

## Partonic collinear structure by quantum computing

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We present a systematic quantum algorithm that combines hadronic state preparation with the evaluation of real-time light-front correlators to investigate the parton distribution function (PDF) and light cone distribution amplitude (LCDA). As a proof of concept, we simulate both the PDF and LCDA in the 1+1 dimensional Nambu-Jona-Lasinio model. The results obtained from exact diagonalization and quantum computation on classical hardware show strong agreement, validating the effectiveness of the proposed algorithm. Our findings highlight the promising potential for calculating partonic collinear structure on current and near-term quantum devices. This quantum algorithm is anticipated to have broad applications in high-energy particle and nuclear physics.

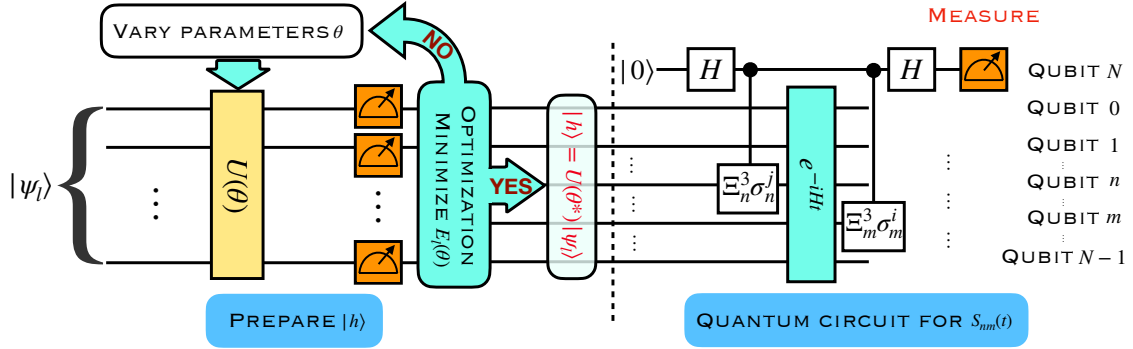
31st International Workshop on Deep Inelastic Scattering (DIS2024)  
8–12 April 2024  
Grenoble, France

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## 1. Introduction

Understanding the partonic structure of hadrons is essential for advancing high-energy particle and nuclear physics. Parton distribution functions (PDFs) and light-cone distribution amplitudes (LCDAs) describe the internal structure of hadrons in both inclusive and exclusive processes. Given the non-perturbative nature of QCD, PDFs and LCDAs are typically extracted through global analyses of experimental data and lattice QCD calculations. However, lattice QCD is unable to compute PDFs and LCDAs directly from light-front correlators due to the sign problem inherent in Monte Carlo algorithms. In this talk, we present a quantum algorithms for direct calculation of PDFs [1] and LCDAs [2] by evaluating light-front correlators on quantum computers. Section 2 introduces the quantum algorithm for PDFs, followed by the quantum calculation for LCDAs in section 3. We conclude with a summary and outlook in the final section.



**Figure 1:** Quantum circuit for calculating PDFs. The left-hand side of the dotted line is the quantum circuit for preparing the hadron state while the right-hand side of the dotted line is the quantum circuit to evaluate the time-dependence two-point functions.

## 2. Quantum computing for parton distribution function

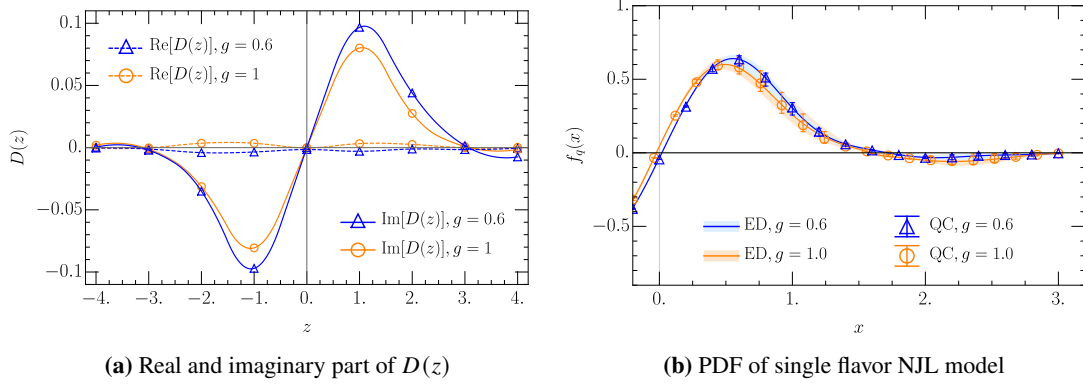
The operator definition of PDFs can be expressed as [3]:

$$f_{q/p}(x) = \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{ix(p \cdot n)z} \langle p | \bar{\psi}(zn)(n \cdot \gamma)W(zn, 0)\psi(0) | p \rangle, \quad (1)$$

where  $p$  is the four-momentum of the hadron,  $(n^\mu = (1, 0, 0, -1))$  is a light-cone basis vector,  $\psi$  is the fermion field,  $W(zn, 0)$  represents the Wilson line, and  $\gamma^\mu$  are the Dirac gamma matrices. As seen in Eq. (1), the PDFs inherently involve time evolution. Consequently, simulating them using the Monte-Carlo-based lattice QCD approach leads to the emergence of the sign problem. One way to circumvent this issue is by employing quantum computing methods.

### 2.1 Nambu–Jona-Lasinio model as a proof of concept study

Simulating the PDFs of QCD poses challenges due to the limited quantum resources available in the noisy intermediate-scale quantum (NISQ) era. As a proof-of-concept study, we simulate the PDFs of the (1+1)-dimensional Nambu–Jona-Lasinio (NJL) model [4, 5] on quantum computers.



**Figure 2:** PDFs of single flavor NJL model, with  $ma = 0.8$ ,  $N = 18$ . ED are the exact diagonalization results while the QC are the quantum computing results.

In the NJL model, there are no gauge fields, so the gauge link can be treated as the identity operator. The fermion fields must be discretized and mapped onto qubits before quantum simulations. We use staggered fermions [6] to discretize the fermion fields, and the Jordan-Wigner transformation [7] to map the discretized fermion fields to qubits. After these steps, the qubit PDF of the NJL model in the hadron rest frame can be written as

$$f(x) = \sum_z \sum_{i,j=0}^1 \frac{1}{4\pi} e^{-ixM_h z} \langle h | e^{iH_{\text{QC}} z} \phi_{-2z+i}^\dagger e^{-iH_{\text{QC}} z} \phi_j | h \rangle \equiv \sum_z \frac{1}{4\pi} e^{-ixM_h z} D(z) \quad (2)$$

where  $|h\rangle$  is the hadron state with zero spatial momentum, and  $\phi_n = \prod_{i<n} \sigma_i^z (\sigma_n^x + i\sigma_n^y)$  represents the staggered fermion field. The qubit Hamiltonian  $H_{\text{QC}}$  for the NJL model can be found in [1]. To obtain PDFs on quantum computers, two additional steps are required: 1. Prepare the hadron state  $|h\rangle$ . 2. Evaluate the real-time dependent correlation functions  $D(z)$ .

## 2.2 Preparation of hadron states on quantum computers

The quantum circuit to prepare the hadron state is shown on the left-hand side of the dotted line in Fig. (1). Suppose the hadron state is the  $k$ -th excited state with quantum number  $l$ , such as charge, spin, baryon number, etc. Then, we need  $k$  orthonormal reference states  $|\psi_{lk}\rangle_{\text{ref}}$ , which share the same quantum number  $l$  as the hadron, as input states for the quantum computers. A parameterized circuit  $U(\theta)$  is required to generate the trial states  $|\psi_{lk}(\theta)\rangle = U(\theta) |\psi_{lk}\rangle_{\text{ref}}$  of the hadron.

To construct  $U(\theta)$ , the Hamiltonian needs to be divided into  $H = H_1 + H_2 + \dots + H_n$  with  $n \geq 2$ , where each  $H_i$  preserves all the symmetries of  $H$  and  $[H_i, H_{i+1}] \neq 0$ . The unitary operator  $U(\theta)$  can then be written as an alternating sequence of evolutions under  $H_i$ ,

$$U(\theta) \equiv \prod_{i=1}^p \prod_{j=1}^n \exp(i\theta_{ij} H_j). \quad (3)$$

We need to optimize the loss function  $E_l(\theta) = \sum_{i=1}^k w_{li} \langle \psi_{li}(\theta) | H | \psi_{li}(\theta) \rangle$  [8] to select the hadron state from the trial wavefunctions. Finally, the hadron state can be prepared as  $|h\rangle = U(\theta^*) |\psi_{lk}\rangle_{\text{ref}}$ , with the optimized parameters  $\theta^*$ .

### 2.3 Evaluation of correlation function $D(z)$

The correlation function  $D(z)$  can be decomposed into the correlation functions of Pauli operators,

$$S_{mn}(t) = \langle h | e^{iHt} \left( \prod_{k<m} \sigma_k^3 \right) \sigma_m^i e^{-iHt} \left( \prod_{l<n} \sigma_l^3 \right) \sigma_n^j | h \rangle. \quad (4)$$

The right-hand side of Fig. (1) illustrates the quantum circuit used to calculate  $S_{mn}(t)$ . This quantum circuit is a special case of the circuit for calculating  $n$ -point correlation functions, as described in [9].

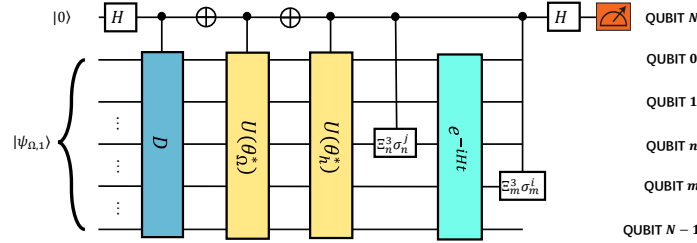


Figure 3: Quantum circuit for calculating LCDAs.

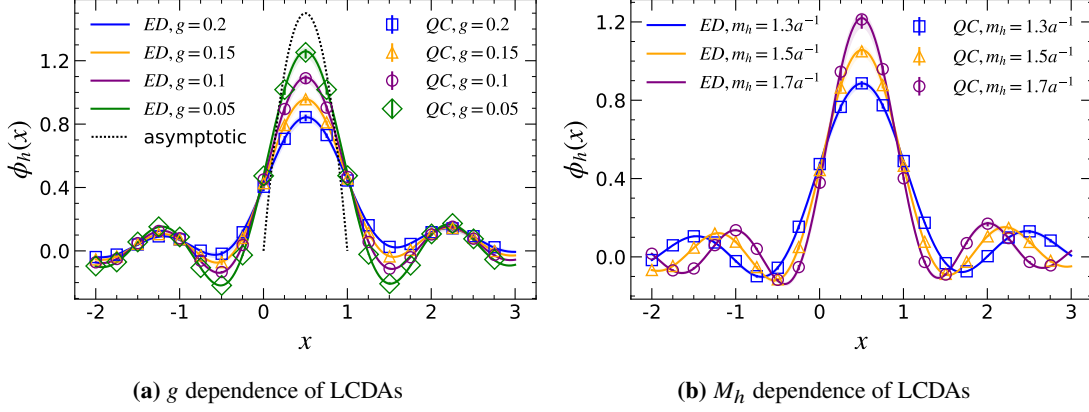
### 2.4 Results of PDFs in NJL model

With the help of Python packages Quspin [10] and ProjectQ [11], we run the quantum circuit on a classical device due to the limitations of using real quantum devices. We calculate the lowest-lying  $ud$ -like hadron mass in two flavors NJL model to test the performance of our VQE algorithm. The result of hadron mass obtained by VQE and numerical exact diagonalization are denoted as  $M_{h,\text{QC}}$  and  $M_{h,\text{NUM}}$ , which are shown in Tab. (1). The agreement between the QC and NUM results shows the good accuracy of our VQE method.

$g$	0.2	0.4	0.6	0.8	1.0
$M_{h,\text{QC}}a$	1.002	1.810	2.674	3.534	4.352
$M_{h,\text{NUM}}a$	1.001	1.801	2.659	3.509	4.342

Table 1: Calculated hadron mass as a function of  $g$  with bare mass  $ma = 0.2$  and number of qubits  $N = 12$ .

After making sure of the effectiveness of the VQE algorithm, we calculate the PDFs of quark anti-quark  $q\bar{q}$  bound state in single flavor NJL model, with  $ma = 0.8$ ,  $N = 18$ . The results of  $D(z)$  in shown in Fig. (2a). The vanishing real part of  $D(z)$  is consistent with the symmetry of PDFs  $f_q(x) = f_{\bar{q}}(x) = -f_q(-x)$  of  $q\bar{q}$  state. The suppression of  $D(z)$  at large  $z$  region shows the bound state behavior of the hadron state. Finally, the PDF  $f_q(x)$  is shown in Fig. (2b). Good agreement between quantum computing and exact diagonalization results show the correctness of our quantum algorithm. We also find that our result is in qualitative agreement with the pion PDFs obtained by QCD global fitting method [12].



**Figure 4:** PDFs of single flavor NJL model, with  $ma = 0.8$ ,  $N = 18$ . ED are the exact diagonalization results while the QC are the quantum computing results.

### 3. Quantum computing for light-cone distribution amplitude

According to the QCD factorization, the form factor  $F(Q^2)$  in the process  $\gamma^* + \gamma \rightarrow \pi^0$  can be factorized into hard coefficients and LCDAs  $\phi_\pi(x)$

$$\phi_\pi(x) = \frac{1}{f_\pi} \int dz e^{-i(x-1)(n \cdot P)z} \langle \Omega | \bar{\psi}(zn)(n \cdot \gamma) W(zn, 0) \psi(0) | p \rangle, \quad (5)$$

where  $f_\pi$  is decay constant of pion and  $|\Omega\rangle$  is the vacuum state. Eq. (5) is the operator definition of pion LCDAs. Fig. (3) shows the quantum circuit for simulating LCDAs (see [2] for more detail).  $U(\theta_\Omega^*)$  and  $U(\theta_h^*)$  in the circuit can prepare vacuum and hadron states from their reference states. We run this quantum circuit on a classical simulator of quantum devices then we obtain the results of LCDAs in Fig. (4). As we expected, good agreement between quantum computing and exact diagonalization was obtained, and reasonable dependence on coupling  $g$  and hadron mass are presented.

### 4. Summary and outlook

In this work, we first present quantum algorithms for computing “PDFs” and “LCDAs” through light-front correlators. As a proof of concept study, we use our quantum algorithm to simulate PDFs and LCDAs in the NJL model. The field is still at the early age of development, so more things need to be done in the future, such as the qubit encoding of gauge field and the extension to higher dimensions for TMDs and spin-dependent processes.

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