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Small x resummation of photon impact factors and the $\gamma^*\gamma^*$ high energy scattering

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I present the renormalization group improved collinear resummation of the photon-gluon impact factors. We construct the resummed cross section for virtual photon-photon ($\gamma^*\gamma^*$) scattering which incorporates the impact factors and BFKL gluon Green's function up to the next-to-leading logarithmic accuracy in energy. The impact factors include important kinematical effects which are responsible for the most singular poles in Mellin space at next-to-leading order. Further conditions on the resummed cross section are obtained by requiring the consistency with the collinear limits. Our analysis is consistent with previous impact factor calculations at NLO, apart from a new term proportional to C_F that we find for the longitudinal polarization. Finally, we use the resummed cross section to compare with the LEP data on the $\gamma^*\gamma^*$ cross section and with previous calculations. The resummed result is lower than the leading logarithmic approximation but higher than the pure next-to-leading one, and is consistent with the experimental data.

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1. Introduction

In this talk I present the calculation of the renormalization-group improved (RGI) photon impact factors within the high-energy factorization approach. Such impact factors, together with the BFKL gluon Green function (GGF), enable us to compute the virtual photon cross section in the high energy limit, resumming to all perturbative orders the leading $(\alpha_s \log s)^n$ and next-to-leading $\alpha_s(\alpha_s \log s)^n$ logarithms of the energy.

The BFKL expansion [1] is known to be affected by large and negative corrections at next-to-leading order, and it is known since long that the main cause of the size of such corrections is due to large collinear terms that need to be resummed as well [2].

In momentum space, the high-energy factorization formula reads

$$\sigma^{(jk)}(s, Q_1, Q_2) = \int \mathrm{d}^2\boldsymbol{k}\; \mathrm{d}^2\boldsymbol{k}'\; \phi^{(j)}(Q_1, \boldsymbol{k})\, G(s, \boldsymbol{k}, \boldsymbol{k}_0)\, \phi^{(k)}(Q_2, \boldsymbol{k}_0)\;, \tag{1}$$

where $j, k \in \{L, T\}$ denote the polarizations of the two photons, q_1, q_2 their momenta and $Q_i^2 \equiv -q_i^2 > 0 : i = 1, 2$ their virtualities.

The gluon Green's function $G(s, \boldsymbol{k}, \boldsymbol{k}_0)$, which depends on the transverse gluon momenta $\boldsymbol{k}$ and $\boldsymbol{k}_0$ and energy squared $s \equiv (q_1 + q_2)^2$, satisfies the evolution equation that can be written in the following form

$$\frac{\partial}{\partial \log s} G(s, \boldsymbol{k}, \boldsymbol{k}_0) = \int \mathrm{d}^2\boldsymbol{k}'\; \mathcal{K}(\boldsymbol{k}, \boldsymbol{k}')\, G(s, \boldsymbol{k}', \boldsymbol{k}_0)\;, \tag{2}$$

where the function $\mathcal{K}$ is the BFKL kernel which has the following perturbative expansion

$$\mathcal{K} = \bar{\alpha}_s \mathcal{K}_0 + \bar{\alpha}_s^2 \mathcal{K}_1 + \dots\;. \tag{3}$$

In the above equation, we introduced the rescaled strong coupling $\bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$ where N_c is the number of colors. In QCD the kernel is known at leading and next-to-leading order. It is customary to use the Mellin transform to obtain the kernel eigenvalue

$$\bar{\alpha}_s \chi(\gamma) = \int \mathrm{d}\boldsymbol{k}'^2 \left(\frac{\boldsymbol{k}'^2}{\boldsymbol{k}^2}\right)^{\gamma} \mathcal{K}(\boldsymbol{k}, \boldsymbol{k}')\,, \tag{4}$$

with the corresponding perturbative expansion corresponding to eq. (3)

$$\chi(\gamma) = \chi_0(\gamma) + \bar{\alpha}_s \chi_1(\gamma) + \dots\;. \tag{5}$$

The leading order kernel's eigenvalue reads

$$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)\;, \tag{6}$$

where $\psi(z) = \Gamma'(z)/\Gamma(z)$ is the polygamma function, and $\psi(1) = -\gamma_E$. The behaviour of the kernel $\mathcal{K}_0(\boldsymbol{k}, \boldsymbol{k}') \sim 1/\boldsymbol{k}^2$ for $\boldsymbol{k} \gg \boldsymbol{k}'$ reflects itself in the presence of a simple pole of $\chi_0(\gamma) \sim 1/\gamma$ for $\gamma \to 0$. At higher orders, on the basis of collinear QCD dynamics [3], one expects that the higher-order kernels behave like $\mathcal{K}_n(\boldsymbol{k}, \boldsymbol{k}') \sim (1/\boldsymbol{k}^2) \log^{n-1}(\boldsymbol{k}/\boldsymbol{k}')$ for $\boldsymbol{k} \gg \boldsymbol{k}'$ and thus $\chi_n(\gamma) \sim 1/\gamma^{n+1}$. However, the actual calculations yield $\chi_n(\gamma) \sim 1/\gamma^{2n+1}$.

The reason for such spurious poles has been found in the "wrong" choice of energy scale s_0 in the BFKL logarithms $\log(s/s_0)$. In fact, if one rewrites the high-energy factorization formula in

Mellin space, where γ is conjugated to virtualities and transverse momenta, while ω is conjugated to the energy s, one diagonalizes the convolutions into simple products:

$$\sigma^{(jk)}(s, Q_1, Q_2) = \frac{1}{2\pi Q_1 Q_2} \int \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} \left(\frac{s}{s_0(p)}\right)^{\omega} \int \frac{\mathrm{d}\gamma}{2\pi\mathrm{i}} \left(\frac{Q_1^2}{Q_2^2}\right)^{\gamma-\frac{1}{2}} \frac{\phi^{(j)}(\gamma; p)\, \phi^{(k)}(1-\gamma; -p)}{\omega - \bar{\alpha}_\mathrm{s}\chi(\gamma; p)} \tag{7}$$

In eq. (7) both impact factors ϕ and eigenvalue function χ are perturbative objects that admit a series expansion in α_s, as in eq. (5); from next-to-leading order on, they depend on the choice of the energy scale:

$$\phi^{(j)}(\gamma; p) = \phi_0^{(j)}(\gamma) + \bar{\alpha}_\mathrm{s}\phi_1^{(j)}(\gamma; p) + O\left(\bar{\alpha}_\mathrm{s}^2\right), \tag{8}$$

$$\chi(\gamma; p) = \chi_0(\gamma) + \bar{\alpha}_\mathrm{s}\chi_1(\gamma; p) + O\left(\bar{\alpha}_\mathrm{s}^2\right). \tag{9}$$

It turns out that also the impact factors have collinear poles whose order exceeds that predicted by the collinear dynamics: $\phi_n^{(n)}(\gamma) \sim \phi_0^{(j)}(\gamma)/\gamma^{2n}$ instead of $\phi_n^{(n)}(\gamma) \sim \phi_0^{(j)}(\gamma)/\gamma^{n}$. Such spurious poles appears when $p \neq 1$, i.e., when the energy scale is different from Q_1^2. However, analogous spurious poles appear at $\gamma = 1$ (anti-collinear limit), unless $s_0 = Q_2^2$. The solution to avoid such spurious poles is to realize that changing $s_0(p)$ amount to an ω-shift in the γ variable:

$$\left(\frac{s}{\boldsymbol{k}\boldsymbol{k}_0}\right)^{\omega} \left(\frac{\boldsymbol{k}^2}{\boldsymbol{k}_0^2}\right)^{\gamma} = \left(\frac{s}{\boldsymbol{k}^2}\right)^{\omega} \left(\frac{\boldsymbol{k}}{\boldsymbol{k}_0}\right)^{\omega} \left(\frac{\boldsymbol{k}^2}{\boldsymbol{k}_0^2}\right)^{\gamma} = \left(\frac{s}{\boldsymbol{k}^2}\right)^{\omega} \left(\frac{\boldsymbol{k}^2}{\boldsymbol{k}_0^2}\right)^{\gamma+\omega/2}. \tag{10}$$

This implies that both eigenvalue function and impact factors should be ω dependent. According to this suggestion, we propose the following RGI factorization formula:

$$\sigma^{(jk)}(s, Q_1, Q_2) = \frac{1}{2\pi Q_1 Q_2} \int \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} \left(\frac{s}{s_0(p)}\right)^{\omega} \int \frac{\mathrm{d}\gamma}{2\pi\mathrm{i}} \left(\frac{Q_1^2}{Q_2^2}\right)^{\gamma-\frac{1}{2}} \tag{11a}$$

$$\times\, \Phi^{(j)}(\omega, \gamma; p)\, \mathcal{G}(\omega, \gamma; p)\, \Phi^{(k)}(\omega, 1-\gamma; -p) \tag{11b}$$

$$\mathcal{G}(\omega, \gamma; p) = \frac{1}{\omega - \bar{\alpha}_\mathrm{s} X(\omega, \gamma; p)}. \tag{11c}$$

The compatibility of the BFKL factorization formula (7) with the RGI one (11b) requires that the one-loop RGI impact factors at $\omega = 0$ are related to the BFKL one [4–6] by the relation

$$\begin{aligned} \Phi_1(0, \gamma) &= \frac{1}{2}\left[\Phi_1(0, \gamma) + \Phi_1(0, 1-\gamma)\right] \\ &= \frac{1}{2}\left[\phi_1(\gamma) + \phi_1(1-\gamma) - \phi_0(\gamma)\partial_\omega X_0(0, \gamma) - \chi_0(\gamma)\big(\partial_\omega\Phi_0(0, \gamma) + \partial_\omega\Phi_0(0, 1-\gamma)\big)\right]. \end{aligned} \tag{12}$$

We extend Φ_1 at $\omega \neq 0$ by requiring the collinear poles to be located at $\gamma = -\omega/2$ and $\gamma = 1 + \omega/2$ and with ω-dependent leading coefficients $M_T(\omega)$ and $\bar{M}_T(\omega)$ to be determined by the collinear analysis of the next section, in the form $[C_0 = \alpha\alpha_\mathrm{s}\left(\sum_q e_q^2\right)\frac{4}{3}T_R\sqrt{2(N_c^2-1)}]$

$$\Phi_1(\omega, \gamma; 0) = \Phi_1(0, \gamma) + C_0\left[\frac{M_T(\omega)}{(\gamma + \frac{\omega}{2})^3} - \frac{M_T(0)}{\gamma^3} + \begin{pmatrix} \gamma \leftrightarrow 1-\gamma \\ M_T \to \bar{M}_T \end{pmatrix}\right]. \tag{13}$$

2. Collinear analysis

Further information for the $\gamma^*\gamma^*$ cross section, somehow complementary to the multi-Regge kinematics, can be inferred by analyzing the collinear limit, i.e., by considering two photons with very different virtualities, say $Q_1 \gg Q_2$. This situation is well described by effective ladder diagrams, like those depicted in fig. 1, where the intermediate propagators are strongly ordered in virtuality (decreasing from left to right). At each QCD vertex, the strong coupling is evaluated at a scale given by the largest virtuality of the connected propagators, while the DGLAP [3] splitting functions $P_{ba}(z_b/z_a)$ describes the fragmentation of the parent parton a (to the right) into a child parton b (to the left) and an emitted on-shell parton (vertical line). The integrals over the ordered longitudinal momentum fractions are convolutions, which can be diagonalized by a Mellin transform in the Bjorken variable $1/x_{Bj} = s/Q_1^2 = s/s_0(p=1)$.

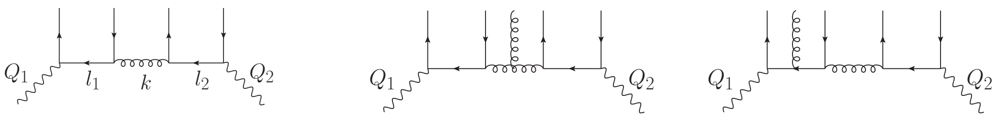

Figure 1: Diagramatics of collinear limit at lowest order (left) and next-to-leading order (center and right) in the high-energy factorization formula.

The cross section in Mellin space (ω, γ) stemming from the left diagram in fig. 1 provides the lowest order integrand of the RGI factorization formula (11b) in the collinear limit $\gamma \to 0$:

$$\sigma_0^{(TT)}(\omega,\gamma;1) \simeq \Phi_0^{(T)}\,\mathcal{G}_0\,\Phi_0^{(T)}\big|_{p=1} \\ = (2\pi)^3\alpha\Big(2\sum_{q\in A} e_q^2\Big)\frac{1}{\gamma}\cdot\frac{\alpha_s}{2\pi}\frac{P_{qg}(\omega)}{\gamma}\cdot\frac{\alpha_s}{2\pi}\frac{P_{gq}(\omega)}{\gamma}\cdot\frac{\alpha}{2\pi}\Big(2\sum_{q\in B} e_q^2\Big)\frac{P_{q\gamma}(\omega)}{\gamma}\,. \tag{14}$$

Considering ladder diagrams with five splittings between the photons (last two diagrams of fig. 1) and the running coupling contribution proportional to $b = \frac{11N_c - 4T_R N_f}{12\pi}$ provides the integrand of the RGI factorization formula at $O\left(\alpha_s^3\right)$ in the collinear limit. Altogether, one obtains the constraint on the coefficients M_T and $\bar{M}_T$ of eq. (13): $M_T(\omega) + \bar{M}_T(\omega) = 2P_{qq}(\omega) - b$.

Analogous considerations can be done for the longitudinal impact factor. In this case we obtain $M_L + \bar{M}_L = \frac{C_F}{T_R}P_{qg}(\omega)\frac{3+\omega}{2} - b$. In the last expression, the term proportional to C_F at $\omega = 0$ is not consistent with the results of ϕ_1 of refs. [6]. We don't know the origin of the mismatch, yet.

By evaluating the cross section with the improved impact factor and GGF for $Q_1^2 = Q_2^2 = 17\ \mathrm{GeV}^2$, and by adding the lowest order perturbative contribution (quark box), we can compare our results with data measured at LEP [7, 8], as can be seen in fig. 2. We also show the LO quark box contribution in this figure. We observe that the RGI NLL improved calculation has a stronger increase over rapidities than the pure NLL one. We also see that our result is significantly higher than the calculation from [6], particularly at high rapidities. The RGI calculation is consistent with the experimental data from LEP within the theoretical and experimental uncertainties.

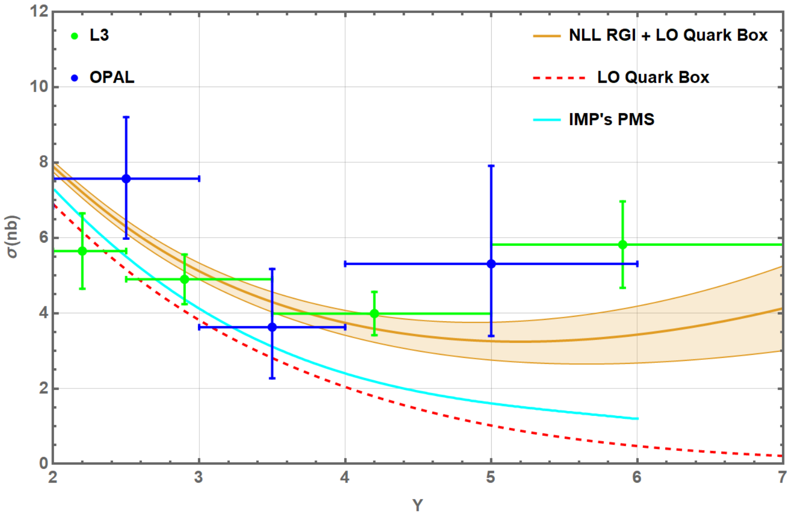

Figure 2: Cross sections for $Q^2 = 17\,\mathrm{GeV}^2$, compared with L3 ($Q^2 = 16\,\mathrm{GeV}^2$) and OPAL ($Q^2 = 17.9\,\mathrm{GeV}^2$) data. The NLL improved curve is the sum of our averaged NLL BFKL resummed scheme and LO quark box contribution. The band represents a combination (in quadrature) of the scheme uncertainty and the renormalization scale uncertainty, The calculation is done for $N_f = 4$ massless flavours. The Ivanov-Murdaca-Papa's (IMP's) PMS optimized curve (solid-cyan) is from [6]. Separately shown is the quark box contribution (dashed red).

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