

## Localization of Dirac modes in a finite temperature SU(2) Higgs model

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Low-lying Dirac modes become localized at the finite-temperature transition in QCD and other gauge theories, indicating a strong connection between localization and deconfinement. This phenomenon can be understood through the “sea/islands” picture: in the deconfined phase, modes become trapped on “islands” of Polyakov loop fluctuations within a “sea” of ordered Polyakov loops. To test the universality of the “sea/islands” mechanism, we investigate whether changes in the localization properties of low modes occur across other thermal transitions where the Polyakov loop becomes ordered, beyond the usual deconfinement transition. The fixed-length SU(2) Higgs model is appropriate for this study. After mapping out the phase diagram, we find that low Dirac modes become localized in the deconfined and Higgs phases, where the Polyakov loop is ordered. However, localization is absent in the confined phase. These findings confirm the “sea/islands” picture of localization.

*The 41st International Symposium on Lattice Field Theory (LATTICE2024)  
28 July - 3 August 2024  
Liverpool, UK*

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## 1. Introduction

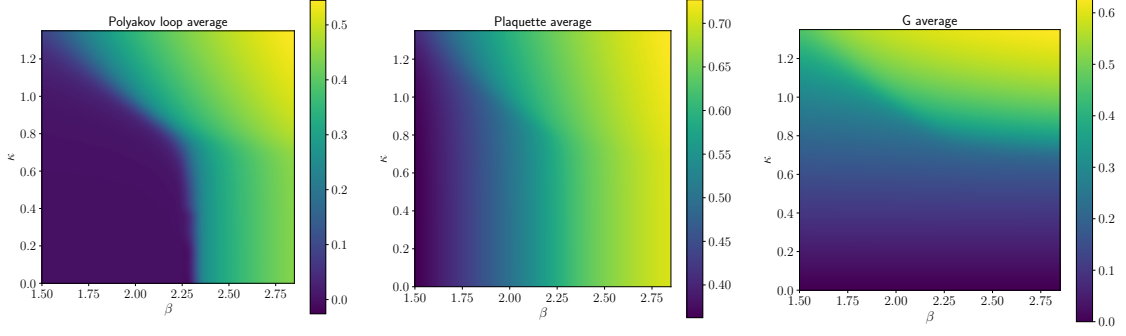
The connection between deconfinement and chiral symmetry restoration at the finite-temperature transition in QCD [1, 2] is still not fully understood. The behavior of the low Dirac modes across the transition could be key in understanding this connection. In fact, chiral symmetry breaking is controlled in the chiral limit by the behavior near zero of the (normalized) spectral density  $\rho(\lambda)$  of Dirac modes,

$$\rho(\lambda) = \lim_{V \rightarrow \infty} \frac{T}{V} \langle \sum_n \delta(\lambda - \lambda_n) \rangle, \quad (1)$$

where  $T$  and  $V$  are the temperature and the volume of the system, respectively, as shown by the Banks-Casher relation  $|\langle \bar{\psi}\psi \rangle| \stackrel{m \rightarrow 0}{=} \pi \rho(0^+)$  [3] for the chiral condensate,  $\langle \bar{\psi}\psi \rangle$ . While chiral symmetry is only approximate for physical quark masses, the effects of its spontaneous breaking in the chiral limit are still visible below the crossover temperature, in particular through a sizeable density of near-zero modes, while its restoration in the high-temperature phase leads to the near-zero spectral region getting depleted. On the other hand, deconfinement affects the localization properties of the low modes: a large amount of evidence shows that in gauge theories with a genuine phase transition, low modes are delocalized in the confined phase and localized in the deconfined phase, up to a critical point in the spectrum,  $\lambda_c$ , known as “mobility edge”, that appears precisely at the critical temperature [4–13] (see Ref. [14] for a review). Also in QCD low modes are delocalized in the low-temperature phase, and become localized in the high-temperature phase, and even though in this case there is no sharply defined critical temperature, the appearance of the mobility edge is well within the crossover region [15–24]. Being sensitive to both chiral symmetry restoration and deconfinement, low Dirac modes could help in unveiling the details of the relationship between the two phenomena.

The connection between localization and deconfinement can be understood qualitatively through the “sea/islands” picture of localization [13, 14, 25–28]. In its original version [25], further developed in Refs. [14, 26–28], this picture explained localization of the low modes in high-temperature QCD in terms of “islands” of Polyakov-loop fluctuations in the “sea” of ordered Polyakov loops, taking values near 1, that characterizes the deconfined phase. These islands are “energetically” favorable for the eigenmodes, supporting eigenvalues below the Matsubara frequency, and are expected to “trap” the corresponding eigenvectors. A refined version of this picture was formulated in Ref. [13], where the role of islands is played more generally by fluctuations that decrease the correlation of gauge fields in the temporal direction, including but not limited to Polyakov-loop fluctuations. This picture applies as well in the quenched case, where at high temperature an exact center symmetry breaks down spontaneously, in the physical sector selected by static fermions; and more generally as long as a strong ordering of the Polyakov loop near 1 takes place. This leads one to expect localization of low Dirac modes in the deconfined phase of a generic gauge theory, an expectation confirmed in all the models investigated so far, both with and without dynamical fermionic matter [4–24].

Further possible tests of this picture include looking at phases of a gauge theory where the Polyakov loop gets ordered, but different from the usual deconfined phase; and looking at theories with a different matter content. One model that allows one to carry out both tests at once is the fixed-length SU(2) Higgs model [29].



**Figure 1:** Expectation values of the Polyakov loop (left), plaquette (center), and gauge-Higgs coupling (right) in the  $(\beta, \kappa)$  plane. Here  $N_s = 20$  and  $N_t = 4$ .

## 2. Fixed-length SU(2) Higgs model

In Ref. [30] we studied Dirac-mode localization and tested the sea/islands picture in the fixed-length SU(2) Higgs model [29]. This model is obtained from the usual SU(2) Higgs model in the limit of infinite Higgs self-coupling, where the magnitude of the doublet of complex Higgs fields does not fluctuate. In this case the properly rescaled Higgs fields can be reorganized in a SU(2) matrix,  $\phi$ . Discretized on a hypercubic  $N_t \times N_s^3$  lattice, this model is defined by the action

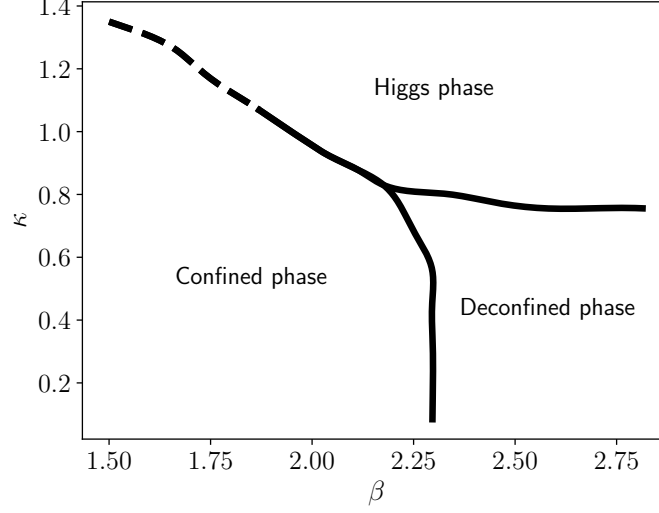
$$S = -\frac{1}{2} \sum_n \left\{ \beta \sum_{1 \leq \mu < \nu \leq 4} \text{tr} U_{\mu\nu}(n) + \kappa \sum_{1 \leq \mu \leq 4} \text{tr} G_\mu(n) \right\}, \quad (2)$$

where  $n$  runs over the lattice sites,  $U_{\mu\nu} = U_\mu(n)U_\nu(n + \hat{\mu})U_\mu(n + \hat{\nu})^\dagger U_\nu(n)^\dagger$  are the usual plaquette variables built out of the link variables  $U_\mu(n)$ ,  $G_\mu(n) = \phi(n)^\dagger U_\mu(n) \phi(n + \hat{\mu})$  couples scalar and gauge fields, and  $\beta$  and  $\kappa$  are real parameters.

The phase diagram of this model was numerically studied in detail at zero temperature in Ref. [31], identifying three phases – a confined phase at low  $\beta$  and  $\kappa$ ; a deconfined phase at large  $\beta$  and low  $\kappa$ ; and a Higgs phase at large  $\kappa$ . This is in agreement with the theoretical expectations of Ref. [29]. The phase diagram was studied also at low temperature resulting in a similar picture, with the transitions between the three phases identified as crossovers. As a preliminary step for our study of localization, we mapped out the finite-temperature phase diagram at fixed temporal extension  $N_t = 4$  in lattice units. Our results for the average Polyakov loop, plaquette, and gauge-Higgs coupling  $\langle G(n) \rangle$ , where

$$G(n) = \frac{1}{8} \sum_{1 \leq \mu \leq 4} (G_\mu(n) + G_\mu(n - \hat{\mu})), \quad (3)$$

are shown in Fig. 1. This confirms our expectations for the phase diagram. We also confirmed the crossover nature of the transition along three lines, one at constant  $\kappa = 0.5$  and two at constant  $\beta = 2.0$  and  $\beta = 2.6$ , crossing over between the confined and the deconfined phase, the confined and the Higgs phase, and the deconfined and the Higgs phase, respectively. Interpolating the peaks of the Polyakov loop, plaquette, and gauge-Higgs coupling susceptibilities with splines, and putting the resulting curves together, we obtained the phase diagram shown in Fig. 2.



**Figure 2:** Phase diagram of the fixed-length SU(2) Higgs model on the lattice for  $N_t = 4$ .

### 3. Localization in the SU(2) Higgs model

To investigate the connection between Polyakov-loop ordering and Dirac-mode localization we probed the gauge configurations of the fixed-length SU(2) Higgs model using external staggered fermions. After generating gauge configurations with a standard heat-bath algorithm, we obtained the low-lying staggered modes with the PRIMME package for sparse matrices [32], and analyzed their localization properties as well as the statistical properties of the spectrum.

The localization properties of staggered eigenmodes are defined by the scaling with the lattice size of the lattice volume that they effectively occupy. The size of mode  $\psi_l$ , corresponding to the staggered eigenvalue  $i\lambda_l$ , is defined using its inverse participation ratio,

$$\text{IPR}_l \equiv \sum_n (\sum_c |\psi_{lc}(n)|^2)^2, \quad (4)$$

where  $c$  is the color index, as

$$\text{size}_l \equiv \text{IPR}_l^{-1}. \quad (5)$$

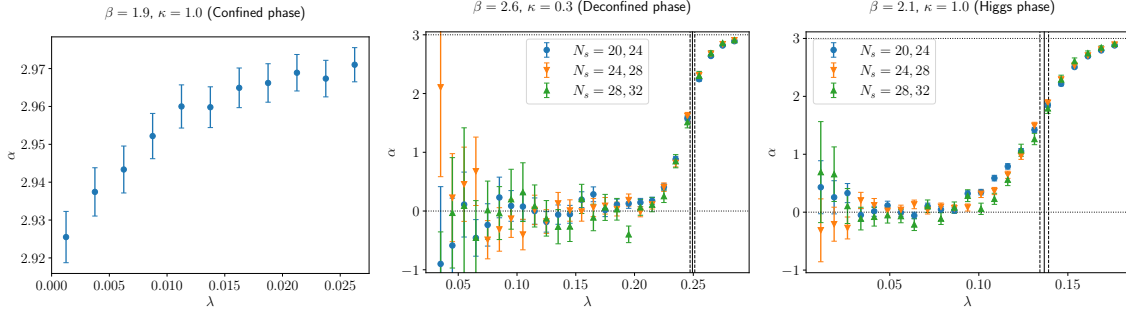
After averaging  $\text{size}_l$  in a small spectral interval and over gauge configurations,

$$\langle \text{size}(\lambda) \rangle \equiv \frac{\langle \sum_l \delta(\lambda - \lambda_l) \text{size}_l \rangle}{\langle \sum_l \delta(\lambda - \lambda_l) \rangle}, \quad (6)$$

one studies its scaling with the linear spatial size  $N_s$  of the lattice. One expects that  $\langle \text{size}(\lambda) \rangle \propto N_s^{\alpha(\lambda)}$  for sufficiently large lattice size, where the fractal dimension  $\alpha(\lambda)$  is  $\alpha = 0$  if in the spectral region near  $\lambda$  the modes are spatially localized, and  $\alpha = 3$  if they are delocalized over the whole lattice. A practical way to estimate  $\alpha$  from numerical data is via

$$\alpha(\lambda; N_{s_1}, N_{s_2}) \equiv \ln \left( \frac{\langle \text{size}(\lambda) \rangle_{N_{s_2}}}{\langle \text{size}(\lambda) \rangle_{N_{s_1}}} \right) \bigg/ \ln \left( \frac{N_{s_2}}{N_{s_1}} \right), \quad (7)$$

in the limit of large (and different)  $N_{s_{1,2}}$ . The dependence of  $\alpha(\lambda; N_{s_1}, N_{s_2})$  on the position in the spectrum is shown for the three phases in Fig. 3. While in the confined phase  $\alpha \approx 3$  (estimated



**Figure 3:** Fractal dimension, Eq. (7), of low staggered modes in the confined (left), deconfined (center), and Higgs phase (right). Vertical solid and dashed lines mark the position of the mobility edge and the corresponding numerical uncertainty, respectively.

using relatively small sizes  $N_{s_{1,2}} = 16, 20$  in the whole near-zero region, in both the deconfined and the Higgs phase one clearly observes a transition from  $\alpha = 0$  near zero to  $\alpha = 3$  higher up in the spectrum. This indicates the presence of a mobility edge,  $\lambda_c$ , between the two spectral regions, where the localization length of the localized modes diverges and a second-order transition (“Anderson transition” [33]) takes place in the spectrum.

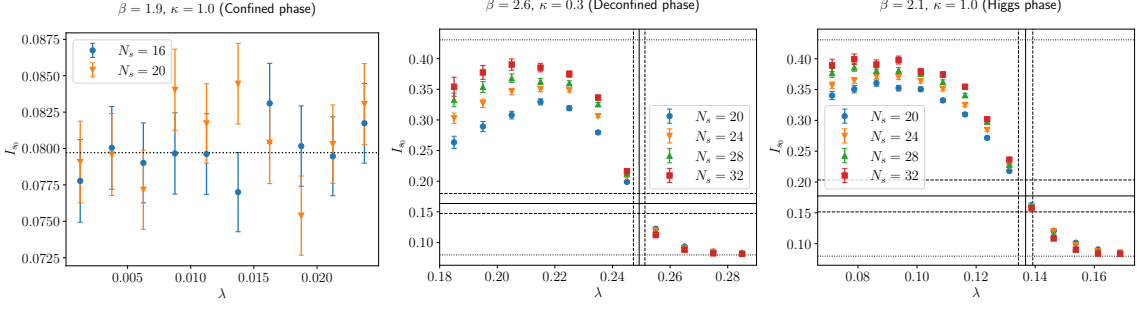
#### 4. Localization and spectral statistics

An alternative way to detect localization, that also allows for a precise determination of  $\lambda_c$ , exploits the connection between the localization properties of the eigenmodes and the statistical properties of the corresponding eigenvalues [34]. Since a local change in the gauge field configuration affects a localized mode only if it takes place within its localization region, eigenvalues corresponding to different localized modes fluctuate independently, and one expects them to obey Poisson statistics. On the other hand, every delocalized mode is affected by any local change in the gauge configuration, and so one expects delocalized modes to be strongly correlated with each other, and to obey the type of random matrix theory (RMT) [35, 36] statistics appropriate for the symmetry class of the system.

A convenient way to exploit this connection is to look at universal statistical properties of the spacing between subsequent eigenvalues, unveiled by means of the so-called unfolding procedure. For large volumes the unfolded level spacings read simply  $s_i \approx (\lambda_{i+1} - \lambda_i)\rho(\lambda_i)V/T$ , and their probability distribution  $p(s)$  is of the form  $p_{\text{Poisson}}(s) = e^{-s}$  for Poisson statistics, while for RMT statistics it is closely approximated by the so-called Wigner surmise,  $p_{\text{RMT}}(s) = a_\beta s^\beta e^{-b_\beta s^2}$ , where the Dyson index  $\beta$  depends on the symmetry class of the system [35]. For the staggered operator in the background of SU(2) gauge field configurations this is the symplectic class, for which  $\beta = 4$  [36]. The coefficients  $a_\beta$  and  $b_\beta$  are fixed by the normalization conditions  $\int_0^\infty ds p(s) = \int_0^\infty ds s p(s) = 1$ .

To identify the mobility edge, one monitors how  $p(s)$ , evaluated locally, changes across the spectrum. This is done in practice by observing some feature of  $p(s)$ , for example the integrated probability density  $I_{s_0}$  [37],

$$I_{s_0} = \int_0^{s_0} ds p(s), \quad (8)$$



**Figure 4:** Spectral statistic  $I_{s_0}$ , Eq. (8), of low staggered modes in the confined (left), deconfined (center), and Higgs phase (right). Vertical solid and dashed lines mark the position of the mobility edge and the corresponding numerical uncertainty, respectively; horizontal solid and dashed lines mark the value of the critical value  $I_{s_0,c}$  and the corresponding numerical uncertainty, respectively.

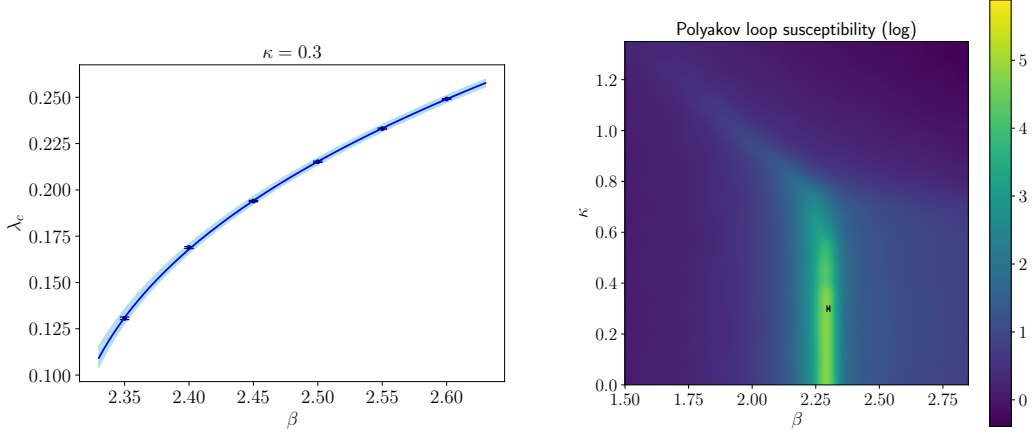
where  $s_0$  is conveniently chosen to maximize the difference between the Poisson and RMT expectations. As one moves from the localized to the delocalized part of the spectrum,  $I_{s_0}$  correspondingly changes from close to its Poisson value to close to its RMT value, getting closer and closer as the lattice size is increased. Since a second-order transition takes place at  $\lambda_c$ , physics should be scale-invariant there, and so  $I_{s_0}$  at  $\lambda_c$  should be volume-independent. This allows one to accurately identify  $\lambda_c$  by means of a finite-size-scaling study [37]; or, less accurately but more practically, by looking at the crossing points of the curves for  $I_{s_0}$  corresponding to different volumes. We followed the latter approach. In any case, since the critical statistics is expected to be universal, this kind of study has to be done only for one set of parameters, and the resulting critical value  $I_{s_0,c}$  of  $I_{s_0}$  can then be used to estimate  $\lambda_c$  elsewhere using a single volume by looking for the point where  $I_{s_0}$  crosses  $I_{s_0,c}$ . For a consistency check, we did the volume study at two points, one in the deconfined and one in the Higgs phase.

Our results are shown in Fig. 4. In the confined phase  $I_{s_0} \approx I_{s_0,\text{RMT}}$  throughout the low end of the spectrum, confirming the delocalized nature of these modes. Both in the deconfined and in the Higgs phase, as the volume increases  $I_{s_0}$  tends to the Poisson value near zero, and to the RMT value higher up in the spectrum, again confirming our results for the fractal dimension. The critical values  $I_{s_0,c}$  obtained in the two phases are consistent with each other.

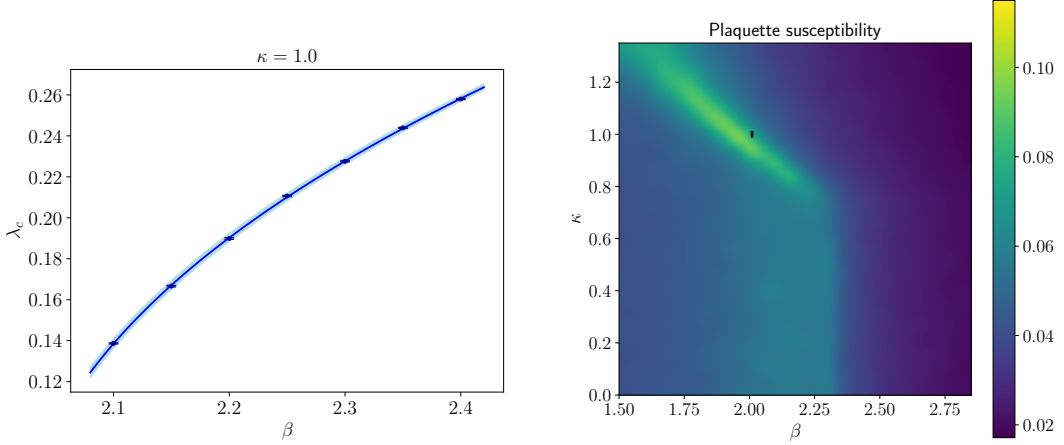
## 5. $\beta$ and $\kappa$ dependence of the mobility edge

According to the sea/islands picture of localization, one expects  $\lambda_c$  to be pushed towards zero as one gets closer to the confined phase, and to disappear somewhere in the crossover region. As explained above, one can efficiently estimate the position of the mobility edge using the critical value  $I_{s_0,c}$ , and so one can verify the validity of the sea/islands picture by studying the dependence of  $\lambda_c$  on  $\beta$  and  $\kappa$  using a single lattice volume for each choice of parameters. In this study we used a fixed spatial size  $N_s = 20$ .

The results of our analysis for the transition between the deconfined and the confined phase at  $\kappa = 0.3$  are shown in Fig. 5 (left panel), together with a plot of the Polyakov loop susceptibility



**Figure 5:**  $\beta$  dependence of the mobility edge at fixed  $\kappa = 0.3$  (left), and Polyakov-loop susceptibility (right). In the left panel we also show a fit with Eq. (9). In the right panel we show the position of the critical point  $(\beta_c(\kappa = 0.3), \kappa = 0.3)$  where the mobility edge disappears.



**Figure 6:**  $\beta$  dependence of the mobility edge at fixed  $\kappa = 1.0$  (left), and plaquette susceptibility (right). In the left panel we also show a fit with Eq. (9). In the right panel we show the position of the critical point  $(\beta_c(\kappa = 1.0), \kappa = 1.0)$  where the mobility edge disappears.

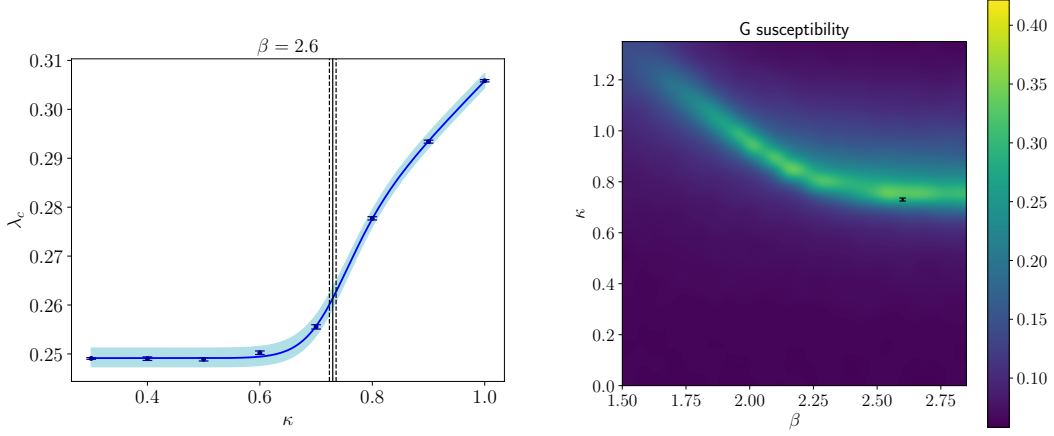
(right panel). Fitting the numerical data for the mobility edge with a power law,

$$\lambda_c(\beta) = a(\beta - \beta_c)^b, \quad (9)$$

we find that  $\beta_c$ , where the mobility edge disappears, is in the crossover region.

Our results for the transition between the Higgs and the confined phase at  $\kappa = 1.0$  are shown in Fig. 6 (left panel), together with a plot of the plaquette susceptibility (right panel). We fitted our numerical data with the same power-law form Eq. (9), finding again that the point where localized modes disappear,  $\beta_c$ , is in the crossover region.

Finally, we show our results for the transition between the deconfined and the Higgs phase at  $\beta = 2.6$  in Fig. 7 (left panel), together with a plot of the gauge-Higgs coupling susceptibility (right panel). In this case the mobility edge never vanishes, but the form of its functional dependence on



**Figure 7:**  $\kappa$  dependence of the mobility edge at fixed  $\beta = 2.6$  (left), and gauge-Higgs coupling susceptibility (right). In the left panel we also show the result of a fit with Eq. (10) with a solid blue line, and the position and uncertainty of the critical value  $\kappa_c(\beta = 2.6)$ , where the functional form of the  $\kappa$  dependence of the mobility edge changes, with vertical solid and dashed black lines, respectively. In the right panel we also show the position of the critical point ( $\beta = 2.6, \kappa_c(\beta = 2.6)$ ).

$\kappa$  clearly changes from almost constant to almost linear across the transition. Fitting the data with the function

$$\lambda_c(\kappa) = a [1 - \sigma(d \cdot (\kappa - \kappa_c))] + (b\kappa + c)\sigma(d \cdot (\kappa - \kappa_c)), \quad \sigma(x) = (1 + e^{-x})^{-1}, \quad (10)$$

we find that the “critical”  $\kappa_c$  is again in the crossover region.

## 6. Conclusions

In this contribution, based on Ref. [30], we have once more confirmed the general expectations of the sea/islands picture of localization, extending the connection between localization and Polyakov-loop ordering in two directions. In fact, the fixed-length SU(2) Higgs model investigated here contains scalar rather than fermionic dynamical matter; and allows one to study a phase where the Polyakov loop is ordered that is qualitatively different from the usual deconfined phase. This further confirms that localization of low Dirac modes is intimately related to the ordering of the Polyakov loop, without particular regard for the matter content or other details of the model.

Our study could be extended in two interesting directions. One is to study the localization properties of the eigenmodes of the covariant Laplacian at finite temperature, extending the zero-temperature results of Refs. [38, 39], looking in particular at how these properties change across the various transitions. Another direction is a detailed study of the low  $\beta$ , large  $\kappa$  region, where at zero temperature the transition line obtained with a gauge-invariant spin-glass-inspired order parameter disagrees with that obtained in a gauge-fixed setting [40], and a study of localization could shed further light on this discrepancy.



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