PROCEEDINGS OF SCIENCE

Matching between TMD and twist-3 factorizations in the transversely polarized hyperon production

Riku Ikarashi,[a] **Yuji Koike**[b] **and Shinsuke Yoshida**[c,d,*]

[a] *Graduate School of Science and Technology, Niigata University, Ikarashi, Niigata 950-2181, Japan*
[b] *Department of Physics, Niigata University, Ikarashi, Niigata 950-2181, Japan*
[c] *Key Laboratory of Atomic and Subatomic Structure and Quantum Control (MOE), Guangdong Basic Research Center of Excellence for Structure and Fundamental Interactions of Matter, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China*
[d] *Guangdong-Hong Kong Joint Laboratory of Quantum Matter, Guangdong Provincial Key Laboratory of Nuclear Science, Southern Nuclear Science Computing Center, South China Normal University, Guangzhou 510006, China*
E-mail: rick.ikatarashi@gmail.com, Koike@phys.sc.niigata-u.ac.jp, shinyoshida85@gmail.com

We focus on the origin of the transverse polarization of Λ hyperon produced in the proton-proton collision which has been a mystery in high energy hadron reactions over the past half century. There are two different perturbative QCD frameworks for the description of the origin with respect to the size of the transverse momentum of the produced hyperon P_{hT}, transverse-momentum-dependent(TMD) factorization in small P_{hT} and the collinear twist-3 factorization in high P_{hT}. We can consider the intermediate region of P_{hT} where both frameworks are valid in semi-inclusive deep inelastic scattering and it was once shown that the TMD Boer-Mulders effect and the twist-3 distribution effect from the initial state proton give consistent cross section formulas in this region. We will here show that the consistency can be also found between the TMD polarizing fragmentation and the twist-3 fragmentation contributions generated by the polarized hyperon. This result supports the idea that the two frameworks describe a unique origin of the transverse polarization in different kinematic regions.

31st International Workshop on Deep Inelastic Scattering (DIS2024)
8–12 April 2024
Grenoble, France

*Speaker

https://pos.sissa.it/

PoS(DIS2024)233

1. Introduction

Large single transverse-spin asymmetries(SSAs) were first observed in the pion production and the transversely polarized Λ hyperon production in the late 70s. The asymmetries in those processes reach a couple of ten percent in the large rapidity region, which is much larger than what was expected by the parton model calculation. The understanding of their origins has been a challenging problem in high energy QCD physics since those first observations. We focus on the origin of the hyperon polarization in semi-inclusive deep inelastic scattering(SIDIS) in this study.

Much experimental data has been accumulated in the past couple decades. Two perturbative QCD frameworks have been well developed as successful frameworks in describing those data. One is the collinear twist-3 factorization framework which is valid when the transverse momentum of a produced hadron P_{hT} is much larger than $\Lambda_{\rm QCD}$, i.e., $\Lambda_{\rm QCD} \ll P_{hT}$. In this framework, the twist-3 multiparton correlation play a role in generating the large SSAs. The other is the transverse momentum dependent(TMD) factorization framework that covers the description of the large SSAs when P_{hT} is much smaller than Q which is the typical hard scale in a process. The intrinsic transverse momenta of partons inside the hadrons cause the large SSAs in this framework. It has been shown that the two frameworks match consistently in the intermediate region of P_{hT}, i.e., $\Lambda_{QCD} \ll P_{hT} \ll Q$, in the pion production in SIDIS[1–3]. In the case of the transversely polarized lambda production, the consistency between the two frameworks has been only established in the Boer-Mulders type contribution from the initial proton[4]. The differential cross section for the twist-3 fragmentation contribution was derived in [5, 6] recently at the leading order (LO) with respect to the QCD coupling constant , which enabled the study of the consistency between the TMD polarizing fragmentation and the twist-3 fragmentation contributions generated by the polarized hyperon. We will here show that the consistency between those two types of contributions in the intermediate region of P_{hT}.

2. Differential cross sections in intermediate transverse momentum region

In the transversely polarized hyperon production, $e(\ell) + p(p) \to e(\ell') + \Lambda^{\uparrow}(P_h, S_\perp) + X$, the differential cross section was derived within the collinear twist-3 factorization formalism,

$$\frac{d^6\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \mathcal{F}_1 \sin\Phi_S + \mathcal{F}_2 \sin\Phi_S \cos(\phi-\chi) + \mathcal{F}_3 \sin\Phi_S \cos 2(\phi-\chi) + \mathcal{F}_4 \cos\Phi_S \sin(\phi-\chi) + \mathcal{F}_5 \cos\Phi_S \sin 2(\phi-\chi), \quad (1)$$

where $Q^2 = -q^2 = -(\ell-\ell')^2$, $x_{bj} = Q^2/(2p\cdot q)$, $z_f = p\cdot P_h/(p\cdot q)$ and $q_T = P_{hT}/z_f$. Φ_S is the azimuthal angle of the transverse spin vector $\vec{S}_\perp$. ϕ and χ are, respectively, azimuthal angles of the lepton plane and the hadron plane. The explicit forms of $\mathcal{F}_{1,2,3,4,5}$ are given in [5, 6]. The twist-3 cross section is valid as long as $\Lambda_{QCD} \ll P_{hT}$ is satisfied. In the intermediate region

$\Lambda_{QCD} \ll P_{hT} \ll Q$, the contribution from $\mathcal{F}_1$ becomes dominant and it is reduced to

$$\mathcal{F}_1 = \frac{-4\alpha_s M_h \sigma_0}{4\pi^3 q_T^3}\Bigg[f_1(x_{bj})\int \frac{dz}{z}\left(A - \frac{1}{4}B\right) + C_F D_{1T}^{\perp(1)}(z_f)\int \frac{dx}{x} f_1(x)\frac{1+\hat{x}^2}{(1-\hat{x})_+}$$
$$+2C_F f_1(x_{bj}) D_{1T}^{\perp(1)}(z_f)\ln\frac{Q^2}{q_T^2}\Bigg], \tag{2}$$

where M_h is the hyperon mass, $\alpha_s = g^2/(4\pi)$ is the strong coupling constant, $C_F = (N^2-1)/(2N)$ and $\sigma_0 = \alpha_{em}^2(1-y+y^2/2)/Q^4$ with the DIS inelasticity parameter $y = p\cdot q/(p\cdot \ell)$. The $f_1(x)$ is the unpolarized twist-2 PDF. A and B are given by

$$A = \frac{D_T(z)}{z}\left(-C_F(1+2\hat{z}) - \frac{1}{2N}\frac{1+\hat{z}^2}{\hat{z}}\right)$$
$$+\left(\frac{\partial}{\partial(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\right)\left(-\frac{1}{2N}\frac{1+\hat{z}^2}{\hat{z}}\right) + D_{1T}^{\perp(1)}(z)\left(C_F\frac{\hat{z}(1+\hat{z})}{(1-\hat{z})_+}\right)$$
$$+\int d\left(\frac{1}{z'}\right)\frac{1/z}{1/z-1/z'}\Im\widehat{D}_{FT}(z,z')\left\{\frac{1}{1/z'}\frac{1}{2N}\frac{2-\hat{z}}{\hat{z}} + \frac{1}{1/z'-1/z_f}\left(C_F+\frac{1}{2N}\right)(1+\hat{z})\right\}$$
$$+\int d\left(\frac{1}{z'}\right)\frac{1/z}{1/z-1/z'}\Im\widehat{G}_{FT}(z,z')\left\{\frac{1}{1/z'}\frac{1}{2N} - \frac{1}{1/z'-1/z_f}\left(C_F+\frac{1}{2N}\right)(1-\hat{z})\right\}, \tag{3}$$

and

$$B = 2C_F z^2\Bigg[\frac{(1-\hat{z})(-2+\hat{z}^2)}{\hat{z}^2}\widehat{G}_T^{(1)}(z) - 2\frac{(1-\hat{z})}{\hat{z}}\Delta\widehat{H}_T^{(1)}(z) + \int d\left(\frac{1}{z'}\right)\frac{1}{1/z-1/z'}\frac{(1-\hat{z})}{\hat{z}^2}$$
$$\times\Im\left\{4(2-3\hat{z}+\hat{z}^2)\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right) + 2(2-3\hat{z})\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right) - 2(2-3\hat{z}+2\hat{z}^2)\widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\}$$
$$+\int d\left(\frac{1}{z'}\right)\frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\frac{(1-\hat{z})}{\hat{z}^2}$$
$$\times\Im\left\{(4-4\hat{z}+\hat{z}^2)\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right) + (4-4\hat{z}+\hat{z}^2)\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right) - 2(2-2\hat{z}+\hat{z}^2)\widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\}$$
$$+\int d\left(\frac{1}{z'}\right)\frac{1}{1/z-1/z'}\frac{(1-\hat{z})}{\hat{z}^2}$$
$$\times\Im\left\{4(2-\hat{z})\widehat{O}_1\left(\frac{1}{z'},\frac{1}{z}\right) + 2(4-3\hat{z}+\hat{z}^2)\widehat{O}_2\left(\frac{1}{z'},\frac{1}{z}\right) + 2(4-3\hat{z}+\hat{z}^2)\widehat{O}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\}$$
$$+\int d\left(\frac{1}{z'}\right)\frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\frac{(1-\hat{z})}{\hat{z}^2}$$
$$\times\Im\left\{(4-4\hat{z}+\hat{z}^2)\widehat{O}_1\left(\frac{1}{z'},\frac{1}{z}\right) + (4-4\hat{z}+\hat{z}^2)\widehat{O}_2\left(\frac{1}{z'},\frac{1}{z}\right) + 2(2-2\hat{z}+\hat{z}^2)\widehat{O}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\}$$
$$+\frac{1}{C_F}\int d\left(\frac{1}{z'}\right)\Im\widetilde{D}_{FT}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)$$
$$\times\left\{-4\frac{(1-\hat{z})}{\hat{z}^2}(2-3\hat{z}+\hat{z}^2) + \frac{1}{N}\frac{1}{z}\frac{1}{1/z-1/z'}\frac{-1}{\hat{z}}(2-\hat{z}) + \frac{1}{N}\frac{1}{1-z_f/z'}(-\hat{z})(-2+\hat{z})\right\}$$
$$+\frac{1}{C_F}\int d\left(\frac{1}{z'}\right)\Im\widetilde{G}_{FT}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\left\{\frac{1}{N}\frac{1/z}{1/z-1/z'} + \frac{1}{N}\frac{1}{1-z_f/z'}(-\hat{z}^2)\right\}\Bigg], \tag{4}$$

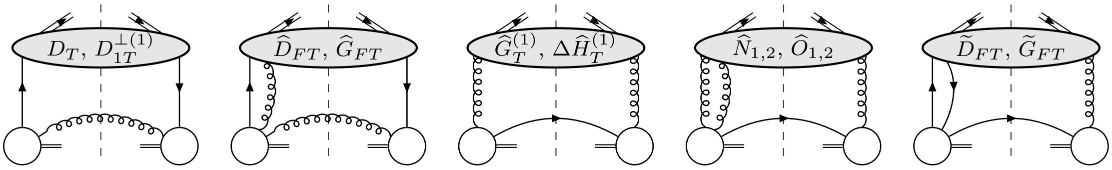

Figure 1: Feynman diagrams contributing to the polarizing FF $D_{1T}^{\perp}$ in $q_T \gg \Lambda_{\rm QCD}$. The mirror diagrams should be also included. Each upper blob denotes the corresponding twist-3 FFs. The white lower blobs represent all of the possible diagrams.

where $\hat{z} = z_f/z$. The definitions of all the twist-3 collinear FFs are introduced in [7, 8]. On the other hand, the polarizing FF $D_{1T}^{\perp}$ contribution in the TMD factorization formalism is given by [9, 10]

$$\frac{d^6\sigma}{dx_{bj}dQ^2dz_fdq_T^2d\phi d\chi} = -z_f^2\sigma_0 \sin\Phi_S \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\lambda}_\perp \delta^2(\vec{k}_\perp - \vec{p}_\perp + \vec{\lambda}_\perp - \vec{P}_{hT}/z_f)$$
$$\times \frac{\vec{p}_\perp^{\,2}}{q_T M_h} f_1(x_{bj}, k_\perp^2) D_{1T}^{\perp}(z_f, z_f^2 p_\perp^2) S^{-1}(\lambda_\perp^2) H(Q^2), \quad (5)$$

where $f_1(x_{bj}, k_\perp^2)$, $S(\lambda_\perp^2)$ and $H(Q^2)$ are the unpolarized TMD PDF, the soft factor and the hard factor, respectively. If the transverse components are large, $\Lambda_{QCD} \ll, k_\perp, p_\perp, \lambda_\perp$, TMD functions can be expressed in terms of the collinear functions[1],

$$f_1(x_{bj}, \vec{k}_\perp^2) = \frac{\alpha_s C_F}{2\pi^2 \vec{k}_\perp^2} \int \frac{dx}{x} f_1(x) \left[\frac{1+\hat{x}^2}{(1-\hat{x})_+} + \delta(1-\hat{x}) \left(\ln \frac{x_{bj}^2 \zeta^2}{\vec{k}_\perp^2} - 1 \right) \right],$$
$$S^{-1}(\vec{\lambda}_\perp^2) = -\frac{\alpha_s C_F}{2\pi^2 \vec{\lambda}_\perp^2} (\ln\rho^2 - 2), \quad (6)$$

where $\zeta^2 = (2v\cdot p)^2/v^2$ and $\rho^2 = (2v\cdot\tilde{v})^2/(v^2\tilde{v}^2)$ with non-lightlike vectors v and $\tilde{v}$ to regulate the light cone singularities. The hard factor takes the form of $H(Q^2) = 1 + O(\alpha_s)$. The P_{hT}-dependence of the polarizing FF $D_{1T}^{\perp}(z_f, P_{hT})$ can be obtained by the perturbative calculation of the diagrams shown in Fig. 1, which gives the factorized expression for $D_{1T}^{\perp}(z_f, P_{hT})$ in terms of the collinear twist-3 FFs as

$$D_{1T}^{\perp}(z_f, P_{hT}^2) = \frac{\alpha_s}{2\pi^2} \frac{2M_h^2 z_f^2}{P_{hT}^4} \left[\int \frac{dz}{z} \left(A - \frac{1}{4}B \right) + C_F D_{1T}^{\perp(1)}(z_f) \left(\ln \frac{\hat{\zeta}^2}{P_{hT}^2} - 1 \right) \right], \quad (7)$$

where $\hat{\zeta}^2 = (2P_h\cdot\tilde{v})^2/\tilde{v}^2$, A and B are, respectively, given in (3) and (4). It is critically important that $D_{1T}^{\perp}$ is written in terms of A and B which appear in $\mathcal{F}_1$ in the collinear twist-3 factorization. The cross section (5) receives the leading α_s^1-contribution from three integral regions

$$\vec{p}_\perp, \vec{\lambda}_\perp \ll \vec{k}_\perp \simeq \vec{q}_T, \quad \vec{k}_\perp, \vec{\lambda}_\perp \ll \vec{p}_\perp \simeq \vec{q}_T, \quad \vec{k}_\perp, \vec{p}_\perp \ll \vec{\lambda}_\perp \simeq \vec{q}_T. \quad (8)$$

Subsituting (6) and (7) into (5), one can finally confirm that the two approaches lead to the same cross section in the intermediate region of the transverse momentum $\Lambda_{\rm QCD} \ll q_T \ll Q$.

3. Summary

We have examined the consistency between the TMD factorization and the collinear twist-3 factorization for the hyperon transverse polarization in SIDIS with respect to the twist-3 quark and gluon FF contributions. We have demonstrated that the collinear twist-3 cross section consistently matches the TMD cross section in the intermediate region of the transverse momentum of the final hyperon. This supports the idea that the two frameworks represent a unique QCD origin for the transverse polarization of the hyperon.

Acknowledgements

This work has been supported by JST, the establishment of University fellowships towards the creation of science technology innovation, Grant Number JPMJFS2114 (R.I.), the Grant-in-Aid for Scientific Research from the Japanese Society of Promotion of Science under Contract No. 19K03843 (Y.K.).

References

[1] X. Ji, J. W. Qiu, W. Vogelsang and F. Yuan, Phys. Lett. B **638**, 178-186 (2006) doi:10.1016/j.physletb.2006.05.044 [arXiv:hep-ph/0604128 [hep-ph]].

[2] Y. Koike, W. Vogelsang and F. Yuan, Phys. Lett. B **659**, 878-884 (2008) doi:10.1016/j.physletb.2007.11.096 [arXiv:0711.0636 [hep-ph]].

[3] F. Yuan and J. Zhou, Phys. Rev. Lett. **103**, 052001 (2009) doi:10.1103/PhysRevLett.103.052001 [arXiv:0903.4680 [hep-ph]].

[4] J. Zhou, F. Yuan and Z. T. Liang, Phys. Rev. D **78**, 114008 (2008) doi:10.1103/PhysRevD.78.114008 [arXiv:0808.3629 [hep-ph]].

[5] Y. Koike, K. Takada, S. Usui, K. Yabe and S. Yoshida, Phys. Rev. D **105**, no.5, 056021 (2022) doi:10.1103/PhysRevD.105.056021 [arXiv:2202.00338 [hep-ph]].

[6] R. Ikarashi, Y. Koike, K. Yabe and S. Yoshida, Phys. Rev. D **105**, no.9, 094027 (2022) doi:10.1103/PhysRevD.105.094027 [arXiv:2203.08431 [hep-ph]].

[7] K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak and M. Schlegel, Phys. Rev. D **93**, no.5, 054024 (2016) doi:10.1103/PhysRevD.93.054024 [arXiv:1512.07233 [hep-ph]].

[8] Y. Koike, K. Yabe and S. Yoshida, Phys. Rev. D **101**, no.5, 054017 (2020) doi:10.1103/PhysRevD.101.054017 [arXiv:1912.11199 [hep-ph]].

[9] P. J. Mulders and R. D. Tangerman, Nucl. Phys. B **461**, 197-237 (1996) [erratum: Nucl. Phys. B **484**, 538-540 (1997)] doi:10.1016/0550-3213(95)00632-X [arXiv:hep-ph/9510301 [hep-ph]].

[10] X. d. Ji, J. p. Ma and F. Yuan, Phys. Rev. D **71**, 034005 (2005) doi:10.1103/PhysRevD.71.034005 [arXiv:hep-ph/0404183 [hep-ph]].