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Radiative corrections to nuclear processes

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This paper reviews early studies of radiative corrections (RC) to the muon and to the beta decays. One loop RC to beta decays of hadrons are significantly enhanced with respect to analogous corrections to the muon decay. The nature of this enhancement is explained. Ultraviolet sensitivity of the beta-decay RC, found in the early studies, is compared to and contrasted with the dependence of Standard Model corrections on the electroweak boson masses.

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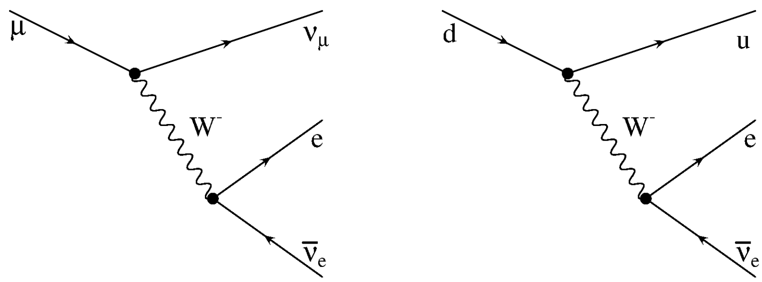
1. Introduction

This contribution to proceedings of the Loops and Legs in Quantum Field Theory (LL2024) results from my effort to understand early determinations of radiative corrections (RC) to the beta decay and to the muon decay, and to reconcile those results with modern literature.

Muon and the beta decays are subjects of very active current research. On the experimental side, there is an abundance of new and upcoming facilities including high-flux reactor and spallation sources of neutrons in Europe [1], China [2], Japan [3], and the USA [4]. Neutron lifetime puzzle, a disagreement between beam and bottle methods (see for example [5] for a recent point of view), persists and motivates new efforts (see for example [6]). Exciting new spectroscopic tools are being developed for low-energy electrons produced in the beta decay, including Cyclotron Radiation Electron Spectroscopy [7].

On the theory side, a very recent study reevaluated RC for superallowed beta decays [8, 9]. Large logs in neutron and more general beta decays have been summed to all orders [10]. It has been proposed to use lattice QCD to help determine a particularly difficult aspect of the neutron RC [11]. Other recent developments have been recently reviewed in [12, 13].

The muon decay, whose amplitude is shown is Fig. 1(a), plays a special role in the Standard Model (SM) as the source of the Fermi constant, $G_F \simeq 10^{-5}/\mathrm{GeV}^2$ [14]. G_F is one of the pillars of the SM precision tests as one of its most precisely determined parameters, together with the fine structure constant $\alpha \simeq 1/137$ and the mass of the Z boson m_Z.



(a) (b)

Figure 1: Tree-level Feynman diagrams describing (a) muon decay and (b) beta decay at the quark level.

Among beta decays, I will focus in particular on the neutron decay whose quark-level amplitude is shown in Fig. 1(b). If we express muon and neutron decay rates as

$$\Gamma\left(\mu \to e\nu_\mu\bar{\nu}_e\right) = c_\mu g^4 \left(1 + \mathrm{RC}_\mu\right), \tag{1}$$

$$\Gamma\left(n \to pe\bar{\nu}_e\right) = c_n g^4 \left(1 + \mathrm{RC}_n\right), \tag{2}$$

the weak coupling constant g can be eliminated and we get approximately

$$\Gamma\left(n \to pe\bar{\nu}_e\right) = \frac{c_n}{c_\mu}\Gamma\left(\mu \to e\nu_\mu\bar{\nu}_e\right)\left(1 + \mathrm{RC}_n - \mathrm{RC}_\mu\right) \tag{3}$$

$$= c_n' G_F^2 \left(1 + \mathrm{RC}_n - \mathrm{RC}_\mu'\right), \tag{4}$$

where c_n, c_μ, c_n' are combinations of experimentally-accessible quantities such as particle masses. We can therefore obtain a precise prediction for the neutron decay rate using the measured muon lifetime and the theoretical expression for the difference of radiative corrections to the neutron and the muon decays, $\mathrm{RC}_n - \mathrm{RC}_\mu'$. The prime on RC_μ' indicates that a part of RC to the muon decay is absorbed in the definition of the Fermi constant.
In this paper I review some aspects of the early studies of RC. A much more extensive historical perspective can be found in the review [15]. An excellent discussion of RC in the Standard Model, in particular to the muon decay, is provided in Ref. [16]. Units in this paper are such that $c = \hbar = 1$.

2. RC for the muon decay

A landmark paper that first determined radiative corrections to general decay processes of charged fermions was Ref. [17], written by two graduate students (Ralph Behrends and Alberto Sirlin) and their supervisor Robert Finkelstein. That work was later refined and extended by Sirlin and collaborators, including Tom Kinoshita.
One aspect of that pioneering study was corrected by Samuel Berman in his doctoral thesis [18, 19] (he was one of only a handful of PhD students working under Richard Feynman). While both groups were in agreement about virtual corrections, Berman found an error in the treatment of the real radiation in [17]. He pointed out that when a finite photon mass is introduced to regulate infrared (IR) divergences, the photon must be consistently treated as a massive particle both in virtual and in real radiation. While the photon was treated as massive in virtual corrections in [17], it was treated as massless in the real radiation (a mass parameter was introduced as a lower limit in the integration over the energy of the photon). As a result, Berman found that the $O(\alpha)$ correction to the muon decay rate is small and negative rather than large and positive.
Since Berman found agreement with [17] for virtual corrections, he did not repeat their formula in his writings. He did present detailed formulas for the real corrections, which he obtained correctly for the first time. These formulas are present both in his 1958 paper in the Physical Review [18] and in his PhD thesis [19]. Unfortunately in both instances there are typographical errors, different in each case. Here is the corrected form,

$$b_1 = 2X + \frac{(\ln\eta + \omega - 1)(1-\eta)\left(\frac{5}{\eta} + 17 - 34\eta\right)}{3(3-2\eta)\eta} + \frac{5(1-\eta)^2}{3(3-2\eta)\eta^2}, \tag{5}$$

$$X = (\ln\eta + \omega - 1)\left(2\ln(1-\eta) - \ln\eta + \omega - 2\ln\frac{\lambda_m}{m_e}\right) + \mathrm{Li}_2(\eta) - \frac{\pi^2}{6} - \frac{1-\eta}{\eta}\ln(1-\eta). \tag{6}$$

Here η denotes the normalized energy of the electron, $\eta = 2E_e/m_\mu$, ω is the logarithm of the ratio of the muon to the electron mass, $\omega = \ln(m_\mu/m_e)$, and λ_m is the small photon mass introduced as the infrared regulator. With respect to the Physical Review [18] version, the only correction is the factor η^2 in the denominator in the last term of Eq. (5). In Eq. (28) of the PhD thesis [19] that factor is partially present as η (it should be η^2); also, the factor η in the denominator of the middle term of Eq. (5) is missing in the PhD thesis. The polylogarithm in Eq. (6) is defined as

$$\mathrm{Li}_2(\eta) = \sum_{k=1}^{\infty}\frac{\eta^k}{k^2}, \tag{7}$$

whereas in the PhD thesis there is k instead of k^2 in the denominator.
The real radiation correction b_1 enters the decay rate formula in the following way,

$$\frac{\mathrm{d}\Gamma\left(\mu \to e\bar{\nu}_e\nu_\mu\right)}{\mathrm{d}\eta} = \frac{G_F^2 m_\mu^5}{96\pi^3}\eta^2\left(3-2\eta\right)\left[1+\frac{\alpha}{2\pi}\left(\alpha_1+b_1\right)\right], \tag{8}$$

where α_1 describes virtual corrections and is given by Eqs. (24b,d) in [17]. The sum $\alpha_1 + b_1$ is free from the dependence on λ_m. Note that the above formulas are valid in the limit of the electron much lighter than the muon; corrections $O\left(m_e/m_\mu\right)$ are not shown.
Berman's corrected treatment of the real radiation inspired the paper by Kinoshita and Sirlin [20] that, in addition to presenting a corrected result for the muon decay, contained the seminal observation of the cancellation of the logarithms of the electron mass in the total decay rate, despite their presence in the differential rate (the energy spectrum). The same cancellation was observed in the asymmetry of the polarized muon decay (the difference of the fraction of electrons emitted along the polarization vector of the muon and in the opposite direction). For the unpolarized decay, integration over $\eta \in (0,1)$ gives the radiative correction to the total decay rate,

$$\Gamma\left(\mu \to e\bar{\nu}_e\nu_\mu\right) = \Gamma^0\left(\mu \to e\bar{\nu}_e\nu_\mu\right)\left[1+\frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right] \simeq \Gamma^0\left[1-0.4\%\right], \tag{9}$$

$$\Gamma^0\left(\mu \to e\bar{\nu}_e\nu_\mu\right) = \frac{G_F^2 m_\mu^5}{192\pi^3}, \tag{10}$$

finite in the limit of the zero electron mass.
Finding the correction to the total rate, which is just a number, is much easier than that to the energy distribution, represented by a function of the energy. Two-loop corrections to the total muon decay rate were determined in the massless-electron approximation in 1998-1999 [21–23]. Effects of the electron mass turned out to be somewhat interesting from the theoretical point of view: initially estimated [22] to enter at the quadratic order $\sim \left(m_e/m_\mu\right)^2$, they turned out [24] to be linear in m_e/m_μ and thus much larger. This is related to using the pole mass of the muon in the calculation. More recently, even the three-loop corrections have been determined [25, 26]. The key to this result was the surprisingly rapidly converging expansion in the *difference* of the muon and electron masses [27]. For the energy distribution, only the two-loop correction has been published so far [28], as well as partial higher-order effects (see [29] and references therein).

3. RC for the neutron decay

3.1 Berman's 1958-1959 work

In addition to the muon decay, Berman studied also the neutron decay, assuming for simplicity that the neutron and the proton have no internal structure [18, 19]. This is the reason why, as we shall see, the mass of a nucleon (m_N, $N = p$ or n) appears in the argument of a large logarithm. For physical nucleons, their structure and the resulting smeared distribution of the electric charge temper the short-distance behavior of photon loops. The mass of the nucleon is then replaced by a hadronic scale $\Lambda \simeq 100$ MeV [10].

Berman's analysis was partially motivated by applications to the beta decay of oxygen-14, an example of a so-called superallowed beta decay,

$$^{14}\mathrm{O} \rightarrow ^{14}\mathrm{N} + e^{+} + \nu_e. \tag{11}$$

Here and in the neutron decay, there is a charged daughter nucleus/nucleon in the final state, in whose Coulomb field the beta particle is emitted. Berman noted that structure-dependent corrections are likely larger in the case of the oxygen decay than in the case of the neutron because possible intermediate excited states might be less separated, leading to larger energy denominators in perturbation theory.

3.2 Berman and Sirlin's 1962 paper

In 1962 Berman and Sirlin wrote their only joint paper [30]. They discovered that in the Landau gauge (also called transversal) real and virtual corrections are separately IR finite. There is then no need to introduce the photon mass. This settled a discussion about the correctness of IR regularization.

Berman and Sirlin presented the one-loop corrected beta-decay rate,

$$\Gamma\left(n \rightarrow p e \bar{\nu}_e\right) = \Gamma^0 \left[1 + \frac{\alpha}{2\pi}\left(6 \ln \frac{\Lambda}{m_p} + 3 \ln \frac{m_p}{E_m} + f\left(E, E_m\right)\right)\right], \tag{12}$$

where Γ^0 is the rate without radiative corrections and f is a "complicated function of the electron energy" whose explicit formula they included. In the context of the then persisting discrepancy between Fermi constant determined from the muon and from the beta decay, the authors stressed that corrections to the beta decay depend logarithmically on an ultraviolet (UV) cutoff Λ, unlike those to the muon decay. This leaves a free parameter in the comparison. If Λ were of the order of the kaon mass, the discrepancy would be removed. We now know that the cutoff is much higher, on the order of the electroweak scale (intermediate boson mass), and the discrepancy is instead removed by the Cabbibo angle [31]. In the next section, the UV cutoff dependence will be related to the result in the present electroweak theory. One might expect that this cutoff should be replaced by an electroweak boson mass. Since the muon decay rate is independent of the UV cutoff (Eq. (9), which is the muon-decay analog of Eq. (12), has no Λ dependence), one might expect the coefficient of $\ln m_{W,Z}$ in the modern theory to be $6\alpha/(2\pi)$. We shall see that this is not the case.

Before discussing this, consider the second log term in Eq. (12). Its argument is the ratio of the mass of the nucleon to the energy released in the decay. This logarithm is large in the case of the neutron decay because the released energy is a tiny fraction of the neutron mass. Namely, with neutron and proton masses being $m_n = 939.6$ MeV and $m_p = 938.3$ MeV, the total energy of the electron and its antineutrino is about 1.3 MeV, and $3\alpha/(2\pi) \ln\left(m_p/E_m\right) = 2.3\%$. In the case of the muon, almost its whole rest energy is converted into the kinetic energy of daughter particles and therefore the logarithm is small [18, 19]. This is the reason why RC are much larger in the decay of the neutron than of the muon.

3.3 Why do RC depend on the UV cutoff in case of the neutron but not muon?

Eqs. (9) and (12) describe one-loop photonic corrections to muon and neutron decays obtained before the modern electroweak theory was discovered. After its formulation (see [32] for a historical overview) and the discovery of its renormalizability [33], a calculation of RC became possible without any UV cutoff. The decay rate of the neutron in terms of the Fermi constant from the muon lifetime [34] is [35–39]

$$\Gamma\left(n \to p e \bar{\nu}_e\right) = \Gamma^0\left(n \to p e \bar{\nu}_e\right)\left[1 + \frac{\alpha}{2\pi}\left(4 \ln \frac{m_Z}{m_p} + \dots\right)\right], \tag{13}$$

where the dots denote terms without large logarithms involving electroweak boson masses. Since corrections to the muon decay (9) have no Λ dependence, it may seem puzzling why $6 \ln \Lambda$ in Eq. (12) has been replaced by the modern $4 \ln m_Z$ in (13). In this section we clarify this issue.

Comparing tree-level diagrams for the muon and the beta decays in Fig. 1 we see that electroweak corrections to the electron line and to the W propagator are the same in both cases and thus cancel in the ratio of the decay rates (are absorbed in G_F determined from the muon decay). The only difference is the 4-momentum transferred through the W to the electron line but it is negligible in comparison with the masses of W, Z, H bosons.

Differences can however be expected in Z and γ corrections to the wave function renormalization constants Z_2 and to the upper vertex, as well as in all box diagrams. To clarify the dependence on the UV cutoff, so it is sufficient to discuss UV divergent diagrams. All results below will be given in the Feynman gauge. Feynman rules are used in the convention defined in [40], whose notation for couplings is adopted here.

Consider first Z loops. The Z boson can connect any fermion line with the W propagator or with the same or different fermion line. Box diagrams are UV finite so they do not have to be considered. Thus we only consider the Z exchanged within the upper fermion line in diagrams in Fig. 1 or connecting that line with the W propagator. While some individual diagrams differ between the beta decay and the muon decay, their sum is the same. This is because the sum is proportional to the difference of Z couplings to left-handed fermions, and that difference is the same for quarks and for leptons, $g_u^- - g_d^- = g_\nu^- - g_\mu^-$,

Similarly, the photon exchange between the upper fermion line and the W propagator gives the same contribution in both processes because the electric charges satisfy $Q_d - Q_u = -1 = Q_\mu$.

The only remaining class of corrections are photon exchanges among the fermion lines. They are shown in Fig. 2, with the W propagator represented by a cross in a circle.

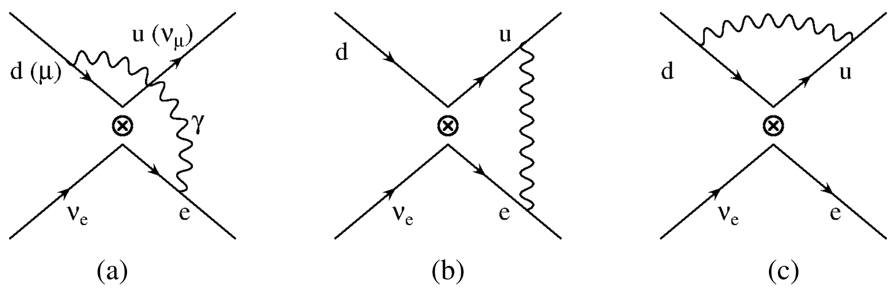


Figure 2: One-loop corrections to the beta (muon) decay in the four-fermion theory.

Neglecting the momentum flowing in the W propagator leads to an effective theory with a 4-fermion (4F) coupling. UV divergences in that "4F" theory are summarized in Table 1. An in-depth discussion of RC in the full Standard Model and the 4F theory is given in [41].

Contribution	Muon decay	Decay $d \to u$
(a)+(b)	$Q_e Q_\mu = 1$	$Q_e\left(Q_d - 4Q_u\right) = 3$
(c)+Z_2	$-\frac{1}{2}\left(Q_\mu^2 + Q_e^2\right) = -1$	$-\frac{1}{2}\left[\left(Q_d - Q_u\right)^2 + Q_e^2\right] = -1$
Total UV divergence in the 4F theory	0	2

Table 1: UV divergences in the 4-fermion (4F) effective theory for the muon and for the beta decays. In column 1, (a,b,c) refer to diagrams in Fig. 2 and Z_2 is the sum of wave function renormalization constants of the charged fermions in each process. In order to find contributions to the decay rate, the integer results should be multiplied by $\frac{\alpha}{2\pi} \cdot \frac{1}{\epsilon} \cdot \mu^{-2\epsilon}$ where μ is a mass scale such as the muon mass in case of the muon decay and ϵ is the parameter of the dimensional regularization in $D = 4 - 2\epsilon$ dimensions.

In particular we see that the correction to the muon decay calculated in the 4F theory is UV-finite. However, Table 1 shows that this finiteness results from a cancellation between contributions of the box in Fig. 2(a) and the wave function renormalization constants. But the box diagram would have been UV finite, had we kept the momentum of the W in its propagator. This means that there are in fact RC to the muon decay that are sensitive to large virtual momenta of the order of the electroweak scale and that in this sense their early evaluations were perhaps somewhat misleading.

Of course, the same remark about the momentum of the W boson applies also to the beta decay. Table 1 shows that the coefficient of the UV divergence in this case is 2. This number means that the low-energy 4F theory makes a UV divergent contribution to the decay rate,

$$\Delta\Gamma_{4F} = \frac{\alpha}{2\pi} \cdot \left(\frac{2}{\epsilon} - 4\ln\mu\right). \tag{14}$$

Assume that the only large mass scale in the full theory is $m_Z \simeq m_W$. In order to cancel the UV divergence of the 4F theory, the large mass scale contribution must be, apart from finite non-logarithmic terms,

$$\Delta\Gamma_{\text{large scale}} = \frac{\alpha}{2\pi} \cdot \left(-\frac{2}{\epsilon} + 4\ln m_Z\right), \tag{15}$$

giving the total correction $\Delta\Gamma = \frac{\alpha}{2\pi} \cdot 4\ln\frac{m_Z}{\mu}$ and thus confirming the large log in the modern prediction (13).

How then was the coefficient $\frac{\alpha}{2\pi} \cdot 6$ of $\ln\Lambda$ arrived at in the early calculations that led to Eq. (12)? Those studies, predating the quark model, considered the nucleon decay $n \to pe\bar{\nu}$, with only two charged particles p, e, just like in the muon decay. In both cases it was therefore possible to use Fierz transformation [42] to bring the decay amplitude to the so-called charge-retention order, where both charged particles form a single fermion line. However, in the muon decay one charged particle is in the initial and the other in the final state, whereas in the neutron case both charged particles are in the final state. As a result, whereas in the muon case the Fierz transformation does not alter the Dirac structure of the vector-minus-axial vertex, in the neutron case it changes it into a combination of a scalar and a pseudoscalar. This combination has an anomalous dimension that results in the UV cutoff dependence. The anomalous dimension of the $V - A$ operator vanishes and thus the muon decay is UV finite in the 4F theory (see also [43]).

4. Summary

Berman's paper [18] and PhD thesis [19] followed the breakthrough *Theory of Fermi interaction* by Feynman and Gell-Mann [44]. That seminal study made it possible to compare the Fermi constant G_F determined from the muon and from beta decays. Before Berman's work, the value from the muon decay was too large in comparison with that from the beta decays.
Berman's work partially alleviated that tension – but not entirely. The small remaining discrepancy was due to the Cabibbo angle θ_C, whose discovery followed about four years later [31]. By partially suppressing the rate of the beta decay, the presence of $\cos\theta_C$ makes the measured coupling seem smaller than it really is (today we would express the suppression factor by a parameter of the Cabibbo-Kobayashi-Maskawa matrix, $|V_{ud}|^2$).
Hopefully the present note will help to preserve the memory of Samuel Berman's work.

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PoS(LL2024)081

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