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Scaling properties of elastic pp cross-section

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We show that the elastic differential pp cross-section has a unique universal property that the ratio of bump-to-dip position is constant from the energies of the ISR to the LHC. We explore this property to compare Geometric Scaling present at the ISR with the recently proposed scaling law at the LHC. We argue that at the LHC, within present experimental uncertainties, there is fact a family of scaling laws.

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1. Introduction

In this report we discuss scaling properties of the differential elastic pp cross-section at low and high energies following Refs. [1, 2]. Differential pp cross-sections have very characteristic structure. One observes a rapid decrease for small $|t|$, then a minimum at $t_{\rm d}$, called a *dip*, followed by a broad maximum at $t_{\rm b}$, dubbed as a *bump*. It turns out that the bump-to-dip cross-section ratio

$$\mathcal{R}_{\rm bd}(s) = \frac{d\sigma_{\rm el}/d|t|_{\rm b}}{d\sigma_{\rm el}/d|t|_{\rm d}}\,, \tag{1}$$

which seems to saturate at LHC energies at a value of approximately 1.8, is rather strongly energy dependent at the ISR (see e.g. Fig. 2 in Ref. [3]).

In Ref. [2] we explored an interesting regularity of $d\sigma_{\rm el}/d|t|$, which, to the best of our knowledge, has not been used in phenomenological studies of the pp elastic scattering. It turns out that the ratio of bump-to-dip *positions* in $|t|$

$$\mathcal{T}_{\rm bd}(s) = |t_{\rm b}|/|t_{\rm d}|\,, \tag{2}$$

is constant, within the experimental uncertainties, at all energies from the ISR to the LHC and equal approximately to 1.355 [2]. It is striking and unexpected that this ratio is constant over such large range of energies.

In the following, w discuss first Geometric Scaling (GS) at the ISR, and then new scaling laws at the LHC. For details see Refs. [1, 2].

2. Geometric Scaling at the ISR

Fifty years ago, in 1973-74, Jorge Dias de Deus proposed and developed the idea of GS in elastic pp scattering [4, 5]. It was based on a phenomenological observation that elastic, inelastic and total cross-sections have (almost) the same energy dependence over the ISR energy span, see Tab. 1.

	elastic	inelastic	total	$\frac{\text{elastic}}{\text{inelastic}}$	ρ
ISR	$W^{0.1142\pm0.0034}$	$W^{0.1099\pm0.0012}$	$W^{0.1098\pm0.0012}$	$W^{0.0043\pm0.0036}$	$0.02-0.095$
LHC	$W^{0.2279\pm0.0228}$	$W^{0.1465\pm0.0133}$	$W^{0.1729\pm0.0163}$	$W^{0.0814\pm0.0264}$	$0.15-0.10$

Table 1: Energy dependence of the integrated cross-sections for the energies $W=\sqrt{s}$ at the ISR [6] and at the LHC [7] and the ρ parameter [6, 8]. Fits from [2].

Elastic cross-sections can be parametrized in terms of the opacity $\Omega(s,b)$ and phase $\chi(b,s)$, which are both real functions of the impact parameter and scattering energy [9, 10]

$$\sigma_{\rm el} = \int d^2\boldsymbol{b}\,\left|1-e^{-\Omega(s,b)+i\chi(s,b)}\right|^2, \qquad \sigma_{\rm inel} = \int d^2\boldsymbol{b}\,\left[1-\left|e^{-\Omega(s,b)}\right|^2\right] \tag{3}$$

and

$$\sigma_{\rm tot} = 2\int d^2\boldsymbol{b}\;{\rm Re}\left[1-e^{-\Omega(s,b)+i\chi(s,b)}\right]. \tag{4}$$

Since the real part of the scattering amplitude (related to the ρ parameter) is small, one can safely neglect χ. GS is a hypothesis that $\Omega(s,b) = \Omega\left(b/R(s)\right)$ where $R(s)$ is the interaction radius [4] increasing with energy. Changing the integration variable from $\boldsymbol{b} \to \boldsymbol{B} = \boldsymbol{b}/R(s)$, one obtains that

$$\sigma_{\text{inel}} = R^2(s) \int d^2\boldsymbol{B} \left[1 - \left|e^{-\Omega(B)}\right|^2\right], \tag{5}$$

where the integral in (5) is an energy independent constant. If we neglect the phase $\chi(s,b)$, both elastic and total cross-sections should scale the same way, which means that their ratios should be energy independent. As can be seen from Table 1, this is indeed the case.

GS of total cross-sections has important consequences for the differential cross-section:

$$\frac{d\sigma_{\text{el}}}{d|t|} \sim \left|\int_0^\infty db^2 A_{\text{el}}(b^2,s)\, J_0\left(b\sqrt{|t|}\right)\right|^2 = \sigma^2_{\text{inel}}(s) \left|\int_0^\infty dB^2 A_{\text{el}}(B^2)\, J_0\left(B\sqrt{\tau}\right)\right|^2 \tag{6}$$

where $A_{\text{el}}(b^2,s)$ is the elastic scattering amplitude and J_0 denotes the Bessel function originating from the Fourier transform. Equation (6) implies that the scaled cross-section

$$\frac{1}{\sigma^2_{\text{inel}}(s)} \frac{d\sigma_{\text{el}}}{d|t|}(s,t) = \Phi(\tau) \tag{7}$$

should be a universal, energy independent function of the scaling variable τ:

$$\tau = R^2(s)|t| \times \text{const.} = \sigma_{\text{inel}}(s)\,|t|\,. \tag{8}$$

GS at the ISR was confirmed in Ref. [5], except for the dip region [11], see also [2].

3. Scaling at the LHC

At the LHC elastic, inelastic and total pp cross-sections have different energy dependence, see Tab. 1, and therefore no GS described in Sect. 2 is expected. Nevertheless, the fact that both (2) and (1) saturate at the LHC energies, suggests that the transformation

$$t \to \tau = f(s)t \tag{9}$$

should align dips and bumps of different energies, and rescaling

$$\frac{d\sigma_{\text{el}}}{dt}(t) \to g(s)\frac{d\sigma_{\text{el}}}{dt}(\tau) \tag{10}$$

should superimpose the cross-section values, at least in the dip and bump regions. At the ISR $g \sim 1/f^2$, but at the LHC both functions f and g seem to be independent [1, 2].

In Ref. [2] we have analyzed TOTEM data [8, 12–15] summarized in [7] with the following result:

$$f(s = W^2) = W^{\beta},\ \ \beta = 0.1686 \quad \text{and} \quad g(s = W^2) = W^{-\alpha},\ \ \alpha \simeq 0.66. \tag{11}$$

The result is plotted in Fig. 1 where the cross-section scaling is clearly seen within the experimental uncertainties.

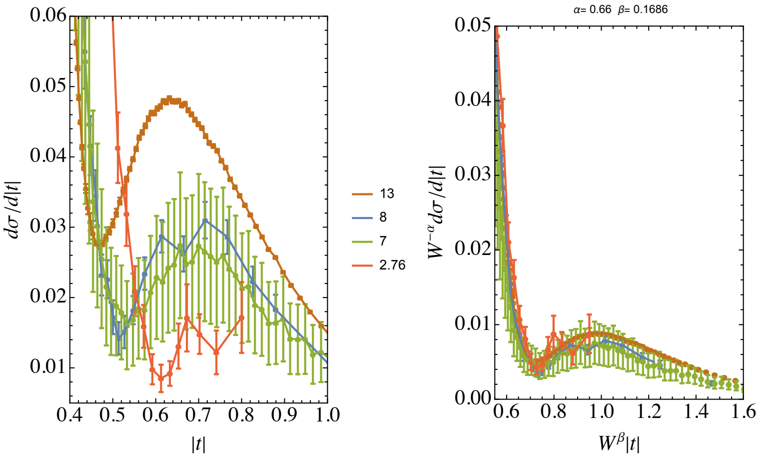

Figure 1: Left: elastic pp cross-section $d\sigma_{\text{el}}/dt$ [mb/GeV^2] at the LHC energies in terms of $|t|$ [GeV^2] in the dip and bump region. Right: scaled cross-section in tems scaling variable $\tau = W^{\beta}\,|t|$. One can see that the cross-sections at different energies are aligned after scaling.

In Ref. [1] a more general scaling variable was proposed

$$\tilde{\tau} = s^a t^b \tag{12}$$

with the result and $a \simeq 0.065$, $b \simeq 0.72$. Aligning dips and bumps alone (rather than the entire cross-sections) requires the value of β given in (11). This imposes a constraint

$$a - b\,\beta/2 = 0, \tag{13}$$

which is roughly satisfied by a and b of Ref. [1]. The assumption of energy independence of $\mathcal{T}_{\text{bd}}$ (2) leads to a family of scaling laws (12) constrained by (13) where the determination of a (or equivalently of b) must follow from the alignment of the points outside of the immediate vicinity of the dip and bump regions. The quality of the present LHC data does not allow for a precise determination of a and b.

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