

## Higgs Self-Coupling and Yukawa Corrections to Higgs Boson Pair Production

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We present a calculation of the Higgs Self-Coupling and Yukawa Corrections, contributing to the electroweak NLO corrections to Higgs boson pair production. The two-loop integrals are calculated numerically with `PYSECDEC` and we show how the choice of master integrals influences the evaluation time of the amplitude. The renormalization of our model, which is obtained from the SM by including only the top quark, Higgs boson and gluon fields, is discussed.

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## 1. Introduction

The production of Higgs boson pairs is the prime process to constrain the trilinear Higgs coupling  $\lambda_{HHH}$  and it will allow to shrink the current limits [1, 2] obtained by the ATLAS and CMS collaboration to  $0.1 < \lambda_{HHH}/\lambda_{HHH}^{SM} < 2.3$  at the high-luminosity LHC [3].

Reaching these limits also requires accurate theoretical predictions and therefore higher-order corrections to the process  $gg \rightarrow HH$ , which has first been calculated at Leading Order (LO) in Refs. [4, 5], need to be included. The Next-to-Leading Order (NLO) QCD corrections including the full top-quark mass dependence have been obtained in Refs. [6–10] and matched to parton showers [11–14]. QCD corrections beyond NLO have been calculated in the heavy-top-limit [15–17], or in a combination of large- $m_t$  and high-energy expansions [18], and combined with the top-mass effects at NLO in Refs. [19–22]. Reducing the large top-mass renormalisation scheme uncertainty [14, 23] of about 20% at NLO, requires the calculation of the top-mass effects at NNLO. First contributions to these three-loop contributions have been calculated recently [24–26].

In addition to higher-order QCD predictions, NLO electroweak (EW) predictions are required for accurate predictions. Various groups have calculated parts of the NLO EW corrections [27–34]. The full NLO EW corrections have been presented in Ref. [35], and in Ref. [36] using a large top-quark mass expansion up to  $1/m_t^8$ .

In this contribution, the calculation of the EW contributions resulting from Higgs Self-Coupling and Yukawa corrections, presented in Ref. [32], is summarized.

## 2. Model and Renormalisation

To calculate the Higgs Self-Coupling and Yukawa corrections to the process  $gg \rightarrow HH$  at NLO, we use the gaugeless limit  $(g, g') \rightarrow (0, 0)$  of the Standard Model and we only consider contributions of the Higgs field, top-quark and gluons. We therefore start from the bare Lagrangian

$$\begin{aligned} \mathcal{L}_0 = & -\frac{1}{4}\mathcal{G}_{0,\mu\nu}\mathcal{G}_0^{\mu\nu} + (D_\mu\Phi_0)^\dagger(D^\mu\Phi_0) + \mu^2\Phi_0^\dagger\Phi_0 + \frac{\lambda_0}{4}(\Phi_0^\dagger\Phi_0)^2 \\ & + i\bar{Q}_{L,0}\not{D}Q_{L,0} + i\bar{t}_{R,0}\not{D}t_{R,0} - (y_{t,0}\bar{Q}_{L,0}\Phi_0^c t_{R,0} + \text{h.c.}), \end{aligned} \quad (1)$$

with

$$Q_{L,0} = \begin{pmatrix} t_{L,0} \\ 0 \end{pmatrix}, \quad (2)$$

After symmetry breaking and using unitary gauge, this leads to

$$\begin{aligned} = & -\frac{1}{4}\mathcal{G}_{0,\mu\nu}\mathcal{G}_0^{\mu\nu} + \frac{1}{2}(\partial_\mu H_0)^\dagger(\partial^\mu H_0) - \frac{m_{H,0}^2}{2}H_0^2 - \frac{m_{H,0}^2}{2v_0}H_0^3 - \frac{m_{H,0}^2}{8v_0^2}H_0^4 \\ & + i\bar{t}_0\not{D}t_0 - m_{t,0}\bar{t}_0t_0 - \frac{m_{t,0}}{v_0}H_0\bar{t}_0t_0 + \text{constant} \end{aligned} \quad (3)$$

with

$$m_{H,0}^2 = 2\mu_0^2, \quad m_{t,0} = \frac{y_{t,0}v_0}{\sqrt{2}} \quad \text{and} \quad v_0^2 = -\frac{2m_{H,0}^2}{\lambda_0}. \quad (4)$$

The bare fields and masses of the top quark and Higgs boson can be related to renormalized quantities via

$$H_0 = \sqrt{Z_H} H = \sqrt{1 + \delta_H} H, \quad t_0 = \sqrt{Z_t} t = \sqrt{1 + \delta_t} t, \quad (5)$$

$$m_{H,0}^2 = m_H^2 (1 + \delta m_H^2), \quad m_{t,0} = m_t (1 + \delta m_t) \quad (6)$$

and we fix the renormalization constants using the on-shell scheme. The inverse of  $v_0$  can be interpreted as the coupling constant of our model. It can be renormalized via

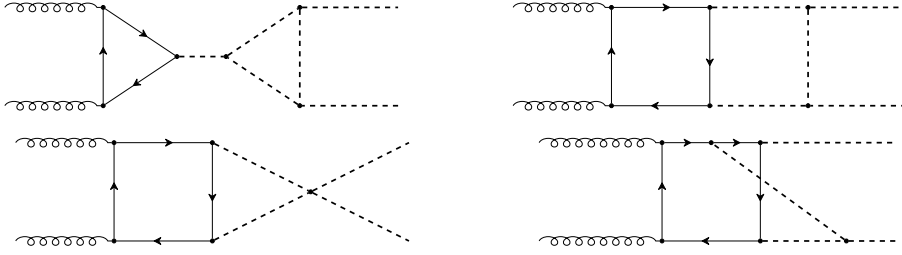
$$v_0 + \Delta v = v(1 + \delta_v) + \Delta v, \quad (7)$$

where we also absorb a shift  $\Delta v$  due to tadpole contributions using the Fleischer-Jegerlehner tadpole scheme [37]. To facilitate comparison of our results with other calculations, we fix  $\delta_v$  according to the  $G_\mu$  scheme [38]. Alternatively, it could be fixed by requiring finiteness of the renormalized vertices, leading to

$$\delta_v|_{\text{UV}} = -\frac{3m_H^4 + 2m_H^2 m_t^2 N_c - 8m_t^4 N_c}{32\pi^2 m_H^2 v^2 \epsilon} \quad (8)$$

in all cases. This, however, would not uniquely fix the finite terms of  $\delta_v$ .

### 3. Calculation



**Figure 1:** Representative two-loop diagrams contributing to the Higgs Self-Coupling and Yukawa Corrections considered here.

To calculate the amplitude, we first generate the Feynman diagrams using QGRAF [39]. Some example diagrams are shown in Fig. 1. We generate a form factor decomposition of the amplitude using ALIBRARY [40] and checked the resulting expressions with an independent calculation using REDUZE 2 [41] and find full agreement.

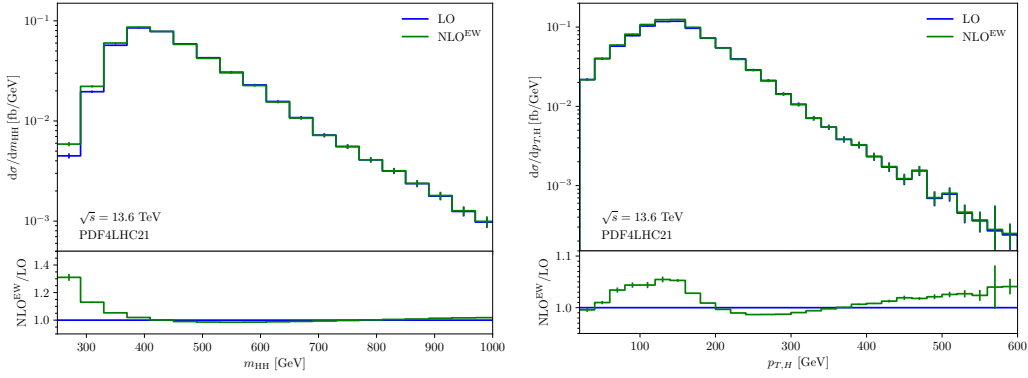
We then reduce the loop integrals to a basis of master integrals using RATRACER [42] and FIREFLY [43, 44] with integration-by-parts identities [45] generated by KIRA [46, 47]. Testing different choices for the master integral basis, we find a basis a finite integrals [48], where the dependence on the space-time dimension  $D$  factorizes from the kinematic dependence in the denominators of the reduction [49, 50], and furthermore, the top-level (7-propagator) integrals, as well as most of the 6-propagator integrals, don't contribute to the poles of the amplitude.

The master integrals are then evaluated numerically using PYSECDEC [51–54] to obtain the amplitude. For the bulk of phase-space points, we find that evaluating the loop integrals takes

approximately five minutes on four Nvidia A100 GPUs to reach our target relative precision of  $10^{-3}$  on the amplitude. It is worth mentioning that with an earlier, non-optimal choice of master integral basis, the integration time was typically  $\mathcal{O}(100\text{h})$  and hence finding a good basis is crucial to obtain a fast evaluation of the amplitude.

No real radiation contributions have to be taken into account, since in our approximation, these correction would involve the radiation of an additional Higgs boson, which can be experimentally distinguished from the  $HH$  final state.

## 4. Results



**Figure 2:** Invariant mass and transverse momentum distributions for Higgs boson pair production at LO and  $\text{NLO}^{\text{EW}}$  including only the Yukawa and self-coupling type corrections. The QCD renormalisation and factorisation scales are set to  $\mu_r = \mu_f = m_{HH}/2$ .

To obtain the total and differential cross section, we reweight  $\sim 7000$  unweighted LO events with the  $\text{NLO}^{\text{EW}}$  contribution obtained from the virtual two-loop amplitude. For the LHC at a center-of-mass energy of  $\sqrt{s} = 13.6$  TeV we find that the  $\text{NLO}^{\text{EW}}$  corrections increase the cross total section by 1%. Differential results for the  $m_{HH}$  and  $p_{T,H}$  distribution are shown in Fig. 2. For the  $m_{HH}$  distribution, we find corrections of up to 15% close to the production threshold, whereas for  $m_{HH} > 400$  GeV the corrections decrease to the  $\pm 1\%$  level. In the  $p_{T,H}$  spectrum, we find a strong phase-space dependence with corrections of up to 5%.

## 5. Conclusion

We have presented a calculation of the subset of EW corrections to Higgs boson pair production stemming from additional Yukawa or Higgs self-interactions. The integrals appearing in the two-loop amplitude have been calculated numerically with `pySECDEC`, after selecting a basis of finite master integrals, which avoids poles in the coefficients of the top-level integrals. We find that the  $\text{NLO}^{\text{EW}}$  corrections increase the total cross section by 1%, and on the differential level, we find up to 15% corrections close to the production threshold.

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