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Non-factorizable corrections to Higgs production in Vector Boson Fusion

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In this report, we present a summary of two recent calculations of the non-factorizable corrections to Higgs production via Vector Boson Fusion at next-to-next-to-leading order in QCD. Our findings include the first power correction to the leading-eikonal approximation. This new sub-leading contribution alters the current estimate of non-factorizable corrections by approximately 20 percent. Additionally, we demonstrate that the significant renormalization scale dependence can be mitigated by incorporating the three-loop fermion-bubble corrections, which account for the effects of the running coupling constant.

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1. Introduction

At the LHC, the Higgs boson can be produced through various channels. The dominant production mechanism is gluon-gluon fusion, followed by Vector Boson Fusion (VBF), which has the second-largest cross section. The current measurement accuracy of the VBF cross section is about 20%, and this precision is expected to improve in future LHC runs and during the high-luminosity phase. This necessitates precise theoretical understanding.

According to whether there are talks between the incoming partons, VBF can be categorized into two kinds of contributions, the factorizable and non-factorizable ones. For the former case, very impressively, QCD corrections are available through N3LO [1–7]. Non-factorizable contributions, including the virtual and real corrections, were previously considered to be color suppressed and thus negligible. However, recent studies have shown that these contributions are enhanced by a factor of π^2, which compensates for the color suppression. This enhancement was observed in the leading-eikonal approximation of virtual non-factorizable corrections to Higgs production in VBF [8]. To better understand and validate the leading-eikonal approximation, further investigation beyond this approximation is necessary [9, 10]. The real corrections were found to be too tiny to cause significant impacts on the phenomenology of Higgs production in VBF [11].

The exact calculation of the two-loop five-point VBF amplitudes involves seven distinct scales, presenting a significant challenge with current techniques. Therefore, we have focused on the subleading effects of the eikonal expansion [9]. Non-factorizable corrections to VBF appear at NNLO for the first time, and their renormalization scale dependence is notably strong. While a complete N3LO calculation of non-factorizable contributions would best reduce this uncertainty, it is currently too complex to perform. As an alternative, we demonstrate that including three-loop fermion-bubble corrections, which account for the running coupling effects, can stabilize theoretical predictions for non-factorizable contributions to the cross section and distributions [10].

2. Kinematics

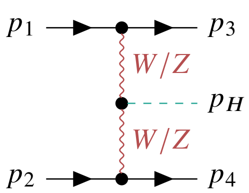


Figure 1: Tree diagram of VBF.

We start with the analysis of the kinematics of VBF and use their features to set up an expansion of the amplitudes around the forward limit. Consider the VBF process (see Figure 1),

$$q(p_1) + q'(p_2) \to Q(p_3) + Q'(p_4) + H(p_H). \tag{1}$$

The upper and lower quark currents are connected by the boson line. The momentum transfers from quark currents to bosons are defined as

$$q_1 = p_1 - p_3, \quad q_2 = p_2 - p_4. \tag{2}$$

We perform the Sudakov decomposition of the outgoing jets,

$$p_i = \alpha_i p_1 + \beta_i p_2 + p_{i,\perp}, \quad i = 3, 4. \tag{3}$$

It follows that

$$\begin{aligned} q_1 &= p_1 - p_3 = \delta_3 p_1 - \beta_3 p_2 - p_{3,\perp}\,, \\ q_2 &= p_2 - p_4 = -\alpha_4 p_1 + \delta_4 p_2 - p_{4,\perp}\,, \end{aligned} \tag{4}$$

where we defined $\delta_3 = 1 - \alpha_3$ and $\delta_4 = 1 - \beta_4$. It is not difficult to find that

$$\delta_3\delta_4 s = m_H^2 + \frac{\mathbf{p}_{3,\perp}^2}{\alpha_3} + \frac{\mathbf{p}_{4,\perp}^2}{\beta_4} + 2\mathbf{p}_{3,\perp}\cdot\mathbf{p}_{4,\perp} - \frac{\mathbf{p}_{3,\perp}^2\mathbf{p}_{4,\perp}^2}{\alpha_3\beta_4 s}\,, \tag{5}$$

where we used

$$\beta_3 = \frac{\mathbf{p}_{3,\perp}^2}{s\alpha_3}, \quad \alpha_4 = \frac{\mathbf{p}_{4,\perp}^2}{s\beta_4}, \quad \text{with } s = (p_1 + p_2)^2. \tag{6}$$

The selection of VBF events requires that the two outgoing jets carry relatively small transverse momenta, forwardly flying into opposite hemispheres. That means α_3 and β_4 are close to one and $|\mathbf{p}_{3,4}|$ are much smaller than the partonic collision energy. That brings us to an important conclusion

$$\delta_3\delta_4 \sim \frac{m_V^2}{s} \sim \frac{m_H^2}{s} \sim \frac{\mathbf{p}_{3,\perp}^2}{s} \sim \frac{\mathbf{p}_{4,\perp}^2}{s} \sim \lambda \ll 1\,, \tag{7}$$

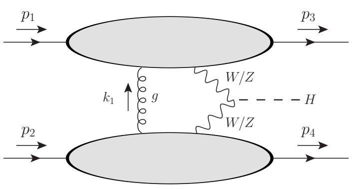
where λ is used to indicate the smallness of those ratios. Moreover, δ_3 and δ_4 should behave similarly, namely,

$$\delta_3 \sim \delta_4 \sim \sqrt{\lambda} \gg \lambda\,. \tag{8}$$

We will see in the following section that the expansion of VBF amplitudes proceeds in a series of $\delta_{3,4}$.

3. Subleading-eikonal corrections

In this section, we describe how to expand the one- and two-loop VBF amplitudes around the forward limit and obtain the first power correction. To demonstrate the idea, we start with the one-loop amplitude. The one-loop non-factorizable VBF amplitude can be constructed as[1]



$$\mathcal{M}_1 = g_s^2 g_w^2 g_{VVH} T^a_{i_3 i_1} T^a_{i_4 i_2} \mathcal{A}_1, \tag{9}$$

where T^a_{ij} are the generators of the $SU(3)$ color group and the color-stripped amplitude $\mathcal{A}_1$ reads

$$\mathcal{A}_1 = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d}\frac{1}{d_1 d_3 d_4} J^{\mu\nu}\tilde{J}_{\mu\nu}, \tag{10}$$

[1]The couplings of the vector boson V to the Higgs boson and V to quarks are assumed to be $ig_{VVH}\ g_{\mu\nu}$ and $-ig_W\gamma^\mu$, respectively. Then the contributions for $V = Z$ and $V = W$ can be reconstructed easily.

	α_1	β_1	$\mathbf{k}_{1,\perp}$	$\mathcal{M}_1$
G	λ	λ	$\sqrt{\lambda}$	λ^{-2}
G-S	$\lambda(\sqrt{\lambda})$	$\sqrt{\lambda}(\lambda)$	$\sqrt{\lambda}$	λ^{-2}
S	$\sqrt{\lambda}$	$\sqrt{\lambda}$	$\sqrt{\lambda}$	λ^{-2}
C	$1(\lambda)$	$\lambda(1)$	$\sqrt{\lambda}$	$\lambda^{-3/2}$
H	1	1	1	λ^0

Table 1: Loop-momentum regions in the forward limit. They are Glauber, Glauber-soft(mixed), soft, colinear and hard regions. The last column is the estimation of the scaling of amplitude $\mathcal{M}_1$ in different regions.

with

$$d_1 = k_1^2 + i0, \quad d_3 = (k_1 + q_1)^2 - m_V^2 + i0, \quad d_4 = (k_1 - q_2)^2 - m_V^2 + i0\,, \tag{11}$$

being the boson propagators and the two quark currents read

$$\begin{aligned} J^{\mu\nu}(k_1, -k_1 - q_1) &= \langle 3| \left[\frac{\gamma^\nu(\hat{p}_1 + \hat{k}_1)\gamma^\mu}{\rho_1(k_1)} + \frac{\gamma^\mu(\hat{p}_3 - \hat{k}_1)\gamma^\nu}{\rho_3(-k_1)} \right] |1]\,, \\ \tilde{J}^{\mu\nu}(-k_1, k_1 - q_2) &= \langle 4| \left[\frac{\gamma^\nu(\hat{p}_2 - \hat{k}_1)\gamma^\mu}{\rho_2(-k_1)} + \frac{\gamma^\mu(\hat{p}_4 + \hat{k}_1)\gamma^\nu}{\rho_4(k_1)} \right] |2]\,, \end{aligned} \qquad \rho_i(k) = (p_i + k)^2 + i0\,. \tag{12}$$

We want to expand the amplitude $\mathcal{A}_1$ around the forward limit. To do it we use the method of regions [12–14]. It provides a way to compute the asymptotic expansion of Feynman integrals under some limits. One starts with the Sudakov parametrization of the loop momentum k_1,

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}\,. \tag{13}$$

Then the loop integration measure becomes

$$\frac{\mathrm{d}^d k_1}{(2\pi)^d} = -\frac{s}{2}\frac{\mathrm{d}\alpha_1}{2\pi i}\frac{\mathrm{d}\beta_1}{2\pi i}\frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{d-2}}\,, \qquad d = 4 - 2\epsilon. \tag{14}$$

It is straightforward to integrate over α_1 or β_1 by taking residues, say α_1. One can then analyze the singularities of the updated integrand which is a product of at most quadratic polynomials of β_1. This analysis will reveal the contributing regions[2] listed in Table 1. Considering that the leading order VBF amplitude $\mathcal{M}_0$ scales as λ^{-2}, to compute the first power correction to the one-loop amplitude, we will need the first three regions to subleading order and colinear to the leading order. The hard region is not relevant. However this naive estimation is not correct. It turns out that, except for the Glauber region, other regions all get a suppression of some negative powers of $\sqrt{\lambda}$. Finally, we only need the Glauber region to subleading order and the mixed region to leading order. The total contribution from these two regions reads

$$\mathcal{A}_1^{\text{G\&G-S}} = -\langle 3|\gamma^\mu|1]\langle 4|\gamma_\mu|2] \int \frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{d-2}} \frac{1}{\Delta_1\Delta_{3,1}\Delta_{4,1}} \times \Phi \times \tilde{\Phi}\,, \tag{15}$$

[2] In this process, $k_{1,\perp}$ is always assumed to be either the order of $p_{3/4,\perp}$ or $\sqrt{s}$.

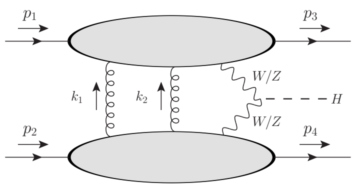
with

$$\begin{aligned}\Phi &= \int \frac{\mathrm{d}\beta_1}{2\pi i} \frac{\Delta_{3,1}}{s\delta_3(\beta_1-\beta_3)+\Delta_{3,1}+i0}\left[\frac{1}{\beta_1+\frac{\Delta_1}{s}+i0}+\frac{1}{-\beta_1+\frac{\Theta_{3,1}}{s\alpha_3}+i0}\right], \\ \tilde{\Phi} &= \int \frac{\mathrm{d}\alpha_1}{2\pi i} \frac{\Delta_{4,1}}{-s\delta_4(\alpha_1+\alpha_4)+\Delta_{4,1}+i0}\left[\frac{1}{-\alpha_1+\frac{\Delta_1}{s}+i0}+\frac{1}{\alpha_1+\frac{\Theta_{4,1}}{s\beta_4}+i0}\right],\end{aligned} \tag{16}$$

and

$$\begin{aligned}&\Delta_i = -\mathbf{k}_{i,\perp}^2\,, \quad \Delta_{3,i} = -(\mathbf{k}_{i,\perp}-\mathbf{p}_{3,\perp})^2-m_V^2\,, \quad \Delta_{4,i} = -(\mathbf{k}_{i,\perp}+\mathbf{p}_{4,\perp})^2-m_V^2\,, \\ &\Theta_{3,i} = -\left(\mathbf{k}_{i,\perp}^2-2\mathbf{k}_{i,\perp}\cdot\mathbf{p}_{3,\perp}\right)\,, \quad \Theta_{4,i} = -\left(\mathbf{k}_{i,\perp}^2+2\mathbf{k}_{i,\perp}\cdot\mathbf{p}_{4,\perp}\right)\,.\end{aligned} \tag{17}$$

It is worthwhile to notice that the integration over the different components of k_1 is factorized. We can easily finish the integration of α_1, β_1 and expand $\Phi, \tilde{\Phi}$ to the first power of $\sqrt{\lambda}$. Before we present the final result, let us briefly discuss the two-loop calculation. It is similar to the one-loop case. The amplitude is constructed as



$$\mathcal{M}_2 = -ig_s^4 g_w^2 g_{VVH}\left(\frac{1}{2}\{T^a,T^b\}\right)_{i_3i_1}\left(\frac{1}{2}\{T^a,T^b\}\right)_{i_4i_2}\mathcal{A}_2, \tag{18}$$

with

$$\mathcal{A}_2 = \frac{1}{2!}\int \frac{\mathrm{d}^d k_1}{(2\pi)^d}\frac{\mathrm{d}^d k_2}{(2\pi)^d}\frac{1}{d_1d_2d_3d_4}J_{\mu\nu\rho}\tilde{J}^{\mu\nu\rho}\,. \tag{19}$$

The quark currents now have more complex structures. We want to expand amplitude $\mathcal{A}_2$ in the forward limit. The complication comes from combinatorics of many components of two loop-momenta k_1 and k_2. Even though not straightforward, it turns out that many regions would not contribute. We only need the Glauber-Glauber and mixed regions. More importantly, the factorization of the integration of different components of loop momenta still holds true. That brings big simplifications for the calculation. To the end, we can write the expanded one- and two-loop VBF amplitudes in a compact and nice form

$$\mathcal{M}_1 = i\frac{g_s^2}{4\pi}\,T^a_{i_3i_1}T^a_{i_4i_2}\,\mathcal{M}_0\,C_1, \quad \mathcal{M}_2 = -\frac{1}{2}\frac{g_s^4}{(4\pi)^2}\left(\frac{1}{2}\{T^a,T^b\}\right)_{i_3i_1}\left(\frac{1}{2}\{T^a,T^b\}\right)_{i_4i_2}\mathcal{M}_0\,C_2, \tag{20}$$

where the functions C_i read

$$\begin{aligned}C_1 &= 2\int\frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{1-2\epsilon}}\frac{(\mathbf{p}_{3,\perp}^2+m_V^2)(\mathbf{p}_{4,\perp}^2+m_V^2)}{\Delta_1\Delta_{3,1}\Delta_{4,1}}\times\Omega_1, \\ C_2 &= 4\int\frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{1-2\epsilon}}\frac{\mathrm{d}^{d-2}\mathbf{k}_{2,\perp}}{(2\pi)^{1-2\epsilon}}\frac{(\mathbf{p}_{3,\perp}^2+m_V^2)(\mathbf{p}_{4,\perp}^2+m_V^2)}{\Delta_1\Delta_2\Delta_{3,12}\Delta_{4,12}}\times\Omega_{12},\end{aligned} \tag{21}$$

with

$$\Omega_i = 1-\delta_3\left(\frac{m_V^2}{\mathbf{p}_{3,\perp}^2+m_V^2}+\frac{m_V^2}{\Delta_{3,i}}\right)-\delta_4\left(\frac{m_V^2}{\mathbf{p}_{4,\perp}^2+m_V^2}+\frac{m_V^2}{\Delta_{4,i}}\right). \tag{22}$$

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The relevant two-dimensional Feynman loop integrals are calculated analytically using the differential equation method [15].

Putting everything together, we can write the non-factorizable partonic differential cross section as

$$\mathrm{d}\hat{\sigma}_{\rm nf}^{\rm NNLO} = \frac{N_c^2 - 1}{4N_c^2}\, \alpha_S(\mu_R)^2\, C_{\rm nf}\, \mathrm{d}\hat{\sigma}^{\rm LO}, \qquad C_{\rm nf} = C_1^2 - C_2\,, \tag{23}$$

where the function $C_{\rm nf}$, which can be shown to be infra-red finite, fully characterizes the non-factorizable corrections.

After performing the phase space integration and convolution with parton-distribution-functions, we obtain the double-virtual non-factorizable contribution to VBF cross section. We set the factorization and renormalization scale dynamically as

$$\mu_F = \mu_R = \frac{m_H}{2}\left[1 + \frac{4\mathbf{p}_{H,\perp}^2}{m_H^2}\right]^{1/4}. \tag{24}$$

We use $m_W = 80.398$ GeV, $m_Z = 91.1876$ GeV and $m_H = 125$ GeV as the masses of the W, Z and Higgs boson. The Fermi constant is taken to be $G_F = 1.16637 \times 10^{-5}$ GeV^{-2}. For 13 TeV at LHC we have

$$\sigma_{VV} = (-3.1 + 0.53)\ \text{fb}. \tag{25}$$

We can see that the new subleading-eikonal correction changes the leading-eikonal one by about 20%. For kinematic distributions, the corrections can even reach a larger value in some kinematic regions. As examples, we present the transverse momentum and rapidity distributions of the leading jet in Figure 2. The first power correction varies from 10% to 50% for these two distributions. A similar observation happens to other kinematic distributions.

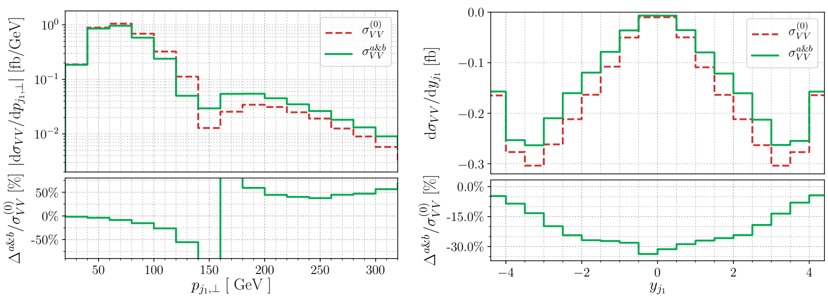

Figure 2: Leading- and subleading-eikonal contributions to the transverse momentum and rapidity distributions of the leading jet. In the upper pane, the leading-eikonal contribution is plotted with a red, dashed line and the subleading-eikonal one with a green, solid line. In the lower pane, we show the ratio of subleading-eikonal corrections $\Delta^{a\&b} = d\sigma_{VV} - d\sigma_{VV}^{(0)}$ to leading-eikonal contributions $d\sigma_{VV}^{(0)}$. The leading-eikonal contributions $d\sigma_{VV}^{(0)}$ cross zero around $p_{j_1,\perp} \sim 2m_W$, causing a rapid change there.

4. Three-loop Fermion-bubble corrections

One can observe from Eq. (23) that the dependence on renormalization scale μ_R completely stems from the strong coupling constant for the double-virtual non-factorizable corrections to Higgs production in VBF. That will bring a significant uncertainty around 20% to the theory prediction. This follows from the fact that non-factorizable corrections start to contribute at NNLO. To reduce such uncertainty, the best way is to extend the calculation to the next order in QCD. This is not possible currently. We instead consider a simpler part of the complete N3LO correction, with fermion bubble inserted into the gluon propagators, and re-write the contribution in the spirit of BLM approach [16] to account for the running coupling effects of α_S, see Figure 3.

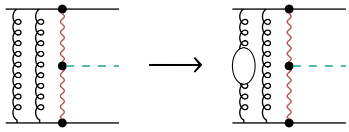

Figure 3: The insertion of a fermion bubble into one of the two gluons in the non-factorizable two-loop diagrams.

We only consider the leading-eikonal approximation. To include the effects of running α_S, replace $\Delta_{1,2}$ in $C_{1,2}$ with

$$\tilde{\Delta}_i = \Delta_i \left(1 + \frac{\beta_0 \alpha_S}{2\pi} \ln \frac{\mathbf{k}_{i,\perp}^2}{\mu_R^2 e^{5/3}}\right), \tag{26}$$

with β_0 the first-order QCD $\beta-$function. Then the function $C_{\rm nf}$ in Eq. (23) becomes

$$\widetilde{C}_{\rm nf} = 4 \int \frac{\mathrm{d}^2\mathbf{k}_{1,\perp}}{(2\pi)} \frac{\mathrm{d}^2\mathbf{k}_{2,\perp}}{(2\pi)} \frac{\Delta_3\Delta_4}{\tilde{\Delta}_1\tilde{\Delta}_2} \left(\frac{\Delta_3\Delta_4}{\Delta_{3,1}\Delta_{4,1}\Delta_{3,2}\Delta_{4,2}} - \frac{1}{\Delta_{3,12}\Delta_{4,12}}\right), \tag{27}$$

which can be shown to be infra-red finite. Expanding and keeping terms up to $\alpha_S\beta_0$ we obtain

$$\widetilde{C}_{\rm nf} = \widetilde{C}_{\rm nf}^{(0)} + \frac{\alpha_S\beta_0}{\pi}\left(\widetilde{C}_{\rm nf}^{(0)} \ln\left(\frac{\mu_R^2 e^{5/3}}{m_V^2}\right) + C_{\rm nf}^{(1)}\right) + O(\alpha_S^2\beta_0^2), \tag{28}$$

where

$$\widetilde{C}_{\rm nf}^{(0)} = \left(C_1^{(0)}\right)^2 - 2C_1^{(1)}, \qquad \widetilde{C}_{\rm nf}^{(1)} = C_1^{(0)}C_1^{(1)} - 3C_1^{(2)} + 2\zeta_3, \tag{29}$$

and the auxiliary function $C_1(\nu)$ is regulated by an analytic regulator ν,

$$C_1(\nu) = -2\int \frac{\mathrm{d}^2\mathbf{k}_{1,\perp}}{2\pi} \frac{\Delta_3\ \Delta_4\ m_V^{2\nu}}{\Delta_1^{1+\nu}\Delta_{3,1}\Delta_{4,1}}. \tag{30}$$

The three coefficients $C_1^{(0),(1),(2)}$ in Eq. 29 can be calculated analytically directly, resulting in expressions in terms of classical polylogarithms through weight 3.

We now present the numerical results. For 13 TeV at LHC, we find

$$\sigma_{\rm nf}^{\rm LO} = -2.97_{+0.52}^{-0.69}\ \text{fb}, \quad \sigma_{\rm nf}^{\rm NLO} = -3.20_{+0.14}^{-0.01}\ \text{fb}, \tag{31}$$

where the first number is the leading-eikonal contribution without fermion-bubble corrections and the second with. We fixed $\mu_F = m_H$ and set $\mu = m_H$ to obtain the central values, and $\mu = m_H/2$ and $\mu = 2m_H$ to obtain values described by superscripts and subscripts in Eq. (31), respectively. We can clearly see from Eq. (31) that the scale variation is reduced very significantly once $O(\beta_0\alpha_S^3)$ contributions are included. We also show the uncertainty reduction in the transverse momentum distribution of the leading jet and Higgs rapidity distribution in Figure 4. The uncertainty band is shrunk after including the fermion-bubble corrections.

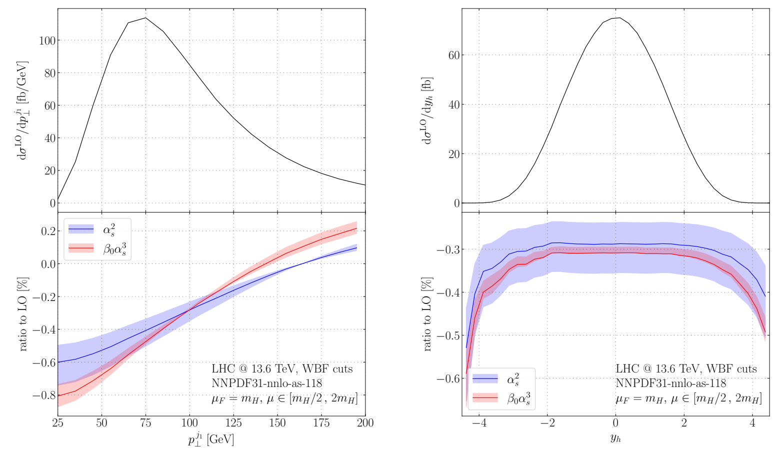

Figure 4: Transverse momenta distribution of leading jet (left panes) and Higgs boson rapidity distribution (right panes) in Higgs boson production in VBF. The upper panes show the LO (tree-level) distributions and the lower panes show the ratio of non-factorizable contributions to LO for corrections of $O(\alpha_S^2)$ (blue) and $O(\beta_0\alpha_S^3)$ (red).

5. Conclusions

We presented two recent calculations that extend beyond the leading-eikonal approximation for non-factorizable corrections to Higgs production in VBF at NNLO QCD. By expanding the complex five-point VBF amplitudes around the forward limit—a highly non-trivial task—we obtained the first power correction in the eikonal expansion. The resulting expressions are surprisingly compact and elegant, benefiting greatly from the special kinematics of VBF. This new sub-eikonal contribution alters the current estimate of NNLO non-factorizable corrections to the VBF cross section by approximately 20%. Furthermore, we addressed the strong renormalization scale dependence of the non-factorizable contribution by calculating the three-loop $O(\beta_0\alpha_S^3)$ corrections. These corrections account for the effects of the running coupling constant, reducing the renormalization scale dependence from $O(20\%)$ to $O(5\%)$, thereby stabilizing the theoretical predictions. These two calculations significantly enhance our understanding of NNLO non-factorizable corrections to VBF.

Acknowledgments

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