

Progress towards High-energy logarithmic resummation at full next-to-leading logarithmic accuracy

Jeppe R. Andersen^{a,*}

^a*IPPP, Department of Physics, University of Durham, South Road, Durham, DH1 3LE, UK*

E-mail: jeppe.andersen@durham.ac.uk, malina.rosca@durham.ac.uk

We discuss briefly recent progress towards a method for obtaining full next-to-leading high-energy logarithmic accuracy for processes while maintaining analytic properties such as crossing symmetry and gauge-invariance of the amplitudes. The additional analytic requirements help constrain the behaviour of the amplitudes away from the asymptotic limit. This aids the accuracy of the predictions beyond the logarithmic precision. Furthermore, the framework ensures full energy and momentum conservation necessary for next-to-leading logarithmic precision not just of the amplitudes but also of the cross sections.

Loops and Legs in Quantum Field Theory (LL2024)
14-19, April, 2024
Wittenberg, Germany

*Speaker

1. Introduction

Recent measurements by ATLAS[1, 2] have highlighted the importance of a precise control of the large logarithmic perturbative corrections for observables which probe regions of large $\log(s/p_t^2)$. For context we will here briefly discuss one of these measurements focusing on the inclusive three-jet to inclusive two-jet ratio[2] for jets (anti- k_t , $R = 0.4$) with $p_t > 60\text{GeV}$. The jet rates were measured differentially for observables based on transverse momenta (p_T , H_{T2}). In these cases the fixed-order calculations converge sufficiently fast that an impressive description is found with both the two-jet and three-jet processes calculated at NNLO[3]. The results were compared also to predictions from general purpose monte carlos which generally all obtain a good prediction.

The two and three jet cross sections were also measured differentially in the rapidity difference and invariant mass between jets (both by the jets of the largest transverse momentum and the jets of largest rapidity difference or invariant mass - the distinction in which two jets are chosen is important of course only in samples with more than two jets). For these observables the sample of predictions from general purpose monte carlos have much more variation. The measurements are also compared to predictions obtained using HEJ[4–7] (see figure 1).

The results are interesting not just because of their relevance for studies for a precise determination of the irreducible QCD background to vector boson fusion and vector boson scattering

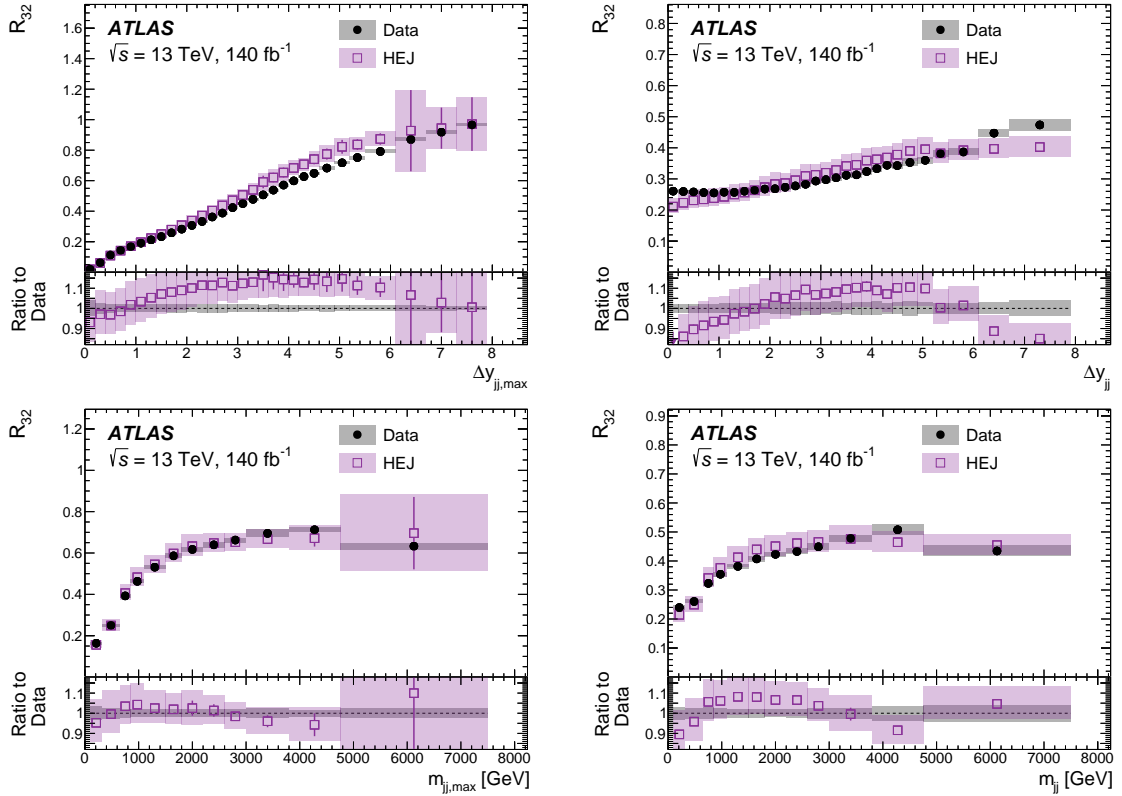


Figure 1: $dR_{32}/d\Delta y_{j_1 j_2}$ (top left), $dR_{32}/d\max(\Delta y_{jj})$ (top right), $dR_{32}/dm_{j_1 j_2}$ (bottom left) and $dR_{32}/d\max(m_{jj})$ (bottom right). Figures taken from ref.[2].

processes. They are interesting also as a measurement of the influence of the high-energy logarithmic corrections on multi-jet processes at the LHC. The influence of the logarithmic corrections on the normalisation of the cross sections largely cancels in the ratio between the two and three jet rates. However, other effects remain. Figure 1 (top left) shows R_{32} as a function of $\max(\Delta y_{jj})$ (i.e. $(d\sigma_{3j}/d\max(y_{jj})) / (d\sigma_{2j}/d\max(y_{jj}))$). $\max(\Delta y_{jj})$ approximates $\log(s/|t|)$, and the results for R_{32} clearly shows that the radiative corrections to the two jet rate (the 3-jet rate is part of this) grows approximately linearly with $\max(\Delta y_{jj}) \sim \log(s/|t|)$. The slope of this line can be crudely calculated in the framework of BFKL, see ref.[8] for details. It is fascinating of course that R_{32} reaches almost unity for large values of $\max(\Delta y_{jj})$.

The logarithmic influence on the corrections are again clearly visible in Figure 1(bottom left) where $\max(m_{jj})$ again is a proxy for $\log(s)$. The right-hand plots are included just to show that the agreement between prediction from HEJ and the measurements is good for observables based both on the two jets with the largest transverse momentum, and the arguably better estimator of the argument of the resummation based on $\max(m_{jj})$ or $\max(\Delta y_{jj})$.

2. Next-to-leading logarithmic corrections

Having established the leading logarithmic behaviour of R_{32} , the measurement are interesting also for the discussion of the impact of sub-leading logarithmic effects. The predictions from HEJ contain both matching to high-multiplicity partonic amplitudes and a resummation obtained by an explicit constructions of the resummation phase space around each multi-jet configuration[6]. The resummation captures the the leading $\log(s/p_T^2)$ corrections to the inclusive two-jet rate - the same logarithms as those from BFKL theory applied to a Mueller-Navelet setup. While the resummation applied so far is just leading logarithmic accurate, all the $2 \rightarrow 3$ next-to-leading logarithmic channels are also included in the resummation (and form a well-defined part of the NLL corrections). The agreement with data is within 10% - and crucially the deviation varies uniformly with δy , which is the chosen resummation variable. The overall size and dependence of the variation indicates this would be changed by NLL effects. We therefore strive to include all the next-to-leading logarithmic contributions.

The scattering amplitudes entering the resummation are different from those in BFKL. In particular they respect crossing symmetry and are fully gauge invariant (not just up to sub-leading terms). The additional analytic requirements on the amplitude are important for constraining the behaviour away from the asymptotic and to obtain a good approximation to the exact scattering amplitude in the region of phase space relevant for the measurements. The requirements also complicate the calculation of the next-to-leading logarithmic corrections. The components necessary for the resummation within the framework of high-energy logarithms are loosely termed *impact factors* and *kernel*. These have long been known in the asymptotic limit relevant for BFKL. The real-emission corrections of the components which would be termed *impact factors* in the language of BFKL were calculated with the additional restrictions on the analytic properties used in HEJ in ref.[7]. The virtual corrections in the framework are identical to the multiplicative corrections extracted in ref.[9]. Both virtual and real corrections to the impact factors have IR poles. What remains to be done is to organise the subtraction such that corrections to the “impact factors” can be included. This requires first a careful check of the cancellation since some non-factorising terms are

dropped compared to the standard fixed-order calculation. In order to maintain the desired analytic properties beyond those controlled by the pure logarithmic accuracy, the retaining or dropping of terms obviously cannot be based just on the logarithmic contribution. It turns out that colour projection operators here are crucial.

Secondly the organisation of the subtraction must be organised. We will base the subtraction on the FKS method. Work is ongoing.

References

- [1] G. Aad *et al.* [ATLAS], “Differential cross-sections for events with missing transverse momentum and jets measured with the ATLAS detector in 13 TeV proton-proton collisions,” [arXiv:2403.02793 [hep-ex]].
- [2] G. Aad *et al.* [ATLAS], “Measurements of jet cross-section ratios in 13 TeV proton–proton collisions with ATLAS,” [arXiv:2405.20206 [hep-ex]].
- [3] M. Czakon, A. Mitov and R. Poncelet, “Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC,” *Phys. Rev. Lett.* **127** (2021) no.15, 152001 [erratum: *Phys. Rev. Lett.* **129** (2022) no.11, 119901] doi:10.1103/PhysRevLett.127.152001 [arXiv:2106.05331 [hep-ph]].
- [4] J. R. Andersen and J. M. Smillie, “Constructing All-Order Corrections to Multi-Jet Rates,” *JHEP* **01** (2010), 039 doi:10.1007/JHEP01(2010)039 [arXiv:0908.2786 [hep-ph]].
- [5] J. R. Andersen and J. M. Smillie, “Multiple Jets at the LHC with High Energy Jets,” *JHEP* **06** (2011), 010 doi:10.1007/JHEP06(2011)010 [arXiv:1101.5394 [hep-ph]].
- [6] J. R. Andersen, T. Hapola, M. Heil, A. Maier and J. M. Smillie, “Higgs-boson plus Dijets: Higher-Order Matching for High-Energy Predictions,” *JHEP* **08** (2018), 090 doi:10.1007/JHEP08(2018)090 [arXiv:1805.04446 [hep-ph]].
- [7] J. R. Andersen, J. A. Black, H. M. Brooks, E. P. Byrne, A. Maier and J. M. Smillie, “Combined subleading high-energy logarithms and NLO accuracy for W production in association with multiple jets,” *JHEP* **04** (2021), 105 doi:10.1007/JHEP04(2021)105 [arXiv:2012.10310 [hep-ph]].
- [8] J. R. Andersen, V. Del Duca, F. Maltoni and W. J. Stirling, “W boson production with associated jets at large rapidities,” *JHEP* **05** (2001), 048 doi:10.1088/1126-6708/2001/05/048 [arXiv:hep-ph/0105146 [hep-ph]].
- [9] V. Del Duca and C. R. Schmidt, “Virtual next-to-leading corrections to the impact factors in the high-energy limit,” *Phys. Rev. D* **57** (1998), 4069-4079 doi:10.1103/PhysRevD.57.4069 [arXiv:hep-ph/9711309 [hep-ph]].
- [10] S. Frixione, Z. Kunszt and A. Signer, “Three jet cross-sections to next-to-leading order,” *Nucl. Phys. B* **467** (1996), 399-442 doi:10.1016/0550-3213(96)00110-1 [arXiv:hep-ph/9512328 [hep-ph]].