PROCEEDINGS OF SCIENCE

QCD corrections to Top-Quark Decay at α_s^3

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We present the first complete high-precision results for the top-quark decay width Γ_t, W-helicity fractions and semi-inclusive distributions for the top-quark decay process to the third order in the strong coupling constant α_s. We find, in particular, that the pure $O(\alpha_s^3)$ correction decreases Γ_t by 0.8% of the previous $O(\alpha_s^2)$ result, exceeding considerably the error estimated by the usual scale-variation prescription. With this critical piece of correction incorporated, our to-date most precise theoretical prediction meets the request by future colliders. This computation is achieved by a very efficient approach, applied recently also to the calculation of $O(\alpha_s^3)$ QCD correction to lepton-pair invariant-mass spectrum in B-meson semi-leptonic decay.

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Introduction

The top-quark t is the heaviest fundamental particle discovered to-date in experiments, and it plays a very important role both in the precision test of the Standard Model (SM), especially the electroweak sector, as well as searching for New Physics. The t-quark mass m_t is currently measured to be $172.69 \pm 0.30\,\text{GeV}$[1], with a precision already less than 2 per-mille, and many of the unique features of t-quark are, one way or another, related to this extraordinary mass value. In particular, this large mass not only sets a sufficiently-large scale to apply the perturbative Quantum Chromodynamics (QCD), but also makes for a unique feature relevant for QCD phenomenology: with a lifetime much shorter than the time needed for the formation of hadrons and resonances, and the spin decorrelation time, the t-quark decays before hadronization and the information about its spin state is preserved in distributions of t-quark decay products. The current world-average value for t-quark decay width Γ_t is $1.42^{+0.19}_{-0.15}\,\text{GeV}$[1]. The anticipated experimental uncertainties for Γ_t at the future hadron and lepton colliders can be reduced to $20 \sim 26\,\text{MeV}$[2–5]. and to fully exploit the very-costly experimental data, the errors for the theoretical predictions shall be reduced as much as possible, ideally to half of the aforementioned experimental error or even less. As we will see in this talk, to this end the complete QCD corrections at next-to-next-to-next-to leading order (NNNLO) in α_s would be needed, which are given in ref. [6], on which this talk is based on.

In addition to the above need from the experimental side, there is also a theoretical motivation for an explicit determination of the NNNLO QCD correction to the t-quark decay process. Although QCD corrections to t-quark decay can and have been calculated perturbatively up to next-to-next-to leading order (NNLO) in the strong coupling α_s, e.g. in refs.[7–17, 17, 18], a prioir it is not clear how large the higher-order QCD corrections might be, since the conventional QCD scale-variation clearly underestimates the perturbative error for Γ_t at the scale m_t[19]. One recalls also that both the definition of the pole mass m_t and decay width Γ_t are sensitive to the infrared-renormalon issue (See e.g. refs.[20–25]).

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The Method

We now give a brief exposition of the techniques employed to accomplish our computation. Aiming for obtaining NNNLO QCD corrections to the semi-inclusive distributions related to the W produced in $t \to b + W^+ + X$, t-quark decay width is written in term of the semi-inclusive hadronic tensor $\mathcal{W}_{tb}^{\mu\nu}$ integrated over the W momentum k as follows,

$$\Gamma_t = \frac{1}{2\,m_t} \int \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1}2E}\, \mathcal{W}_{tb}^{\mu\nu} \sum_{\lambda}^{L,R,0} \varepsilon_\mu^*(k,\lambda)\, \varepsilon_\nu(k,\lambda)\,, \tag{1}$$

with fixed t-quark momentum p. Anticipating potential infrared-soft and/or collinear (IR) singular behavior of the integrand $\mathcal{W}_{tb}^{\mu\nu}$ in the phase-space integration over k, in the region where the W-energy E reaches its maximum, we have introduced the dimensional regularization (DR) with the spacetime dimension denoted by $d \equiv 4 - 2\epsilon$. For an unpolarized on-shell W with mass m_W, there is the well-known polarization-sum $\sum_\lambda \varepsilon_\mu^*(k,\lambda)\,\varepsilon_\nu(k,\lambda) = \left(g^{\mu\nu} - k^\mu k^\nu\right)/m_W^2$. For W with a definite helicity λ, the corresponding projectors can be found in ref. [26], and see also ref. [27], which we quote for the sake of readers' convenience:

$$\begin{aligned}
\varepsilon^{*\mu}(k,\lambda_0)\,\varepsilon^{\nu}(k,\lambda_0) &= \frac{\left(m_W^2 p^\mu - p\cdot k\, k^\mu\right)\left(m_W^2 p^\nu - p\cdot k\, k^\nu\right)}{m_t^2\, m_W^2\, \vec{k}^2} \equiv \mathcal{P}_{\rm L}^{\mu\nu} \\
\varepsilon^{*\mu}(k,\lambda_L)\,\varepsilon^{\nu}(k,\lambda_L) &= \left(\mathcal{P}_{\rm tot}^{\mu\nu} - \mathcal{P}_{\rm L}^{\mu\nu} + \mathcal{P}_{\rm A_{FB}}^{\mu\nu}\right)/2 \\
\varepsilon^{*\mu}(k,\lambda_R)\,\varepsilon^{\nu}(k,\lambda_R) &= \left(\mathcal{P}_{\rm tot}^{\mu\nu} - \mathcal{P}_{\rm L}^{\mu\nu} - \mathcal{P}_{\rm A_{FB}}^{\mu\nu}\right)/2
\end{aligned}$$

where the difference between the two transversal helicity-state projectors $\mathcal{P}_{\rm A_{FB}}^{\mu\nu} \equiv -i\epsilon^{\mu\nu}_{p\,k}/\left(m_t|\vec{k}|\right)$ is known as the forward-backward-asymmetry projector.

There are simple diagrammatic representations for QCD corrections to $\mathcal{W}_{tb}^{\mu\nu}$ in terms of the t-quark self-energy diagrams; they contain exactly one W-propagator interacting with the external t-quark plus arbitrary QCD loops, subject to all possible final-state cuts going through at least the W propagator and a bottom-quark propagator. We compute $\mathcal{W}_{tb}^{\mu\nu}$ via form-factor decomposition, which involves 5 linearly-independent Lorentz-tensor structures:

$$\mathcal{W}_{tb}^{\mu\nu}(p,k) = W_1\, g^{\mu\nu} + W_2\, p^\mu p^\nu + W_3\, k^\mu k^\nu + W_4\left(p^\mu k^\nu + k^\mu p^\nu\right) + W_5\, i\epsilon^{\mu\nu}_{p\,k} \qquad (2)$$

where $\epsilon^{\mu\nu}_{p\,k}$ appears due to the chiral structure of the weak tbW-vertex. Since the γ_5 from the tbW-vertex always appears on an open fermion chain of the contributing QCD amplitudes, γ_5 can be treated fully anticommutively [28–30] in a straightforward manner. In the massless bottom-quark limit, the QCD corrections to the axial part of the inclusive decay width Γ_t becomes the same as the vector counterpart.

Each form factor W_i is a function of m_t, m_W and the W-energy E, as well as m_b in case of a massive bottom-quark, and receives both virtual and real-radiation type QCD corrections. The loop integration are completed by the integration-by-parts reduction [31] with the resulting master integrals computed using the differential equation method [32] based on series expansion (See e.g. refs.[33, 34]). To fix the boundary conditions for these differential equations, the auxiliary mass flow method [35–38] implemented in `AMFlow` [39] are used. The phase-space integrals over the momenta of the aforementioned cut propagators, except for k, are treated in the same manner as loop integrals by means of the reverse unitarity [40–42]. To give an idea of the complexity encountered in our calculation, the number of integrals in the $O(\alpha_s^3)$ corrections to $\mathcal{W}_{tb}^{\mu\nu}$ is about 7×10^4, which are reduced to linear combinations of in total 2988 master integrals. For each master integral, a piecewise series expansion representation (PSE), deeply-expanded up to about 200 orders in E is obtained using the differential equation solver in `AMFlow`. Consequently, we obtain a high-precision result for $\mathcal{W}_{tb}^{\mu\nu}$ in the form of deeply-expanded PSE.

According to the Frobenius series solution for the differential equations of dimensionally-regularized loop integrals, these integral functions admit the following type of series expansion in a representative kinematic variable x around the point of interest parameterized at $x = 0$:

$$f(\epsilon,x) = \sum_{a,b\in S} x^a \ln^b(x)\, T_{ab}(\epsilon,x) = \sum_{a,b\in S} x^a \ln^b(x)\left(\sum_{n=0}^{\infty} C_{abn}(\epsilon)\, x^n\right)$$

where $a = a_0 + a_1\epsilon$ with rational a_0, a_1 and non-negative integer b, belonging to a finite set S. We derive the following term-wise integration formula:

$$\int_0^u x^a \ln^b(x)\,{\rm d}x = \begin{cases} \frac{\ln^{1+b}(u)}{1+b} & \text{if } a=-1 \\ u^{1+a}\frac{1}{(1+a)^{1+b}} \sum_{i=0}^b (-1)^{b-i}\frac{b!}{i!}(1+a)^i \ln^i(u) & \text{if } a\neq -1 \end{cases} \qquad (3)$$

where the contribution from $x = 0$ is dropped by the virtue of DR. The phase-space integration of $\mathcal{W}_{tb}^{\mu\nu}$ over the IR-dangerous regions in k is done using its PSE by (3) with ϵ assigned with non-zero numbers to regularize the potential IR divergences. The fit regarding the ϵ-dependence is done only at the very end for the final finite physical objects of interest, which can be Γ_t and $f_{\mathrm{L,R,0}}$ as well as distributions. This treatment saves us from the need to manipulate Laurent expansions in ϵ during the intermediate stages, which significantly reduces the computational time. The treatment above is validated using Γ_t, by comparing the numerical results composed from $\mathcal{W}_{tb}^{\mu\nu}$ against those calculated by applying directly the optical theorem (using the reversed unitarity [40–42]): we found a perfect agreement between the two within our numerical precision.

Results for the inclusive Γ_t

We are now ready to discuss our numerical results for the top quark decay process and begin with Γ_t. The QCD effects on Γ_t in SM can be parameterized as the following truncated series in a_s:

$$\Gamma_t \quad = \quad \Gamma_0 \left(\mathbf{c}_0 + \alpha_s \, \mathbf{c}_1 + \alpha_s^2 \, \mathbf{c}_2 + \alpha_s^3 \, \mathbf{c}_3 + O(\alpha_s^4) \right) \tag{4}$$

where we introduced a prefactor $\Gamma_0 \equiv \frac{G_F \, m_W^2 \, m_t \, |V_{tb}|^2}{12\sqrt{2}}$ with G_F the Fermi constant, the CKM matrix element V_{tb} taken to be 1 in the following numerical results. The perturbative coefficients $\mathbf{c}_i$ are functions of the kinematic variable m_W^2/m_t^2 in the limit $m_b = 0$, and also the renormalization scale μ if α_s and/or m_t are renormalized in the $\overline{\mathrm{MS}}$ scheme. Unlike what is typically chosen in literature, we take $\mu = m_t$ as our central scale, motivated by the fact that it is close to the kinetic energy of the QCD system. We obtain the following numbers,

$$\begin{aligned} \Gamma_t &= 1.48642 - 0.140877 - 0.023306 - 0.007240 \text{ GeV} \\ &= 1.31500 \text{ GeV}, \end{aligned} \tag{5}$$

decomposed according to the α_s order, where m_t renormalized in the on-shell scheme and α_s is $\overline{\mathrm{MS}}$-renormalized with 5 massless quark flavors.[1] We see clearly the QCD corrections continue to decrease the Born-level result for Γ_t up to $O(\alpha_s^3)$, eventually down to a value about 1.31500 GeV. Furthermore, by examining the successive ratios $\mathbf{c}_{i+1}/\mathbf{c}_i$ starting from $i = 1$, we begin to see the hint that the convergence of the perturbative series seems to deteriorate as the perturbative order goes higher. This is not unexpected and thought to be related to the well-known infrared-renormalon sensitivity of the on-shell (pole) mass definition for quarks (see, e.g. refs.[20–25]). Up to $O(\alpha_s^3)$, the rate of growth of the perturbative coefficients recedes a bit once m_t is rewritten in terms of the $\overline{\mathrm{MS}}$-mass $\overline{m}_t$.

In figure 1 we plot the μ-dependence of our fixed-order results for Γ_t at $m_t = 172.69$ GeV for $\mu/m_t \in [0.1, 1]$ up to NNNLO.First of all, the scale dependence (span of these curves in the vertical direction) is reduced as one goes to higher orders in perturbation, as expected. However, due to the turning point of the NNLO green curve (at about $\mu/m_t = 0.14$), its scale variation can never cover the NNNLO result, the red curve, at any scales less than $\mu/m_t = 0.6$, including our central

[1] The SM input parameters are $G_F = 1.166379 \times 10^{-5}$ GeV^{-2} and $\alpha_s(m_t/2) \approx 0.1189$ at scale $\mu = m_t/2$, obtained by solving the renormalization-group equation for the running α_s at four-loop order [43–46] with input $\alpha_s(m_Z) = 0.1179$ at the Z-pole mass $m_Z = 91.1876$ GeV.

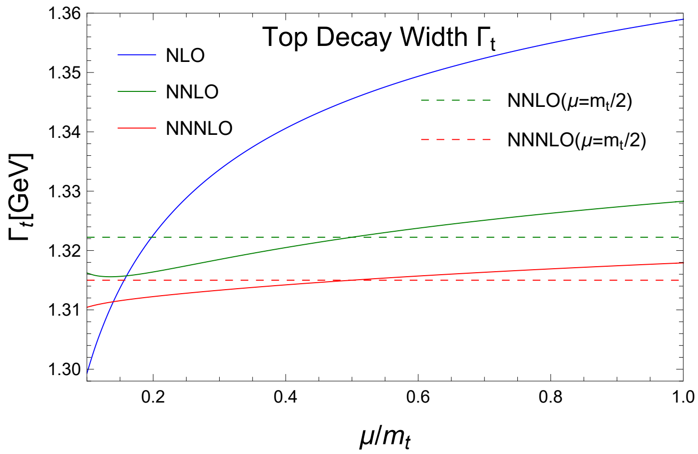

Figure 1: The scale dependence of the fixed-order results for Γ_t in $\mu/m_t \in [0.1, 1]$

value $\mu/m_t = 0.5$. Therefore, the NNLO results can underestimate the theoretical error by simply studying the μ dependence, and thus determining the $O(\alpha_s^3)$ corrections explicitly is very important.

With our efficient method, we investigated furthermore the so-called off-shell W effect in the top decay width, up to $O(\alpha_s^3)$, through the replacement $\frac{1}{k^2-m_W^2+i\epsilon} \to \frac{1}{k^2-m_W^2+im_W\Gamma_W}$ in the following formula [47]:

$$\begin{aligned} \Gamma_t(m_W) \to \tilde{\Gamma}_t &= \int_0^{m_t^2} \frac{\mathrm{d}\,k^2}{2\pi} \frac{2m_W\Gamma_W}{(k^2-m_W^2)^2+(m_W\Gamma_W)^2} \Gamma_t(m_W^2 \to k^2) \\ &= \tilde{\Gamma}_0 \left[\tilde{\mathbf{c}}_0 + \frac{\alpha_s}{\pi}\mathbf{c}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \mathbf{c}_2 + \left(\frac{\alpha_s}{\pi}\right)^3 \mathbf{c}_3 + O(\alpha_s^4)\right]. \end{aligned}$$

$\tilde{\Gamma}_t$ assumes a similar perturbative parametrization as in (4) but with the i-th order coefficient denoted as $\tilde{\mathbf{c}}_i$ for distinction. With W total decay width $\Gamma_W = 2.085$ GeV, $\delta_i \equiv (\tilde{\mathbf{c}}_i - \mathbf{c}_i)/\mathbf{c}_i$ are found to be quite small and decrease, albeit very slowly, as the α_s-order increases: δ_i takes $-1.54‰, -1.53‰, -1.39‰, -1.23‰$ respectively for $i = 0, 1, 2, 3$.

For the finite b-quark mass effect, we denote the i-th order coefficient for Γ_t with a non-zero m_b as $\mathbf{c}_i^{m_b}$. With $m_b = 4.78$ GeV we find that $(\mathbf{c}_1^{m_b} - \mathbf{c}_1)/\mathbf{c}_1 \approx (\mathbf{c}_2^{m_b} - \mathbf{c}_2)/\mathbf{c}_2 \approx -1.47‰$. This strongly indicates that the small non-zero m_b effect at $O(\alpha_s^3)$ may observe a similar small ratio, well-below sub-per-mille level for the total Γ_t.

Taking into account the aforementioned finite m_b and off-shell W effects, as well as the NLO electroweak corrections [48–51] which we re-derived and included by a multiplicative K-factor 1.0168, our final result for Γ_t reads

$$\Gamma_t = 1.3148^{+0.003}_{-0.005} \times |V_{tb}|^2 + 0.027\,(m_t - 172.69)\,\text{GeV}, \tag{6}$$

where the second term is introduced to parameterize the main source of error on Γ_t originated from the experimental uncertainty of the input t-quark mass value; furthermore V_{tb} is restored explicitly for completeness.

Results for the W-helicity fractions and angular distributions

Our final results for $f_{\mathrm{L,R,0}}$, with the QCD corrections up to α_s^3 as well as the aforementioned miscellaneous effects, read

$$f_0 = 0.686^{+0.002}_{-0.003}\,,\;\; f_{\mathrm{L}} = 0.312^{+0.001}_{-0.002}\,,\;\; f_{\mathrm{R}} = 0.00157^{+0.00002}_{-0.00002}\,. \tag{7}$$

The errors for the above results, given in numbers in super- and sub-script are obtained by combining the errors of the following main sources: the conservative QCD-scale-uncertainty error for Γ_t determined in (6) and the errors induced by the input t-quark mass 172.69 ± 0.30 GeV and b-quark mass $4.78^{+0.02}_{-0.03}$ GeV, as well as that of $\alpha_s(m_Z) = 0.1179 \pm 0.0009$ [1] in the case of $f_{\mathrm{R}}^{[3]}$. The helicity fractions $f_{\mathrm{L,R,0}}$ in (7) are related to the normalized semi-inclusive $\cos\theta^*$ angular distribution [52] via

$$\frac{1}{\Gamma_t}\frac{\mathrm{d}\Gamma_t}{\mathrm{d}\cos\theta^*} = \tfrac{3}{4}(\sin^2\theta^*)\, f_0 + \tfrac{3}{8}(1-\cos\theta^*)^2\, f_{\mathrm{L}} + \tfrac{3}{8}(1+\cos\theta^*)^2\, f_{\mathrm{R}}$$

where θ^* is the angle between the momentum direction of the charged lepton from the W-decay and the reversed momentum direction of the b-quark from the t-quark decay both calculated in the W-rest frame. In figure 2 we plot our results for the $\cos\theta^*$ angular distribution up to NNNLO in QCD, confined to the window $\pi/4 < \theta^* < 3\pi/4$ for a better display. Reference [53] suggested a

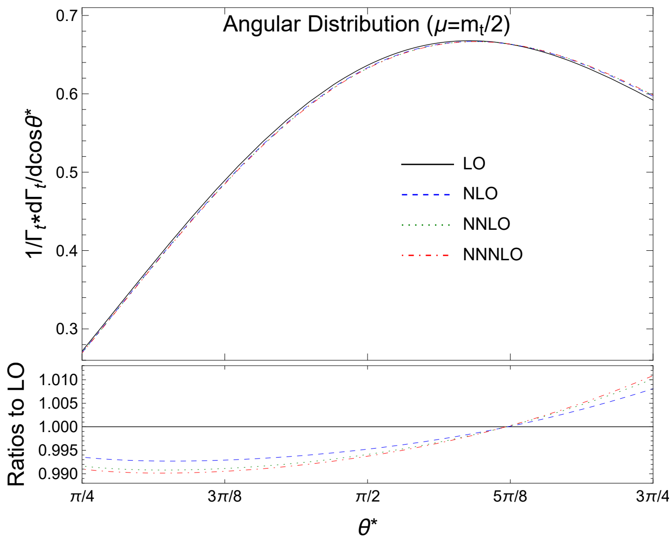

Figure 2: Angular distribution of the charged lepton from t-quark decay in the W rest frame.

generalized angular asymmetry based on this $\cos\theta^*$ distribution, $\mathcal{A}_z = \frac{\mathrm{N}(\cos\theta^*>z)-\mathrm{N}(\cos\theta^*<z)}{\mathrm{N}(\cos\theta^*>z)+\mathrm{N}(\cos\theta^*<z)}$ where $\mathrm{N}(\cos\theta^* > z)$ denotes the number of charged leptons in t-quark decay observed with $\cos\theta^*$ larger than a given value z, for the sake of reducing measurement uncertainties when extracting results on $f_{\mathrm{L,R,0}}$. Our results can be readily used to determine the QCD corrections to these quantities up to NNNLO. The so-called forward-backward asymmetry of the charged lepton in θ^* with respect to $\theta^* = \pi/2$ corresponds to the special case of $\mathcal{A}_z$ at $z = 0$, which equals to $\frac{3}{4}\left(f_{\mathrm{L}} - f_{\mathrm{R}}\right)$. Interestingly, we observe that the *total* NNNLO QCD corrections to $\mathcal{A}_z$ with $z > 0.6$ drops below 1 per-mille. In particular for $z = \cos(\pi/4)$ it is just a few 10^{-4}, completely negligible as far as the experimental precision reachable in the near future is concerned.

Results for the W-energy distribution

Having the results for $\mathcal{W}_{tb}^{\mu\nu}$, we are able to calculate the NNNLO QCD corrections to the W-energy distribution in t-quark decay observed in the t-quark rest frame, shown in figure. 3. The

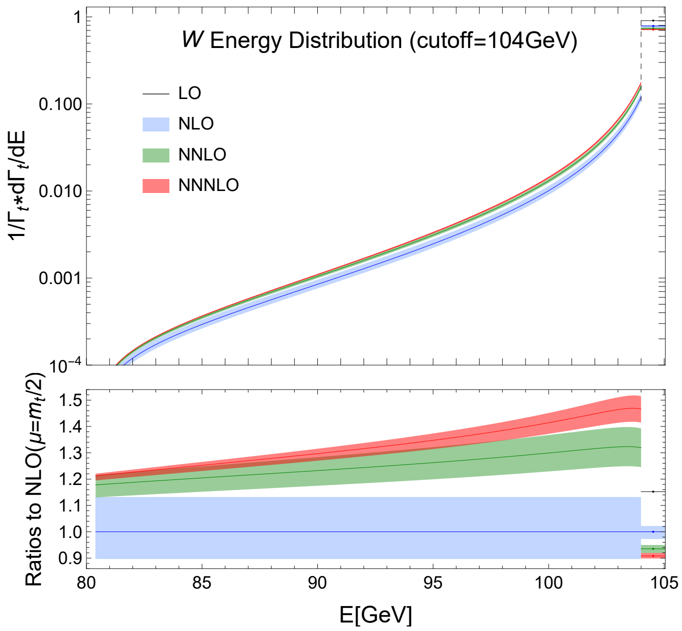

Figure 3: The W-energy distribution in t-quark decay observed in the t-quark rest frame.

distribution of W-energy E increases as E becomes larger, and becomes singular in the limit of E reaching its maximum $E_{\rm max}$ where the QCD radiations have to be either soft and/or collinear to the b-quark. The fixed-order prediction breaks down in this region, and we take an average over a 1 GeV-bin in the rightmost end of the distribution, where the QCD corrections up to $O(\alpha_s^3)$ decrease the Born-level result in a way similar to the Γ_t case. From the lower panel of figure. 3, one sees that the QCD corrections are positive and quite sizable, in particular, the pure $O(\alpha_s^3)$ correction modifies the lowest order result by about $7 \sim 14\%$ for $E \in [94, 104]$ GeV. The scale variation of the next-to-leading (NLO) curve is solely determined by the change of α_s, and thus the blue band is independent of E. Starting from NNLO, the appearance of large logarithm $\ln\left((E_{\rm max} - E)/E\right)$ leads to the increase of corrections as E approaches $E_{\rm max}$. At NNNLO, the change in the scale-variation band in this region becomes more visible due to poor convergence of the perturbative series contaminated by the aforementioned large logarithmic structure.

Applications to $b \to u\, e^+ \nu_e$

It shall be clear that the computational set-up we have can be applied straightforwardly to the calculation of another heavy quark decay process, the semi-leptonic B-decay, more specifically $b \to u\, e^+ \nu_e$. We obtain the following numbers for the perturbative QCD corrections to the decay width $b \to u\, e^+ \nu_e$ determined in the $\overline{\rm MS}$ scheme for b-quark mass (where all particles involved are taken massless): $\overline{\Gamma}_0 \left(1 + 0.3036075 + 0.1365820 + 0.06841766\,.\right)$ at the scale of $\mu = \bar{m}_b$, where $\overline{\Gamma}_0 \equiv \frac{G_F^2\, |V_{ub}|^2\, \bar{m}_b(\bar{m}_b)^5\, A_{\rm ew}}{192\pi^3}$ and the Fermi constant $G_F = 1.166379 \times 10^{-5}\,\mathrm{GeV}^{-2}$, and the CKM matrix element $V_{ub} = 3.82 \times 10^{-3}$; $A_{\rm ew} = 1.014$ denotes the known electroweak K-factor [54]. We have observed a nice pattern that every one more perturbative order higher in this $\overline{\mathrm{MS}}$ result, the size of the term is reduced roughly by 1/2. Based on this simple geometric power series, one would expect the 4-th order QCD correction would be 0.034184. The results for the lepton-pair invariant-mass spectrum in $b \to u\, e^+ \nu_e$ at various orders in QCD can be computed using the functions provided in the associated supplementary file of ref.[6].

To summarize, our results for the top quark decay process represent the to-date most precise theoretical prediction for Γ_t, and will be useful for the purpose of testing the SM and probing New Physics. Moreover, the W-energy distribution can serve as an ingredient to arrive at a fully differential calculation of any infrared-safe observables for t-quark decay process at NNNLO in QCD where only NNLO IR-subtraction terms are needed. The method in use has been applied in studies of other heavy-to-light quark decays, in particular the B-meson semi-leptonic decay.

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