ scattering length, as discussed in Ref. [2]. Secondly, following the discovery of the first pentaquark state P_c by the LHCb Collaboration in 2015 [3], a comprehensive understanding of its nature and properties has become a pressing issue. Achieving this goal requires precise knowledge of the N-$c\bar{c}$ interactions, which serve as critical inputs in coupled-channel analyses. Moreover, accurate N-$c\bar{c}$ interactions are also crucial for predicting the binding energy of charmonium-nucleus bound states, given large discrepancies remain among various phenomenological methods [4]. Thirdly, since N and $c\bar{c}$ do not have common quarks, the interaction between them arises from multiple gluon exchange, which is likely dominated by two-pion exchange at long range [5–7]. It is important to investigate this hypothesis from lattice QCD.

Experimentally, the low-energy N-$c\bar{c}$ interaction is inferred from the J/ψ photonproduction off the proton, which gives rise to a scattering length of $O(1 \sim 10) \times 10^{-3}$ fm [8] when assuming the vector meson dominance and $O(1)$ fm [9] when constrained by the low-energy unitarity. Previous lattice claudications [10–14] are limited to quenched approximation, heavy pion mass, or large uncertainties.

Given the situation, a realistic lattice QCD study on the low-energy N-$c\bar{c}$ interaction has been performed recently in Ref. [15], and we report the results in this conference.

2. Methodology

The starting point in HAL QCD method [16, 17] is the following R-correlator,

$$\begin{aligned} R(\boldsymbol{r},t) &= \sum_{\boldsymbol{x}} \langle 0|N(\boldsymbol{x}+\boldsymbol{r},t)O_{c\bar{c}}(\boldsymbol{x},t)\overline{\mathcal{J}}(0)|0\rangle \,/e^{-(m_N+m_{c\bar{c}})t} \\ &= \sum_n a_n \psi_{E_n}(\boldsymbol{r}) e^{-(\Delta E_n)t} + O(e^{-(\Delta E^*)t}), \end{aligned} \tag{1}$$

where $\mathcal{J}(0)$ and $a_n = \langle N, c\bar{c}; E_n|\overline{\mathcal{J}}(0)|0\rangle$ are a source operator and the corresponding overlapping factor to n-th eigenstate, respectively. $\Delta E_n = E_n - (m_N + m_{c\bar{c}})$ and $\Delta E^* \sim m_\pi$ are the eigenenergy and the inelastic threshold with respect to the N-$c\bar{c}$ threshold. $R(\boldsymbol{r},t)$ at large t satisfies the following integrodifferential equation,

$$\begin{aligned} &\left[\frac{1+3\delta^2}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 + O(\delta^2\partial_t^3)\right] R(\boldsymbol{r},t) \\ &= \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t), \quad \mu = \frac{m_N m_{c\bar{c}}}{m_N + m_{c\bar{c}}}, \quad \delta = \frac{m_N - m_{c\bar{c}}}{m_N + m_{c\bar{c}}}. \end{aligned} \tag{2}$$

The nonlocal potetential defined above can be expanded as $U(\boldsymbol{r},\boldsymbol{r}') = V(r)\delta(\boldsymbol{r}-\boldsymbol{r}')+\sum_{n=1} V_n(\boldsymbol{r})\nabla^n\delta(\boldsymbol{r}-\boldsymbol{r}')$, leading to a local potential at the leading order, which is accurate for describing near-threshold scattering,

$$V(r) = R^{-1}(\boldsymbol{r},t)\left[\frac{1+3\delta^2}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 + O(\delta^2\partial_t^3)\right] R(\boldsymbol{r},t), \tag{3}$$

where the $O(\delta^2\partial_t^3)$ term is found to be consistent with zero within statistical uncertainties, and is neglected in our study.

3. Lattice setup

Local sink operators are adopted in our calculation. They are defined as,

$$N_\alpha(x) = \epsilon_{ijk}[u^i(x)C\gamma_5 d^j(x)]u^k_\alpha(x), \tag{4}$$

$$J/\psi_\mu(x) = \delta_{ij}\bar{c}^i(x)\gamma_\mu c^j(x), \tag{5}$$

$$\eta_c(x) = \delta_{ij}\bar{c}^i(x)\gamma_5 c^j(x), \tag{6}$$

with ijk being color indices, α (μ) being spinor (vector) index. At source, we perform wall-type smearing and Coulomb gauge fixing.

We use (2 + 1)-flavor lattice QCD configurations generated on a 96^4 lattice with a spacing of $a \simeq 0.0846$ fm ($a^{-1} \simeq 2333$ MeV), resulting in a physical volume of $La \simeq 8.1$ fm. The Iwasaki gauge action at $\beta = 1.82$ and the nonperturbatively $O(a)$-improved Wilson quark action with stout smearing are employed at nearly physical quark masses [18]. For the charm quark, the relativistic heavy quark (RHQ) action is used to eliminate cutoff errors up to next-to-next-to-leading order [19]. Two sets of RHQ parameters (set 1 and set 2) [20], tuned close to the physical charm quark mass, allow interpolation (0.385× set 1 + 0.615× set 2) to reproduce the spin-averaged $1S$ charmonium dispersion relation. Shown in Table 1 is the mass for relevant hadrons in our study.

Table 1: Hadron masses with statistical errors obtained from lattice QCD together with the experimental values (isospin-averaged). Two values for J/ψ and η_c are from set 1 and set 2 parameters of RHQ action, respectively.

Hadron	Lattice [MeV]		Expt. [MeV]
π	146.4(4)		138.0
K	524.7(2)		495.6
N	954.0(2.9)		938.9
J/ψ	3121.1(1)	3076.3(1)	3096.9
η_c	3022.8(1)	2976.3(1)	2984.1

4. Numerical results

4.1 Potential

In Fig. 1, we show the N-$c\bar{c}$ potentials extracted at $t \simeq 1.2$ fm. We find: (i) Potentials show weak t dependence, meaning systematic errors associated with the inelastic states and the truncation of the derivative expansion are small. (ii) The N-$c\bar{c}$ potential is attractive at all distances, qualitatively similar as N-$s\bar{s}$ potential [22]. (iii) The long range potentials for all three channels are very similar, indicating a common underlying mechanism dictates these three interactions at large distances.

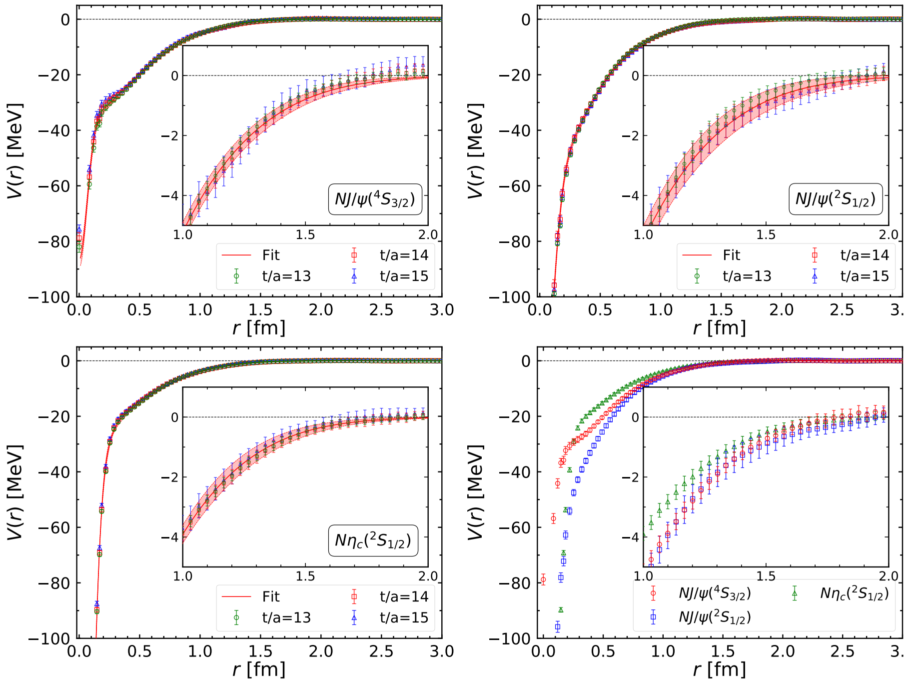

Figure 1: The N-$c\bar{c}$ potential extracted at $t/a = 13$, 14, and 15 for N-J/ψ with $^4S_{3/2}$ (upper left), with $^2S_{1/2}$ (upper right), and N-η_c with $^2S_{1/2}$ (lower left). The red bands show the fit results with phenomenological three-range Gaussians at $t/a = 14$. The three potentials at $t/a = 14$ are also shown in (lower right) for a direct comparison. A magnification is shown in the inset for each panel. Figures are taken from [15].

4.2 Two-pion exchange

As discussed before, the potential between N and $c\bar{c}$ initially generated through multiple gluon exchange, is expected to transform into a two-pion exchange mechanism at long distances, which takes the following term in coordinate space $V(r) = -\alpha\frac{e^{-2m_\pi r}}{r^2}$ [7]. Theoretically, this potential closely resembles the van der Waals interaction generated from multiple photon exchange in QED, which decreases in power-law in coordinate space $V(r) = -\alpha/r^7$. Shown in Fig. 2 is the best fit to the long range potential ($0.9 < r < 1.8$ fm) by $V(r) = \alpha\frac{e^{-2m_\pi r}}{r^2}$ (bands) with a fit parameter $\alpha = 22(2)$, 23(3), and 16(2) MeVfm^2 for N-J/ψ with spin 3/2, with spin 1/2, and N-η_c, respectively. This result implies that the long-range N-$c\bar{c}$ potentials are consistent with the two-pion exchange interaction.

4.3 Phase shifts and scattering parameters

In order to convert the lattice potential to physical observables, we fit the potentials in Fig. 1 with three-range Gaussians $V_{\rm fit}(r) = -\sum_{i=1}^{3} a_i e^{(-r/b_i)^2}$, and corresponding fitting results are shown by red bands in Fig. 1. The scattering phase shits are calculated with $V_{\rm fit}(r)$ and shown in Fig. 3.

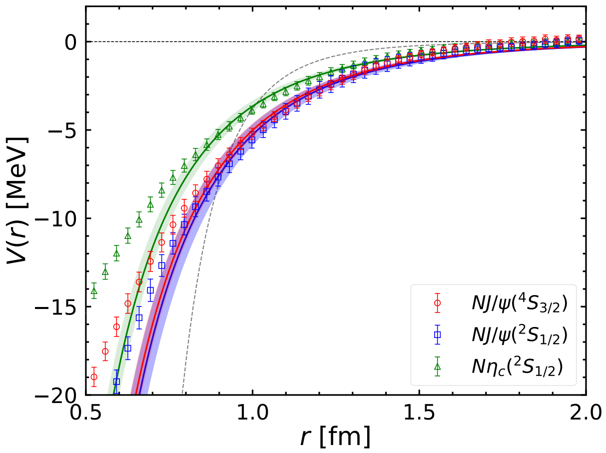

Figure 2: The bands show the fit with the TPE function $V(r) = -\alpha e^{-2m_\pi r}/r^2$ to the long-range N-$c\bar{c}$ potentials. The gray dashed line is the best fit with $V(r) = -\alpha/r^7$ for comparison. Taken from [15].

Scattering length (a_0) and effective range (r_{eff}) are extracted from the near threshold phase shifts through $k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + O(k^4)$, and tabulated in Table 2.

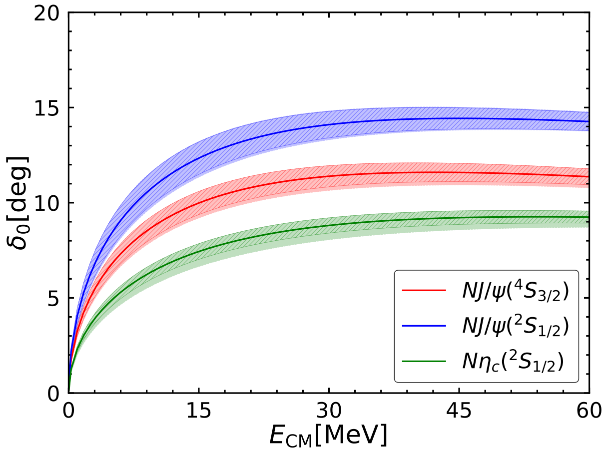

Figure 3: The N-$c\bar{c}$ scattering phase shifts. The central value and statistical error obtained with $V_{\text{fit}}(r)$ at $t/a = 14$ are shown by solid lines and inner bands. The outer bands show total uncertainty obtained by adding statistical error and systematic error in quadrature. Taken from [15].

Using the scattering length in Table 2, we are able to predict the J/ψ mass reduction in normal nuclear medium with density $\rho_{\text{nm}} = 0.17\ \text{fm}^{-3}$,

$$\delta m_{J/\psi} \simeq \frac{2\pi(m_N + m_{J/\psi})}{m_N m_{J/\psi}} a_{J/\psi}^{\text{spin-av}} \rho_{\text{nm}} = 19(3)\ \text{MeV}, \tag{7}$$

where the N-J/ψ spin-averaged scattering length is defined as $a_{NJ/\psi}^{\text{spin-av}} = \frac{2a_0^{(3/2)} + a_0^{1/2)}}{3}$.

In Table 3, we summarize the spin-averaged N-J/ψ scattering length in the literature, which involves various approaches including: photoproduction reaction, QCD multipole expansion, QCD sum rule, coupled channel analysis, and lattice QCD. Previous studies in one way or another surfer

Table 2: The N-$c\bar{c}$ scattering length a_0 and effective range $r_{\rm eff}$ with statistical error (1st parentheses) and systematic error (2nd parentheses).

channel	a_0 [fm]	$r_{\rm eff}$ [fm]
N-$J/\psi(^4S_{3/2})$	$0.30(2)\left(^{+0}_{-2}\right)$	$3.25(12)\left(^{+6}_{-9}\right)$
N-$J/\psi(^2S_{1/2})$	$0.38(4)\left(^{+0}_{-3}\right)$	$2.66(21)\left(^{+0}_{-10}\right)$
$N\eta_c(^2S_{1/2})$	$0.21(2)\left(^{+0}_{-1}\right)$	$3.65(19)\left(^{+0}_{-6}\right)$

from either badly fixed parameters in corresponding models or lacking of some important dynamics of QCD in corresponding calculations, which also highlights the importance of performing realistic lattice QCD calculations as what we did in the present study.

Table 3: The spin-averaged N-J/ψ scattering length in the literature compared with the result from the present study shown in the last line.

$a^{\rm spin\text{-}av}_{NJ/\psi}$ [fm]	Year	Author	Method
0.046(5)	2016	Gryniuk-Vanderhaeghen [23]	Photoproduction (VMD)
$3 \sim 25 \cdot 10^{-3}$	2021	Pentchev-Strakovsky [8]	Photoproduction (VMD)
$O(1)$	2023	JPAC [9]	Photoproduction (unitarity)
0.05	1992	Kaidalov-Volkovitsky [24]	QCD multipole expansion
0.24	1997	Brodsky-Miller [25]	QCD multipole expansion
≥ 0.37	2005	Sibirtsev-Voloshin [26]	QCD multipole expansion
$0.2 \sim 3 \cdot 10^{-3}$	2020	Du-Baru-Guo et. al. [27]	Coupled channel
0.10(2)	1999	Hayashigaki [2]	QCD sum rule
0.71(48)	2006	Yokokawa-Sasaki et al. [10]	LQCD (quenched)
0.1(7)	2008	Liu-Lin-Orginos [13]	LQCD (full, extrapolate to phys. pt.)
0.33(5)	2010	Kawanai-Sasaki [11]	LQCD (quenched)
0.47(1)	2019	Sugiura-Ikeda-Ishii [12]	LQCD (m_π = 700 Mev)
≈ 0	2019	Skerbis-Prelovsek [14]	LQCD (m_π = 266 Mev)
0.33(4)	2024	**The present work**	LQCD (m_π = 146 Mev)

5. Summary

We present a realistic study on the low-energy N-$c\bar{c}$ interaction based on $(2+1)$-flavor lattice QCD simulations with nearly physical light quark masses (m_π = 146 MeV) and physical charm quark mass. The potential derived by the HAL QCD method is attractive for all distances, and possesses a characteristic long-range behavior consistent with the two-pion exchange interaction. The resulting phase shifts and scattering parameters are shown in Fig. 3 and Table 2.

We are under way to perform physical-point calculations with the (2 + 1)-flavor lattice QCD configurations generated by the HAL QCD Collaboration [28]. Also, we plan to investigate the universality of the long-range potential across other hadron pairs.

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