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Describing the incoherent exclusive diffraction t-spectrum with hotspot evolution

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We present a model of proton geometry where the number and size of gluon density hotspots in the proton's thickness function evolves with the resolution scale of the event, given by the Mandelstam t variable in exclusive diffractive ep collisions. We use the impact-parameter dependent saturation dipole model bSat (IPsat), as well as its linearised (non-saturated) version bNonSat. In the latter the proton thickness has a clear interpretation as a thickness and in the former it is directly related to the saturation scale. The resulting phenomenological model for the splitting of hotspots, making full use of earlier experimental and phenomenological studies, is able to describe the entire incoherent t-spectrum for $1.1 < |t| < 25$ GeV^2 with a single phenomenological parameter. As such, this is a useful model for investigating saturation physics at large t at future colliders such as the EIC, LHeC, or μIC for both proton and nuclear targets.

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Exclusive diffractive processes in Deep Inelastic Scattering (DIS) are the only processes which offer direct access to the Mandelstam t-spectrum in both ep and eA collisions. This gives us access to the transverse spatial gluon structure of the proton or nucleus, via a Fourier transform from $\Delta = \sqrt{-t}$ to impact parameter b. Furthermore, as diffraction at small values of the Bjorken x variable is mediated by (at least) two gluons in the perturbative regime, these processes are also an excellent probe for the saturated gluon regime. At small x the dipole picture becomes relevant, where the virtual photon fluctuates into a quark-antiquark pair, which interacts with the target after which the dipole recombines into a vector meson final state. In the dipole model we can model the b-dependence of the target's gluonic wavefunction as a thickness function $T(\vec{b})$. However, it is only at leading twist, which is relevant outside of the saturated regime, that the thickness function can be interpreted as the target's thickness at a given impact parameter. In the all twist saturation model it may instead be interpreted in terms of the saturation scale such that $Q_S(x, \vec{b}, \vec{r}) \sim T(\vec{b})$.

At large $|t|$ the spectrum is dominated by incoherent scattering, where the target gets excited in the interaction with the dipole and subsequently fragments. In the Good-Walker picture, the incoherent cross-section is given by the fluctuations in the gluon wave-function. These fluctuations are probed at an area resolution given by $1/t$, which means that the large $|t|$ part of the spectrum is sensitive to small length scale fluctuation, and vice versa.

In order to model proton dissociation at HERA, Mäntysaari and Schenke (MS)[1, 2] developed a hotspot model of the proton's wave-function. In this model, the proton's wavefunction is concentrated in three hotspots whose positions fluctuate event by event. While successful, the model is firmly non-perturbative and thus cannot be applied for $|t| \gtrsim 1\text{GeV}^2$. In an attempt to understand what kind of thickness function may be able to describe the large part of the t-spectrum, we extended the hotspot-model by adding hotspots within hotspots [3]. We found that a self-similar pattern emerged, and that the entire spectrum could be well described up to $|t| \leq 25\text{GeV}^2$. While this approach gave insights into the proton's thickness function it required many, albeit highly correlated, parameters to describe the measurements.

In this paper, we present a hotspot evolution model which takes the MS hotspots model as an initial state and evolves it by letting the hotspots split into smaller hotspot based on the resolution scale. The resulting model only has one extra parameter and is able to describe the entire t-spectrum.

1. The Colour Dipole Model

In the colour dipole model, the amplitude is expressed as a convolution of the three processes: the splitting of the virtual photon into a quark-antiquark dipole, the interaction between the dipole and the target, and the recombination of the dipole into the final state vector meson [4]:

$$\mathcal{A}_{T,L}^{\gamma^* p \to J/\Psi p}(x_{I\!P}, Q^2, \mathbf{\Delta}) = i \int \mathrm{d}^2\mathbf{r} \int \mathrm{d}^2\mathbf{b} \int \frac{\mathrm{d}z}{4\pi} (\Psi^* \Psi_V)_{T,L}(Q^2, \mathbf{r}, z) e^{-i[\mathbf{b} - (\frac{1}{2} - \mathbf{z})\mathbf{r}].\mathbf{\Delta}} \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, x_{I\!P})$$

where T and L denote the polarizations of the virtual photon, $\mathbf{r}$ is the size of the dipole, and $\mathbf{b}$ is the impact parameter. The variable z is the momentum fraction taken by the quark. The term $(\Psi^* \Psi_V)$ is the wave-function overlap between the virtual photon and the produced vector meson. We use the boosted Gaussian wave function for J/ψ with parameter values from [5]. The dipole cross section, $\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, x_{I\!P})$, characterizes the strong interaction between the dipole.

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We consider here the saturated bSat, and non-saturated bNonSat versions of the dipole cross-section. The bSat model is given by: [4]:

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}}(\mathbf{b},\mathbf{r},x_{I\!P}) = 2\left[1-e^{-\Omega(x_{I\!P},\mathbf{t},\mathbf{b})}\right] = 2\left[1-e^{-\frac{\pi^2}{2N_C}\mathbf{r}^2\alpha_s(\mu^2)x_{I\!P}g(x_{I\!P},\mu^2)T_p(\mathbf{b})}\right] \tag{1}$$

where Ω is the opacity. This defines the saturation scale $Q_S(r, x_{I\!P}, \mathbf{b}) = 2/r_S(r, x_{I\!P}, \mathbf{b})$ where r_S is the value of r which solves $\Omega(x_{I\!P}, \mathbf{t}, \mathbf{b}) = 1/2$. Note that $Q_S^2 \sim T_p(b)$. The strong coupling α_s and gluon density are evaluated at $\mu^2 = \mu_0^2 + \frac{C}{r^2}$ and the gluon density at the initial scale μ_0 is parametrised as $xg(x, \mu_0^2) = A_g x^{-\lambda_g}(1-x)^6$, where the parameters A_g, λ_g, C, and m_f are determined through fits to inclusive DIS reduced cross section measurements [6].

The linearised dipole cross section, the bNonSat model, is given by:

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}}(\mathbf{b},\mathbf{r},x_{I\!P}) = \frac{\pi^2}{N_C}\mathbf{r}^2\alpha_s(\mu^2)x_{I\!P}g(x_{I\!P},\mu^2)T_p(\mathbf{b}) \tag{2}$$

which does not include any saturation effects and gives only the leading twist contribution. Currently, both models effectively describe HERA F_2 and exclusive data [5, 8], while the studies of exclusive vector meson production in ultra-peripheral collisions of lead nuclei at the LHC exhibit a clear preference for the bSat model [6]. The exclusive cross section receives large corrections from the real part of the amplitude and from the skewedness of the two-gluon exchange (discussed in detail in [2, 7]).

In the Good-Walker picture, the incoherent cross-section is given by:

$$\frac{\mathrm{d}\sigma_{\text{incoherent}}}{\mathrm{d}t} = \frac{1}{16\pi}\left(\left\langle\left|\mathcal{A}(x_{\mathbb{P}},Q^2,\Delta)\right|^2\right\rangle_{\hat{\Omega}} - \left|\left\langle\mathcal{A}(x_{\mathbb{P}},Q^2,\Delta)\right\rangle_{\hat{\Omega}}\right|^2\right) \tag{3}$$

where $\hat{\Omega}$ denotes an initial state gluon configuration.

The Hotspot model

In the hotspot model, the proton is characterized by a lumpy configuration consisting of hotspots of gluonic density at small-x. The gluon density within these hotspots is assumed to be isotropic and smooth, implying a single relevant scale corresponding to the hotspot width for incoherent diffraction. In the independent scattering approximation, the transverse profile of the proton in the hotspot model is given by [2]:

$$T_p(\mathbf{b}) = \frac{1}{N_{hs}}\sum_{i=1}^{N_{hs}} T_{hs}(\mathbf{b}\text{-}\mathbf{b}_i), \quad \text{with} \quad T_{hs}(\mathbf{b}) = \frac{1}{2\pi B_{hs}}\exp\left[-\frac{\mathbf{b}^2}{2B_{hs}}\right] \tag{4}$$

Here, N_{hs} is set to 3, and $\mathbf{b}_i$ represents the locations of hotspots, sampled from a normalised Gaussian of width B_{qc}. The width of the hotspots is B_{hs}. The parameters B_{qc} and B_{hs} govern the degree of fluctuations in the proton wave function, and their values are constrained by both coherent and incoherent data. It should be noted that since the saturation scale $Q_S^2 \sim T(\mathbf{b})$, the saturation scale is larger in the hotspots, and that for some configurations of hotspots we may get a rather large saturation scale compared to what is possible for a spherical proton.

2. The Hotspot Evolution model

A measurement at a certain Mandelstam variable t at small x constitute a measurement of the combined gluonic wave function at an areal resolution $1/|t|$. Therefore, the measurement causes the gluon wave function to collapse into an area $B \sim 1/|t|$. At larger $|t|$ we would therefore measure a smaller area of the gluon wave-function. This manifests as a splitting of a hotspot existing at one $|t|$ into two hotspots at larger $|t|$. In order to capture this resolution picture of the proton's gluonic substructure, we suggest the following hotspot evolution process: We let the MS hotspot model act as an initial state for the evolution at $|t| \leq |t_0|$, then we evolve the initial state by probabilistically splitting the hotspots based on the resolution for $|t| > |t_0|$. For this one needs to define a splitting function. We know from the self-similarity found in [3] that the hotspot structure should scale with $\ln|t|$. We are using the following hotspot evolution model:

$$\frac{\mathrm{d}\mathcal{P}_{\text{split}}}{\mathrm{d}t} = \frac{\alpha}{|t|}\frac{t-t_0}{t}, \quad \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t} = \frac{\alpha}{t}\frac{t-t_0}{t}\exp\left[-\alpha\left(\frac{t_0}{t} - \ln\frac{t_0}{t} - 1\right)\right] \tag{5}$$

where t_0 is the initial scale and α is the evolution parameter which is determined by fits to the measured t-spectrum. Here, the splitting probability at t is multiplied by the probability of having no splittings between t_0 and t. The algorithm for the evolution of the initial state is as follows: 1. Offspring hotspots i and j are created at distance $d_{ij} = |\mathbf{b}_i - \mathbf{b}_j|$ sampled from the parent hotspot with widths $B_{i,j} = \frac{1}{|t|}$ GeV^{-2}, where the t value is sampled from eq.(5). 2. We impose probe and geometry resolution criteria by demanding that $d_{ij} > 2\sqrt{B_{i,j}}$. If this is not the case, the two offspring hotspots cannot be resolved by the probe and therefore, the splitting is rejected. This results in an effective hotspot repulsion. The total thickness function of the proton is calculated as:

$$T_p(\mathbf{b}) = \frac{1}{N_{hs}}\sum_{i=1}^{N_{hs}} T_i(\mathbf{b}\text{-}\mathbf{b}_i) \tag{6}$$

where N_{hs} is the total number of resolved hotspots at that scale. We make sure that the total normalisation of this thickness function is conserved throughout the evolution. In the limit of large momentum transfers our model approaches the the color glass condensate description of the proton initial state where its structure is characterized by point-like color charges superimposed on the geometric hotspot structure [9]. The hotspot evolution adds a transition from the large hotspots to the limit of pointlike sources. To compute cross sections, we generate 800 proton configurations from our model. This model contains several different kinds of event-by-event fluctuations, namely hotspot width, hotspot number, and normalization fluctuations. For the bSat model we use a modified profile for hotspots which was proposed in [3], in order to retain the coherent cross section while introducing fluctuations.

3. Results

In fig. 1 we show the resulting hotspot evolution models, both for bSat and bNonSat, compared to the measured t-spectrum at for J/ψ photoproduction at HERA. We let $|t_0| = 1.1\text{GeV}^2$, and have checked that the result is not sensitive to this choice for $0.8 < |t| < 1.2\text{GeV}^2$. We use $\alpha = 18.5$. We see that there is no discernible saturation effects in the HERA data.

4. Discussion and Conclusions

We have presented a model for the proton geometry which computes the incoherent cross section for vector meson production at large $|t|$. The resulting model is consistent with previous observations that the ep t-spectrum can be described with a self-similar gluon wave-function of the proton[3], and that the small x partons are maximally entangled[12, 13]. It has also been observed that extra sources of fluctuations are needed to describe the exclusive diffractive data [14]. Our model naturally introduces several such sources: In the number of hotspots, and in their sizes and normalisations. As the thickness function is directly related to the saturation scale in the bSat model, this naturally leads to event-by-event fluctuations in the saturation scale. The model only has one extra parameter α for describing the entire spectrum $1 < |t| < 25\text{GeV}^2$. The only further assumptions is that of the Gaussian shape of hotspots. It would be desirable as a future project to calculate these splitting functions as well as the hotspot shapes from first principle, which may be possible since the evolution is within the perturbative realm of QCD. Such a first principle evolution model would then be used to restrict the parameters of the initial state hotspot model, such as the number of hotspots, and their widths and shapes.

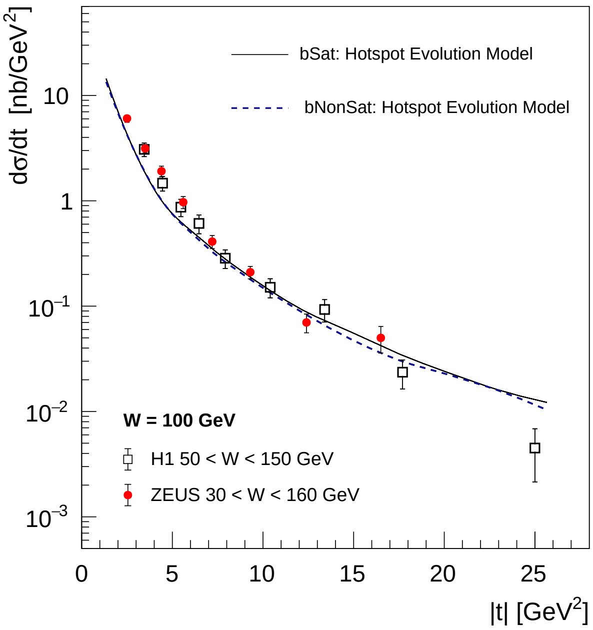

Figure 1: The resulting t-spectrum from hotspot evolution in the bSat and bNonSat models, compared to measurements from the H1[10] and ZEUS [11] experiments.

We noted that there are no discernible difference between the saturated and non-saturated dipole models in their description of the measured data. This situation is expected to improve in the eA collisions planned at the electron-ion collider (EIC) [15, 16]. The EIC has the potential to measure the large $|t|$ spectrum in ePb collisions. It will be an important future study to extend these results to eA collisions at the EIC and as well as the LHeC [17] and a future Muon Ion Collider [18, 19]. An important check of the universality of our (as well as any other) approach is to confront it with data at different Q^2 as well as different final states, such as ρ-, ϕ-, and Υ-mesons. Currently there are no measurements of these processes at large $|t|$, which is also a situation that the EIC will remedy.

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