PROCEEDINGS OF SCIENCE

Dipole picture diffractive structure function at NLO

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We compute transverse and longitudinal diffractive structure functions to full next-to-leading order accuracy in the dipole picture of deep inelastic scattering. Our calculation uses the standard light-cone perturbation theory method for the partonic content of the virtual photon, together with the Color Glass Condensate description of the target color field. Our result includes as a subset the contribution calculated earlier. We show that there is a rapidity divergence that can be factorized into the BK evolution of the target Wilson lines, and that all other divergences cancel. We emphasize the robustness of a fixed invariant mass as the definition of the final state, leading to a specific pattern of divergence cancellation between real and virtual contributions. Unlike for jets or identified hadrons, there is no fragmentation function or jet definition to absorb collinear divergences for this inclusive observable.

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1. Introduction

The process of interest for this contribution is Deep Inelastic Scattering (DIS) in the high energy saturation regime. We discuss a recent calculation [1] of diffractive structure functions to next-to-leading (NLO) accuracy in the dipole picture of DIS. After a short introduction to the dipole picture we discuss the kinematics of diffractive DIS (DDIS). The full NLO calculation is rather complicated algebraically. Thus we will here focus on the overall structure in terms of contributing diagrams, together with some technical remarks about the calculation. The full details can be found in Ref. [1], with earlier partial results and some useful discussions already presented in Ref. [2].

The basis of the dipole picture is the idea that we want to study the dense gluon field of the target by sending through it a dilute colored probe. At high energy the interaction of this probe is eikonal, meaning that the transverse coordinate of the probe is conserved during the interaction with the gluonic "shockwave". For example the amplitude for a quark is given by a *Wilson line*, a path ordered exponential in the color field

$$\mathbb{P}\exp\left\{-ig\int \mathrm{d}x^+ A^-(x^+,x^-,\mathbf{x}_T)\right\} = V(\mathbf{x}_T) \in \mathrm{SU}(N_\mathrm{c}). \tag{1}$$

In deep inelastic scattering the leading colored quantum state in the virtual photon is a quark-antiquark *color dipole*, whose scattering amplitude, the dipole amplitude, is given by a trace of the product of a Wilson line for the quark and a conjugate Wilson line for the antiquark

$$\mathcal{N}\left(r = |\mathbf{x}_T - \mathbf{y}_T|\right) = 1 - \frac{1}{N_\mathrm{c}}\mathrm{Tr}\, V^\dagger(\mathbf{x}_T)V(\mathbf{y}_T). \tag{2}$$

This quantity interpolates between zero at $r = 0$ (color transparency) and unity for large dipoles $r \gg 1/Q_\mathrm{s}$, a phenomenon known as gluon saturation. Since the Wilson line resums an infinite number of interactions, this picture is at the same time based on weak coupling and nonperturbative.

In the leading order dipole picture [3–6] the DIS scattering is factorized into the $\gamma^* \to q\bar{q}$ splitting in vacuum and an eikonal interaction of the $q\bar{q}$ dipole with the target. At Leading Log accuracy one adds to this dipole a *soft* (i.e. carrying a small fraction of the large longitudinal momentum of the photon) gluon. The integral over the longitudinal momentum of the gluon leads to a large logarithm $\sim \ln 1/x$, which is absorbed into a high energy evolution of the target, via the Balitsky-Kovchegov [7, 8] evolution equation. The topic of our present discussion is going beyond this limit, and including the same gluon with full kinematics (with, however, still eikonal interactions with the target). This corresponds to the NLO contribution to the cross section.

2. Diffractive DIS

In inclusive DDIS we are interested in the process $\gamma^* + p/A \to X + p/A$, with a rapidity gap separating the remnants of the target and of the photon. In terms of the photon remnants one measures differentially in terms of their total invariant mass M_X, but sums over all hadronic states with this constraint. The Lorentz invariant kinematical variables characterizing this process are the momentum transfer $t = (P - P')^2$ to the target, the size of the rapidity gap $\ln 1/x_\mathbb{P}$, which tells us how far in rapidity the BK evolution of the target should go, the virtuality Q^2 and the diffractive system mass M_X, which is often parametrized by $\beta = Q^2/(Q^2 + M_X^2)$, with $x_\mathrm{Bj} = x_\mathbb{P}\beta$.

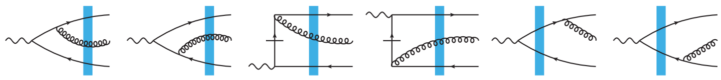

Figure 1: Radiative diagrams for the amplitude, with gluon emission before or after the target shockwave, represented by the light blue bar.

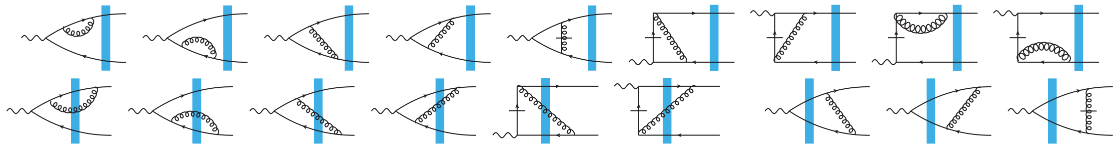

Figure 2: Virtual diagrams for the amplitude.

Depending on the value of β, one can discuss three qualitatively different regimes for the diffractive structure function, i.e. the diffractive cross section. At large $\beta \to 1$, i.e. small M_X the cross section is dominated by longitudinal virtual photons interacting via their leading $q\bar{q}$ Fock states. At medium $\beta \sim 0.5$, i.e. ${M_X}^2 \sim Q^2$ the dominant contribution comes from transversally polarized photons, but the small invariant mass restricts them still to the leading $q\bar{q}$ Fock state, at leading order in the coupling constant. Finally, at small $\beta \ll 1$, i.e. large ${M_X}^2$, higher Fock states in the photon are needed, starting with $q\bar{q}g$. In our power counting these contributions are a part of the NLO DDIS cross section. In this talk we do not consider the regime of parametrically large diffractive masses with $\alpha_s \ln 1/\beta \sim 1$ where all higher Fock states would need to be resummed.

A rather good description of HERA diffractive structure function data within this LO picture supplemented with a large Q^2 approximation of the $q\bar{q}g$ Fock states with a factorized impact parameter dependence was already achieved in Ref. [9]. More recently [2] the corresponding result was derived in full kinematics, with a general impact parameter dependence. The result of [2] for the $q\bar{q}$ contribution can be written as a coordinate space convolution of the dipole amplitude for the $q\bar{q}$ state crossing shockwave and calculable "transfer functions" that relate the coordinates at shockwave to the momentum transfer $t = -{\mathbf{\Delta}_T}^2$ and the invariant mass of the final state M_X.

3. Diffractive DIS at NLO

Let us first discuss the different kinds of contributions that are needed for the NLO diffractive cross section. Firstly there are the radiative corrections with an emission before the target, shown as the first four diagrams in Fig.1. The square of these contributions forms a self-contained subset of the NLO cross section, and was already calculated in Ref. [2]. In the full calculation these have to be complemented with the emissions after the target (last diagrams of Fig.1). The squares of these latter diagrams, and the interference terms, need to be combined with the virtual diagrams to cancel all the divergences and get a finite result.

The virtual correction diagrams for the amplitude are depicted in Fig. 2. The corrections to the photon vertex (the first line in Fig. 2) are known from the earlier calculations of the $\gamma \to q\bar{q}g$ wavefunction. Then there are the contributions where the gluon crosses shockwave, but not the cut (first 6 diagrams on the second line of Fig. 2). These involve tree level light cone wave functions, but represent loop corrections to the amplitude. They involve 3-point correlators of Wilson lines, and

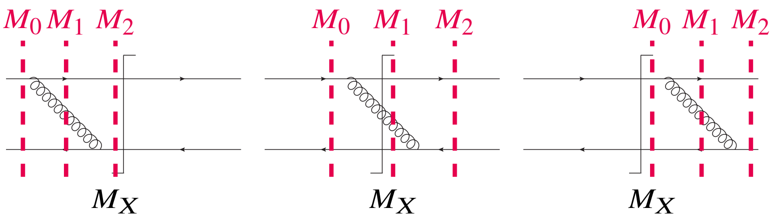

Figure 3: Diagrams with gluon exchange in the final state.

include divergences that must be included in the BK evolution of the LO amplitude. The technically most difficult contributions are the final state interaction diagrams (last three in the Fig. 2), which have several soft and collinear divergences that need to be canceled with other contributions. Propagator correction (final state wavefunction renormalization) diagrams after the shockwave are zero in dimensional regularization, where infrared and ultraviolet divergences cancel. In [1] we have calculated all these contributions.

We use a regularization procedure where transverse momentum integrals are performed in $2-2\varepsilon$ dimensions leading to $\frac{1}{\varepsilon}$ divergences which can be collinear or UV ones. The longitudinal momentum k^+ is regularized by a cutoff $k^+ > \alpha$, $\alpha \to 0$. This can lead to $1/\alpha$, $\ln^2\alpha$ or $\ln\alpha$ divergences. The UV $\frac{1}{\varepsilon}$ and $\frac{1}{\varepsilon}\ln\alpha$ divergences cancel between loop corrections to $\gamma^* \to q\bar{q}$ vertex, virtual diagrams where the gluon crosses the shockwave, and wavefunction renormalization contributions. Collinear $\frac{1}{\varepsilon}$ divergences cancel between wavefunction renormalization and final state emission contributions. The power $1/\alpha$ cancels between normal and instantaneous exchange diagrams. Double logs $\ln^2\alpha$ cancel between final state exchange and emission contributions in a way that is different from jet production, because it is important that we are fixing the final state invariant mass M_X. The remaining $\ln\alpha$ divergence is then absorbed into BK evolution.

The technically most difficult aspect of the calculation is separating these different types of divergences. For this it is essential to perform the calculation at the cross section, rather than amplitude level. For example the virtual diagrams with gluon exchanges in the final state in the amplitude or conjugate amplitude, and the real diagram with the corresponding gluon emission (depicted in Fig. 3) share the same gluon emission vertices and only differ by their energy denominators, and by which invariant mass is fixed to be the produced one M_X. They can be combined together by reverting the delta function for M_X to a propagator-like form using the representation

$$\delta({M_X}^2 - M^2) = \frac{i}{2\pi}\left[\frac{1}{{M_X}^2 - M^2 + i\delta} - \frac{1}{{M_X}^2 - M^2 - i\delta}\right], \quad \delta \to 0. \tag{3}$$

One then expresses the scalar products of momenta in the numerator, resulting from the gluon emission vertices, in terms of the different invariant masses M_0^2, M_1^2, M_2^2 of the intermediate states. This allows one to combine the diagrams before integrating over momenta and split them into separate terms involving divergences of different types.

In conclusion, we have calculated the diffractive DIS total cross section, i.e. the diffractive structure functions $F_2^{;}F_L^D$, in the dipole picture fully at NLO. Our result has a general impact parameter dependence and reproduces earlier large M_X, $\ln Q^2$ limits used so far in phenomenology.

In the future it will be interesting to develop a numerical implementation of these results and include them in global NLO dipole picture fits of DIS data.

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