Forward η_Q production impact factor at NLO

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The High-Energy Factorisation (HEF) coefficient function or impact factor for the hadroproduction of the state $^1S_0^{[1]}$ of a heavy quark-antiquark pair ($Q\bar{Q}$) at forward rapidities is computed at NLO in α_s. This impact factor is an integral part of the resummation procedure for higher-order QCD corrections to the hadroproduction cross sections of η_c or η_b mesons at forward rapidities in collinear factorisation (CF), enhanced by logarithms of partonic center of mass energy. The NLO computation results of which are presented in this contribution, is an important step towards an extension of aforementioned resummation beyond the Leading Logarithmic Approximation(LLA).

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1. Introduction

Recently, the importance of resummation of higher-order QCD corrections enhanced by logarithms of partonic center-of-mass energy ($\hat{s}$) in the inclusive heavy-quarkonium hadroproduction [1] and photoproduction [2] computations in CF had been pointed out. The goal of the present contribution is to report the result of the computation of the NLO coefficient function (or impact factor) of HEF, which is necessary to extend such computations beyond Doubly-Logarithmic Approximation (DLA), used in Refs. [1, 2]. Due to the lack of space in this proceedings, only the final result of the computation will be presented, postponing the discussion of it's derivation and numerical results obtained with it for a more complete publication. To put this computation in context, one considers the quarkonium transverse momentum ($\mathbf{p}_T$) and rapidity (y)-differential cross section of the process $p(P_1) + p(P_2) \to Q\bar{Q}[^1S_0^{[1]}](p) + X$, where $P_1^\mu = \sqrt{S}n_-^\mu/2$, $P_2^\mu = \sqrt{S}n_+^\mu/2$ (with Sudakov vectors $n_\pm^\mu = (1, 0, 0, \mp 1)^\mu$ in the pp center-of-momentum frame and $p^2 = M^2$), which in CF can be written as:

$$\frac{d\sigma}{dyd\mathbf{p}_T^2} = \frac{M^2}{S} \int\limits_0^{\eta_{\max}} d\eta \int\limits_0^1 dz \, f_i(x_1, \mu_F) f_j(x_2, \mu_F) \frac{d\hat{\sigma}_{ij}(\eta, z, \mathbf{p}_T^2)}{dzd\mathbf{p}_T^2}, \tag{1}$$

where $x_1 = \frac{M_T e^y}{\sqrt{S}z}$, $x_2 = \frac{M^2 z(1+\eta)}{M_T\sqrt{S}}e^{-y}$, $M_T = \sqrt{M^2 + \mathbf{p}_T^2}$, $f_i(x_{1,2}, \mu_F)$ are collinear PDFs of a parton $i = q, \bar{q}, g$ with four-momentum $x_{1,2}P_{1,2}^\mu$ and factorisation scale μ_F, $z = (n_+ \cdot p)/(x_1 n_+ \cdot P_1) = p_+/(x_1 P_1^+)$, $\eta = (\hat{s} - M^2)/M^2$ with $\hat{s} = x_1 x_2 S$ being the squared center-of-mass energy in the partonic subprocess of CF, $d\hat{\sigma}_{ij}$ is the partonic coefficient function of a said subprocess and $\eta_{\max} = S/M^2 - 1$. With the aim to resum LL ($\propto \alpha_s^n \ln^{n-1}(1+\eta)$) and NLL ($\propto \alpha_s^{n+1} \ln^{n-1}(1+\eta)$) corrections to the $d\hat{\sigma}$ in Eq. (1) at $\eta \gg 1$, one writes the following HEF-resummation formula in the gluon channel[1]:

$$\frac{d\hat{\sigma}_{ig}^{(\text{HEF, NLLA})}}{dzd\mathbf{p}_T^2} = \frac{1}{2M^2} \int \frac{d^2\mathbf{q}_T}{\pi} C_{ig}\left(1 + \eta, \mathbf{q}_T^2, \mu_F, \mu_R\right)$$

$$\times \left[\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)\delta(1-z)\delta(\mathbf{q}_T^2 - \mathbf{p}_T^2) + \frac{\alpha_s(\mu_R)}{2\pi}\mathcal{H}_{gg}^{(\text{NLO})}(\mathbf{q}_T, z, \mathbf{p}_T)\right], \tag{2}$$

where the resummation factor C_{ig} in the LLA is defined e.g. in [1] and should be extended with NLLA corrections to obtain the complete NLLA result for the $d\hat{\sigma}$. The LO HEF coefficient function is: $\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T) = \frac{32\pi^3\alpha_s^2}{N_c^2(N_c^2-1)} \frac{\left\langle O\left[^1S_0^{[1]}\right]\right\rangle}{M^3} \frac{M^4}{(M^2+\mathbf{p}_T^2)^2}$, with the Long-Distance Matrix Element of Non-Relativistic QCD factorisation [3] being $\left\langle O\left[^1S_0^{[1]}\right]\right\rangle = 2N_c|R(0)|^2/(4\pi)$. The NLO correction to the HEF coefficient function in Eq. (2) in the gluon channel can be decomposed as follows:

$$\mathcal{H}_{gg}^{(\text{NLO})}(\mathbf{q}_T, z, \mathbf{p}_T) = \mathcal{H}_{gg}^{(\text{real, fin.})}(\mathbf{q}_T, z, \mathbf{p}_T) + \mathcal{H}_{gg}^{(\text{NLO, analyt.})}(\mathbf{q}_T, z, \mathbf{p}_T). \tag{3}$$

2. The real-emission contribution

The real-emission contribution to the NLO impact factor (3) in the gluon channel comes from the process $g(x_1P_1) + R_-(\mathbf{q}_T) \to Q\bar{Q}[^1S_0^{[1]}](p) + g(k)$, where R_- is the Reggeised gluon, the

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[1]Results for $\hat{\sigma}_{iq}$ will be presented elswere.

exact matrix element of this process can be written as:

$$\tilde{H}_{Rg} = \frac{C_A \left(M^2 + \mathbf{p}_T^2\right)^2}{s^2 t^2 u^2 \left(M^2 - s\right)^2 \left(M^2 - t + t_1\right)^2 \left(M^2 - u\right)^2} \sum_{n=-1}^{3} (1-z)^n w_n(s,t,u). \qquad (4)$$

where $t_1 = \mathbf{q}_T^2$, Mandelstam variables $s = M^2\rho - \mathbf{q}_T^2$, $t = -[M^2(\rho z - 1) - \mathbf{p}_T^2]/z$, $u = -[M^2(1 - z) + \mathbf{p}_T^2]/z$, $\rho = \frac{(\mathbf{p}_T - \mathbf{q}_T)^2}{M^2(1-z)} + \frac{M^2 + \mathbf{p}_T^2}{M^2 z}$, and coefficients:

$$w_{-1} = s t t_1 u \left(t + t_1 + u\right)\left(s + 2t_1 + u\right)\left(s^2[t_1 - u][t + t_1 + u] + t[t + t_1][t_1 + u][2t_1 + u]\right.$$
$$\left. + s[t_1^3 - t_1 u^2 + t^2(t_1 + u) + 2t t_1(2t_1 + u)]\right), \qquad (5)$$

$$w_0 = -u\left\{-s^5 t t_1\left(t + t_1 + u\right)^2 - t^2 u\left(t + t_1\right)^2\left(t + t_1 + u\right)^2\left(2t_1 + u\right)^2 - s^4\left[t + t_1 + u\right]\left[-u(t_1 + u)^3\right.\right.$$
$$\left.-u\right] + t(t_1 + u)^2[2t_1 + t^3[5t_1 + u] + t^2[t_1 + u][10t_1 + u]] - st\left[t + t_1\right]\left[t + t_1 + u\right]\left[2t_1 + u\right]$$
$$\times\left[2t^3 u + t_1^2 u[t_1 + u] + tu[2t_1 + u][5t_1 + 2u] + 2t^2[t_1 + u][4t_1 + 3u]\right]$$
$$+ s^2\left[t + t_1 + u\right]\left[-t^5 u + t_1^2 u(t_1 + u)^3 + t t_1[t_1 + u][2t_1 + u][t_1^2 - t_1 u + 2u^2] - t^4[16t_1^2 + 21t_1 u + 7u^2]\right.$$
$$\left.- t^3[2t_1 + u][22t_1^2 + 29t_1 u + 8u^2] - 2t^2 t_1[5t_1^3 + 24t_1^2 u + 19t_1 u^2 + 4u^3]\right]$$
$$+ s^3\left[2t_1 u(t_1 + u)^4 - 2t^5[2t_1 + u] - 2t^2 t_1[t_1 + u][10t_1^2 + 22t_1 u + 7u^2] - t^4[32t_1^2 + 33t_1 u + 8u^2]\right.$$
$$\left.\left.+ t(t_1 + u)^2[t_1^3 + 2t_1^2 u + 7t_1 u^2 + 2u^3] - t^3[49t_1^3 + 101t_1^2 u + 54t_1 u^2 + 8u^3]\right]\right\}, \qquad (6)$$

$$w_1 = su\left[s + t + t_1\right]\left[t + t_1 + u\right]\left[s^3\left(t + u\right)\left(2t^2 + t t_1 - 2(t_1 + u)^2\right) + t\left(2t_1 + u\right)\left(t t_1 u[-4t_1 + u]\right.\right.$$
$$+ 2t^3[2t_1 + u] - t_1^2 u[t_1 + 2u] + 2t^2[-4t_1^2 + t_1 u + u^2]) + s^2\left(2t^4 - 4t_1 u(t_1 + u)^2 + 2t^3[3t_1 + 5u]\right.$$
$$\left.+ t^2[-2t_1^2 + t_1 u + 6u^2] - t[5t_1^3 + 14t_1^2 u + 14t_1 u^2 + 2u^3]\right)$$
$$+ s\left(-2t_1^2 u(t_1 + u)^2 + 4t^4[2t_1 + u] + t^3[-4t_1^2 + 20t_1 u + 10u^2] + t^2[-8t_1^3 - 18t_1^2 u + t_1 u^2 + 2u^3]\right.$$
$$\left.\left.- t t_1[2t_1^3 + 10t_1^2 u + 14t_1 u^2 + 3u^3]\right)\right] \qquad (7)$$

$$w_2 = s\left\{s^5\left(t + t_1 + u\right)^2\left[t - u\right]\left[t + u\right] + s^4\left[t + t_1 + u\right]\left[-2u^3[t + 2t_1] + 2t^2[t + t_1][t + 3t_1]\right.\right.$$
$$+ u^2[2t^2 - 5t t_1 - 4t_1^2] + tu[6t^2 + 9t t_1 + t_1^2]] + s^2\left[t\, t_1 u^5 + 4t^2 t_1(t + t_1)^3[t + 3t_1]\right.$$
$$+ 2u^4[t^3 + 6t^2 t_1 - 2t_1^3] + 2u^3[4t^4 + 27t^3 t_1 + 29t^2 t_1^2 - 6t t_1^3 - 4t_1^4]$$
$$+ tu[t + t_1][2t^4 + 32t^3 t_1 + 90t^2 t_1^2 + 51t t_1^3 + 5t_1^4] + u^2[8t^5 + 73t^4 t_1 + 163t^3 t_1^2 + 84t^2 t_1^3 - 6t t_1^4 - 4t_1^5]\right]$$
$$+ s\left[4t^2 t_1^2(t + t_1)^4 + 2t t_1 u^5[t + t_1] + u^4[t^4 + 17t^3 t_1 + 24t^2 t_1^2 + 4t t_1^3 - t_1^4]\right.$$
$$+ 2t\, t_1 u[t + t_1][2t^4 + 20t^3 t_1 + 36t^2 t_1^2 + 11t t_1^3 + t_1^4] + u^3[2t^5 + 33t^4 t_1 + 101t^3 t_1^2 + 64t^2 t_1^3 - 4t t_1^4 - 2t_1^5]$$
$$ \qquad (8)$$

$$+u^2[t^6 + 22t^5t_1 + 117t^4t_1^2 + 172t^3\,t_1^3 + 57t^2t_1^4 - 4tt_1^5 - t_1^6]\big] + tt_1u\Big[t + t_1\Big]\Big[t + t_1 + u\Big]\Big[2\,t_1 + u\Big]$$

$$\times\Big[t_1u[-t_1 + u] + 4t^2[2t_1 + u] + tu[6t_1 + u]\Big] + s^3\Big[t^6 - 6t_1^2u^2(t_1\ + u)^2 + 4t^5[3t_1 + 2u]$$

$$+2tt_1u[t_1 + u][2t_1^2 - 7t_1u - 2u^2] + t^4[34\,t_1^2 + 53t_1u + 15u^2] + t^3[36t_1^3 + 87t_1^2u + 61t_1u^2 + 8u^3]$$

$$+t^2t_1[13t_1^3 + 46t_1^2\,u + 41t_1u^2 + 14u^3]\Big]\Big\}, \tag{9}$$

$$w_3 = stt_1u\Big(s + t + t_1\Big)\Big(s + 2t_1 + u\Big)\,\Big(s^2[t - u][t + t_1 + u] + t_1[t + t_1 + u][2tt_1 + u(t + t_1)]$$

$$+st[(3\,t_1 - u)(t_1 + u) + t(3t_1 + u)]\Big). \tag{10}$$

The integral of the Eq. (4) over phase space of emitted gluon ($g(k)$) is divergent at $\mathbf{k}_T \to 0$ due to soft and collinear singularities and at $z \to 1$ due to the rapidity divergence. In the contribution $\mathcal{H}_{gg}^{(\text{real, fin.})}$ to the NLO impact factor (3):

$$\mathcal{H}_{gg}^{(\text{real, fin.})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int\limits_0^{2\pi} \frac{d\phi}{2\pi}\left[\frac{\tilde{H}_{Rg}(\mathbf{q}_T, z, \mathbf{p}_T)}{z(1-z)\mathbf{q}_T^2} - \mathcal{J}_{Rg}^{(\text{sub.})}(\mathbf{q}_T, z, \mathbf{p}_T, 0)\right], \tag{11}$$

these divergences are subtracted by the following local subtraction term, similar to one used in Ref. [4]:

$$\mathcal{J}_{Rg}^{(\text{sub.})}(\mathbf{q}_T, z, \mathbf{p}_T, r) = \frac{2C_A}{\mathbf{k}_T^2}\left[\frac{1 - z}{(1-z)^2 + r\frac{\mathbf{k}_T^2}{(x_1 P_1^+)^2}} + \Delta p_{gg}(\mathbf{q}_T, z, \mathbf{p}_T)\right], \tag{12}$$

where $\Delta p_{gg}(\mathbf{q}_T, z, \mathbf{p}_T) = z(1-z) + 2[\mathbf{k}_T^2\mathbf{q}_T^2 - (\mathbf{k}_T\mathbf{q}_T)^2]/(z\mathbf{k}_T^2\mathbf{q}_T^2) - [3\mathbf{k}_T^2\mathbf{q}_T^2 - 2(\mathbf{k}_T\mathbf{q}_T)^2]/(\mathbf{k}_T^2\mathbf{q}_T^2)$, $\mathbf{k}_T = \mathbf{q}_T - \mathbf{p}_T$ and $r \ll 1$ is the regularisation parameter for rapidity divergence at $z \to 1$, which is set to zero in Eq. (11). In this way the Eq. (11) can be safely integrated over z with the PDF in Eq. (1) and over $\mathbf{q}_T$ with the C_{gg} in Eq. (2) in two dimensions for $\mathbf{q}_T$ and at $r = 0$, while the integral of the subtraction term (12) in dimensional regularisation and at $0 < r \ll 1$ had been computed analytically and included into the contribution described in the next section.

3. One-loop contribution and integrated subtraction term

One-loop correction to the amplitude $g(x_1 P_1) + R_-(\mathbf{q}_T) \to Q\bar{Q}[^1S_0^{[8]}]$ is computed in Ref. [5] using the same "tilted-Wilson-line" regularisation for rapidity divergence as in Eq. (12). The one-loop correction is combined with: i) integral of Eq. (12) over phase-space of a gluon ($g(k)$); ii) the ultraviolet counterterms; iii) the transition terms to the regularisation scheme for rapidity divergence via the cutoff in the target light-cone momentum component k_- ("the HEF scheme", Sec. 6.2 of Ref. [5]). After that, all IR, UV and rapidity divergences ($\sim \ln r$) cancel with an exception of the collinear divergence, proportional to the DGLAP splitting function $P_{gg}(z)$, which is absorbed into the collinear PDF of the projectile gluon $g(x_1 P_1)$. After these cancellations, the following finite

remainder is left, which contributes to the Eq. (3):

$$\mathcal{H}_{gg}^{(\text{NLO, analyt.})}(\mathbf{q}_T, z, \mathbf{p}_T) = \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int d^2\mathbf{k}_T K_{\text{BFKL}}(\mathbf{q}_T, \mathbf{k}_T, \mathbf{p}_T)$$

$$\times \left[\frac{1}{(1-z)_+} + \Delta p_{gg}(\mathbf{q}_T, z, \mathbf{p}_T) + \delta(1-z) \ln\left(\frac{M^2 + \mathbf{p}_T^2}{\mathbf{k}_T^2} \right) \right]$$

$$+ \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \delta(\mathbf{q}_T^2 - \mathbf{p}_T^2) \left\{ -\ln\frac{\mu_F^2}{\mathbf{p}_T^2} P_{gg}(z) + 2C_A \frac{1-z}{z} \right.$$

$$\left. + \delta(1-z) \left[-\frac{\pi^2}{2}C_A + \frac{8}{3}C_A - \frac{5}{3}\beta_0 - 2C_F\left(2 + \frac{3}{2}\ln\frac{4\mathbf{p}_T^2}{M^2} \right) + \beta_0 \ln\frac{\mu_R^2}{\mathbf{p}_T^2} + F_{{}^1S_0^{[1]}}(\mathbf{p}_T^2/M^2) \right] \right\},$$

where $K_{\text{BFKL}} = \frac{2C_A}{\mathbf{k}_T^2}\left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2}\delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T) \right]$, $\beta_0 = 11C_A/3 - 2n_F^{(\text{massless})}/3$. The function $F_{{}^1S_0^{[1]}}(\tau) = C_F C[gR \rightarrow {}^1S_0^{[1]}, C_F] + C_A C[gR \rightarrow {}^1S_0^{[1]}, C_A]$ is the finite part of the one-loop impact factor correction, computed in the Ref. [5], Eqns. (5.13) – (5.17).

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