PROCEEDINGS OF SCIENCE

Doubly Charmed H-like dibaryon $\Lambda_c\Lambda_c$ scattering from Lattice QCD

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To explore the properties of H-like dibaryon $\Lambda_c\Lambda_c(0^+)$, we proceed ab-initio calculation on lattice. Two Wilson-Clover ensembles are used with the same setup at $m_\pi \approx 303$ MeV and lattice spacing $a \approx 0.07746$ fm. We find the coupling between $\Lambda_c\Lambda_c$ and $\Xi_{cc}N$ is quite small. Considering the discretization effect from lattice, a modification on Lüscher equation is firstly proposed in this work. Scattering length a_0 given by Effective Range Expansion method(ERE) equals to $-1.43(49)$ fm. Under this quark mass, we find no bound state relying on our calculation.

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1. Introduction

Simulation of dibaryons system on lattice has been a hot topic in Hadron Physics [1–7], such as NN, $\Lambda\Lambda$ which is also known as H dibaryon suggested by Jaffe in Ref[8]. However, until now, the Deuteron bound state have not been confirmed in the lattice calculation, thus, it is necessary to make further effort on this challenging job.

According to kinetic analysis, in the heavy quark sector, baryons may be easier to form a binding structure. On the other hand, there is no data about doubly charmed H-like dibaryon $\Lambda_c\Lambda_c$ from lattice calculation yet. These motivation inspire us to perform a lattice calculation for $\Lambda_c\Lambda_c$.

In this work, we make a lattice calculation for the $\Lambda_c\Lambda_c$ system, $I(J^P) = 0(0^+)$, at the A_1^+ irreducible representation(irrep). Lüshcer finite-volume method [9] is adopted for scattering analyses and a modification caused by discretization effect is firstly proposed in this work. Two Wilson-Clover lattice ensembles are used with the same mass of π and lattice spacing but different volume. The detail information for these two ensembles are present in following Table.1.

ensemble	$(L/a)^3 \times T/a$	β	a(fm)	m_π(MeV)	m_K(MeV)	$m_\pi L$	N
F32P30	$32^3 \times 96$	6.41	0.07746(18)	303.2(1.3)	524.6(1.8)	3.81	567
F48P30	$48^3 \times 96$	6.41	0.07746(18)	303.4(0.9)	523.6(1.4)	5.72	201

Table 1: Ensemble details[10]. These two ensembles share the exact same parameters except volume.

2. Formalism

2.1 Operators construction

Since momentum on lattice is discretized, three-dimension momentum becomes $\vec{k} = \frac{2\pi}{L}\vec{n}$, where $\vec{n}$ is a three-dimension integer. General baryon operators [1, 11] with Fourier Transform read as

$$B(x) = \sum_x P_+ \varepsilon_{abc} r_{a,x} \left[s^T_{b,x} (C\gamma_5) t_{c,x} \right] e^{-i\vec{k}\cdot x}, \tag{1}$$

where r, s, t are quark fields with certain flavors. P_+ is parity projection operator. Based on Dirac basis [12], only two spinor components of quark r is non-zero. The so-called di-quark st composed through the charge conjugation matrix C and γ_5 has total angle momentum J=0 and total isospin $I = 0$. In terms of Ref. [5], the two baryons operators can be written as:

$$\phi^\Lambda_{B_1B_2}(|\vec{k}|) = \sum_j c^\Lambda_j \; B^T_1(\vec{k}_j) C\gamma_5 B_2(-\vec{k}_j), \tag{2}$$

where c^Λ_j is the Clebsch-Gordan coefficients(CGs) satisfying the irrep Λ in cubic group O_h. $\pm\vec{k}$ are the back-to-back momentum of two baryons respectively.

2.2 Correlation function

The single baryon correlation function can be written as:

$$C_B(\vec{k}, t) = \sum_{\substack{t_{snk} \\ t_{src}}} \langle B(\vec{k}, t_{snk}) \; B(\vec{k}, t_{src})^\dagger \rangle \simeq A e^{-\omega t}, \tag{3}$$

where each $\Delta t = t_{snk} - t_{src} \equiv t$ traverses all the cases. ω is the energy of single baryon. During constructing these correlation functions, we perform the calculation under Distillation method [13, 14]. Then we turn to two-particle correlation matrix:

$$C_{ij}(t) = \langle\, \phi^{\Lambda}_{B_1B_2}(|\vec{k}_i|)\phi^{\Lambda}_{B_1B_2}(|\vec{k}_j|)^{\dagger}\,\rangle, \tag{4}$$

and diagonalize it via GEVP method,

$$C(t)\cdot v_{\alpha}(t,t_0) = \lambda_{\alpha}(t,t_0)C(t_0)\cdot v_{\alpha}(t,t_0), \tag{5}$$

where $t > t_0$. The eigenvalues $\lambda_{\alpha}(t,t_0)$ of correlation matrix have the form as $e^{-E_{\alpha}(t-t_0)}$, where E_{α} is the two-particle system energies. During fitting the effective mass, we choose the fitting range where the fitting energy level becomes stable and could provides a reasonable $\chi^2/dof \simeq 1$. Usually, E_{α} of dibaryon are all very close to and even crossed with their corresponding free energies. Therefore, the ratio fitting method is needed,

$$C^{\alpha}_{R}(t) = \frac{C_{ij}(t)}{C_{thr.}(t)} \simeq A_{\alpha}e^{-\Delta E_{\alpha}t}, \tag{6}$$

where $C_{thr.} = C^2_{\Lambda_c}(\vec{k} = 0)$, and ΔE_{α} is the difference between E_{α} and the $\Lambda_c\Lambda_c$ threshold. Parameter A_{α} plays a role of stabilizing the fitting goodness.

2.3 Overlap

To visualize the overlap linear between each operators, a cross-correlation matrix $\tilde{C}(t)$ [15] at a particular time slice t is defined as:

$$\tilde{C}_{ij}(t) = C_{ij}(t)/\sqrt{|C_{ii}(t)C_{jj}(t)|}, \tag{7}$$

then, the diagonal matrix elements could be normalized to unity. The smaller the value of off-diagonal elements $\tilde{C}_{ij}(t)$, the smaller overlap between the two operator ϕ_i and ϕ_j. Another choice called 'Overlap Factor' [16] can also give the overlap:

$$Z^{\alpha}_{i} \equiv \langle\alpha|\phi_i|0\rangle = \sqrt{2E_{\alpha}}e^{E_{\alpha}t_0/2}v^{\alpha *}_{j}C_{ji}(t_0), \tag{8}$$

where E_{α} and v^{α} are solved from GEVP method.

2.4 Discretization effect

We can define a scattering-dependence momentum q^2 through

$$E^*(q) = \omega_1(q) + \omega_2(q) = \sqrt{m_1^2 + Z_1q^2} + \sqrt{m_2^2 + Z_2q^2}, \tag{9}$$

where $Z_{1,2}$ is calculated from two single-particle dispersion relation instead of being set to one. In most cases, the slope $Z_{1,2}$ may not be equal to 1 perfectly due todiscretization from lattice. Therefore, a modification on the Lüscher equation is needed. Based on the equation (2.38)~(2.40) in Ref [17], consider a transfer of the limit:

$$\begin{aligned}
&\lim_{k^*\to q}\frac{1}{4\omega_1(k^*)\omega_2(k^*)}\,\frac{q-k^*}{E^*(q)-\omega_1(k^*)-\omega_2(k^*)} = \lim_{k^*\to q}\frac{1}{4q[Z_1\omega_2(q)+Z_2\omega_1(q)]}\\
\Rightarrow &\frac{1}{4\omega_1(k^*)\omega_2(k^*)}\,\frac{1}{E^*(q)-\omega_1(k^*)-\omega_2(k^*)} = \frac{1}{2[Z_1\omega_2(q)+Z_2\omega_1(q)]}\,\frac{1}{q^2-k^{*2}}.
\end{aligned} \tag{10}$$

Finally, S wave zeta function [18] is modified as

$$q cot\delta_0(q) = \frac{2}{\sqrt{\pi}L}\mathcal{Z}_{00}(1;\tilde{q}^2) \times \frac{\omega_1(q) + \omega_2(q)}{Z_1\omega_2(q) + Z_2\omega_1(q)}, \tag{11}$$

Comparing to the original Lüscher equation, there is an additional factor, $\frac{\omega_1+\omega_2}{Z_1\omega_2+Z_2\omega_1}$, which indeed reflects the effect of $Z_{1,2} \neq 1$. Here, $\tilde{q} = qL/2\pi$.

3. Result and discussion

In this work, single baryon operators $\Lambda_c\ 0(\frac{1}{2}^+)$, $\Xi_{cc}\ \frac{1}{2}(\frac{1}{2}^+)$, $N\ \frac{1}{2}(\frac{1}{2}^+)$ and $\Sigma_c\ 1(\frac{1}{2})^+$ are contained to calculate related threshold. For two baryons operators, only spin-zero case is taken into account. For total isospin $I = 0$ and positive parity of $\Lambda_c\Lambda_c$, S wave operators in A_1^+ irrep read as

$$\begin{aligned} \Lambda_c\Lambda_c^{I=0} &= \Lambda_c^+\Lambda_c^+ \\ \Xi_{cc}N^{I=0} &= \frac{1}{2}\left(p\Xi_{cc}^+ + \Xi_{cc}^+ p - n\Xi_{cc}^{++} - \Xi_{cc}^{++} n\right). \end{aligned} \tag{12}$$

The anti-operators could be given by changing the quarks with anti-quarks. The number of Laplacian eigenvectors $N_{ev} = 100$ is used in F32P30 and $N_{ev} = 200$ in F48P30.

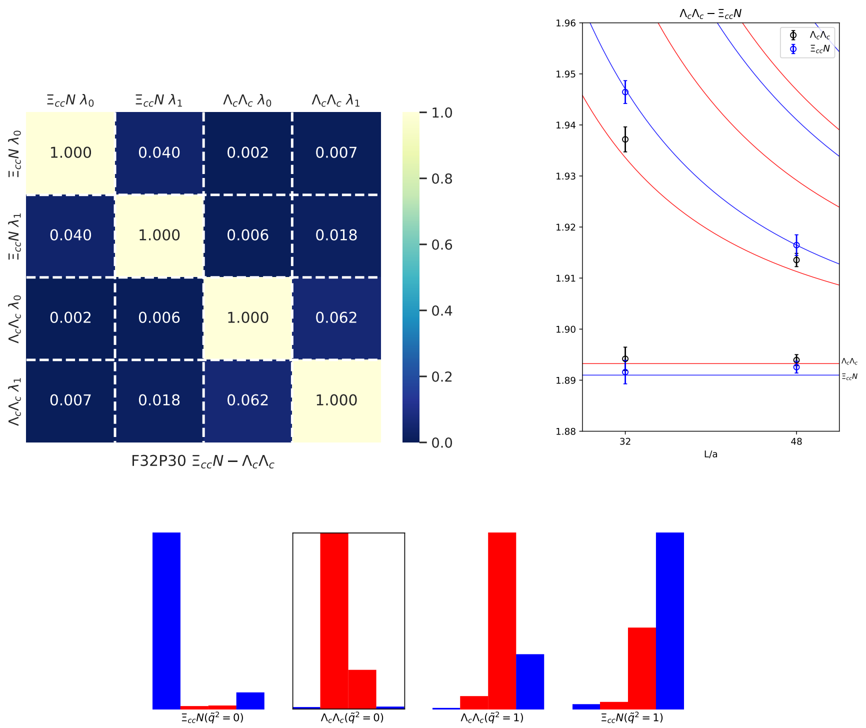

Figure 1: Left-upper panel: Cross-correlation matrix. Lower panel: Overlap factor. The one within box represents that the operator component of $\Lambda_c\Lambda_c$ ground state energy is dominated by the $\Lambda_c\Lambda_c\ \lambda_0$. Right-upper panel: $\Xi_{cc}N - \Lambda_c\Lambda_c$ couple channel spectrum.

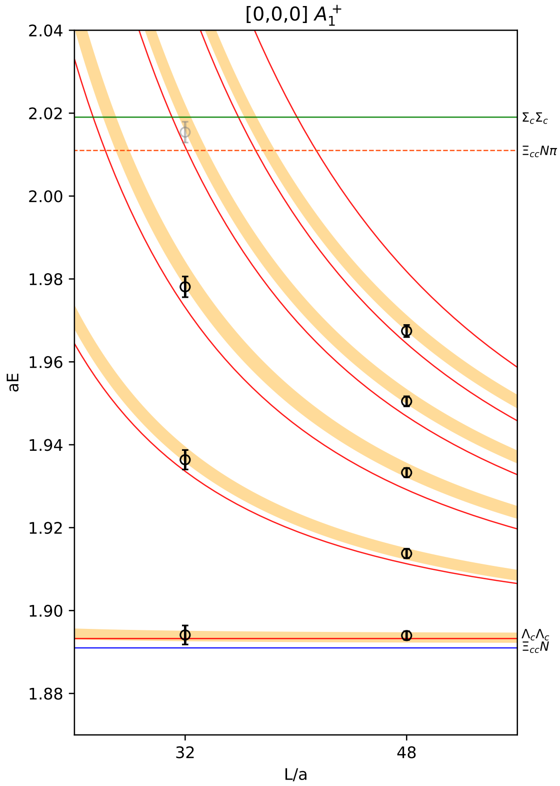

Figure 2: Single channel $\Lambda_c\Lambda_c$ spectrum in rest frame. Total eight energy levels plotted with black are contained in scattering analyses while the highest energy level has exceeded the inelastic scattering threshold. The orange band is the solutions of Lüscher equation

The interaction between $\Lambda_c\Lambda_c$ is repulsive. According to our couple channel analyses, the coupling with $\Xi_{cc}N$ is weak. Also, ground-state energy fitting result of $\Lambda_c\Lambda_c$ won't be lower than the threshold and it still shows a repulsive interaction. To simplify the finite-volume scattering analysis, only single channel $\Lambda_c\Lambda_c$ is contained. Total eight data points plotted in Fig .2 are contained in the improved Lüscher equation Eq. (11). The two ensembles with same lattice set must be consistent, and a global fitting is performed with ERE parameterization. We should notice that ERE method will not describe the energy levels well when the q^2 is far away from the threshold. Refs. [19, 20] give a left-hand-cut below the threshold from One-Boson-Exchange, i.e.

$$k_{lhc}^2 \simeq \frac{1}{4}\left[(\Delta M)^2 - m^2\right], \tag{13}$$

where ΔM is the difference of two particles and m is the mass of exchanging particle. $\Lambda_c\Lambda_c$ case could exchange ω boson instead of pion [21]. Therefore, $k^2 = -\frac{m_\omega^2}{4}$ on the left hand (also called t-cut [5]) is taken in the fitting. In Fig. 3, we plot the fitting band of $kcot\delta_0 = \frac{1}{a_0}$. The scattering length is $-0.143(49)$ fm, with $\chi^2/dof = 0.86$. The χ^2 here is redefined [15, 22] with a given set of parameters $\{p_i\}$,

$$\chi^2(\{p_i\}) = \sum_L [E_{latt.}(L) - E_{sol.}(L;\{p_i\})]_m \; C(L)^{-1}_{mn} \, [E_{latt.}(L) - E_{sol.}(L;\{p_i\})]_n \,, \tag{14}$$

where $E_{latt.}$ is the fitting energy in the center-mass frame, while $E_{sol.}$ is the solution of Lüscher equation when parameters are given. C is the covariance matrix.

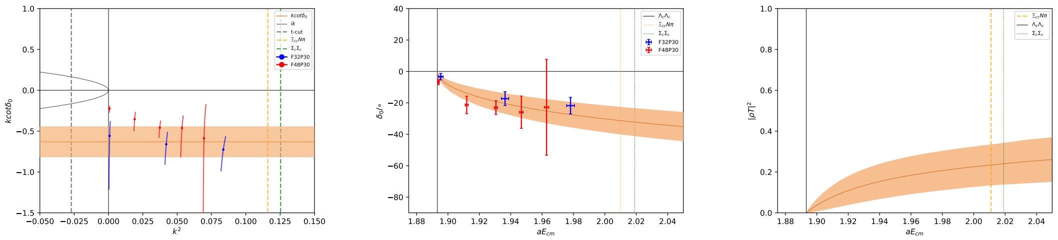

Figure 3: Phase shifts in rest frame. Red points are solved form the configuration F48P30, while blue ones are from F32P30. Here, t-channel cut($k^2 = -m_\omega^2/4$) is the valid boundary of ERE.

Continue the discussion from a different perspective on phase shift δ_0, which can be translated directly with Eq. (11). As we can see in the middle panel of Fig. 3, the fitting band of δ_0 shows a repulsive interaction and it can cover our lattice data set well. In the right panel of Fig. 3, the $|\rho T|^2$ is plotted.

4. Summary

The first calculation on $\Lambda_c\Lambda_c$ system is performed in this work. According to couple channel analyses, the coupling between $\Lambda_c\Lambda_c$ and $\Xi_{cc}N$ is weak, and Single channel S wave $\Lambda_c\Lambda_c$ is contained. Finite-volume energy levels of $\Lambda_c\Lambda_c$ are all above their corresponding free energy, which shows a repulsive interaction. Because of the discretization effect, a modification on Lüscher equation is proposed firstly in this work. The scattering phase shift translated from the Lüscher equation with ERE parameterization is below zero. Also, scattering length can be given by our fitting as $a_0 = -0.143(49)$ fm. These results all indicate that $\Lambda_c\Lambda_c$ system is only a common scattering state, i.e. there doesn't exist a bound state under this quark mass.

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