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Correction of partial snake resonances with betatron coupling at the Brookhaven AGS

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Polarization of proton beam in the Brookhaven Alternating Gradient Synchrotron is preserved during acceleration by use of two helical dipole partial snake magnets. These magnets modify the spin tune to avoid strong intrinsic resonances associated with the vertical betatron motion, but in the process excite many weaker resonances associated with the horizontal betatron motion. Since these resonances occur at the same frequencies as resonances from betatron coupling, they may be corrected by introducing coupling resonances of equal and opposite amplitude. Fifteen pulsed skew quadrupoles have been added to the AGS lattice in order to implement such a correction scheme. We describe the correction principle, its implementation and commissioning results.

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1. Polarized proton beams in the RHIC injectors

The Relativistic Heavy Ion Collider (RHIC) provides polarized proton beams at beam energies up to 255 GeV. A polarized H$^-$ beam is produced by an optically pumped polarized ion source (OPPIS) and accelerated to 200 MeV via Linac. The beam is strip-injected into the Booster. From there the beam is accelerated in a sequence of synchrotrons: the Booster (to 2.5 GeV total energy), the Alternating Gradient Synchrotron (to 23.8 GeV) and finally in the two RHIC rings as high as 255 GeV.

Polarization from the OPPIS source, measured at the end of the Linac, is 78-84% during operation. In the Booster the beam passes through two imperfection resonances at $G\gamma$ =3,4 which are crossed at >99% polarization transmission efficiency with closed orbit manipulations (γ is the relativistic Lorentz factor, G the anomalous gyromagnetic ratio). At extraction energy in the AGS the polarization in operation is 68-70% at a per bunch intensity of 2×10^{11}.

Partial snakes have been used in the AGS for preservation of proton polarization since 1994 [1]. Currently, the AGS uses a pair of helical dipole partial snakes, separated by 1/3 of the ring circumference, that rotate the proton spin about the longitudinal direction by 18°and 10.8°, respectively. These magnets allow avoidance of the strongest depolarizing resonances associated with the vertical beam motion. In the presence of horizontal betatron motion, however, partial snakes can themselves drive numerous weak horizontal intrinsic resonances which collectively can lead to substantial polarization loss. Polarized protons are accelerated in the AGS from $G\gamma$= 4.5 to 45.5 which requires passing through 82 horizontal intrinsic resonances where $G\gamma = N \pm \nu_x$ (N is an integer). Left uncorrected, the horizontal resonances in the AGS result in a 15-20% relative loss in proton beam polarization [2]. Since 2011, a fast tune jump has been used to increase the resonance crossing rate, but this only prevents about half the polarization loss, leaving an 8-10% drop [3, 4, 5].

Here we describe a method for suppressing these horizontal intrinsic resonances using betatron coupling. Since resonances from the partial snakes and resonances from betatron coupling occur at the same resonant frequencies, a coupling resonance can be excited which exactly cancels each snake-driven resonance. A set of 15 fast ramping skew quadrupoles has been installed in the AGS ring to excite these resonance terms. We will discuss the theory of the correction, implementation details of the magnets and commissioning results from the 2024 RHIC run.

2. Theory

In an ideal, planar ring with a single partial snake, the spin vector of a particle on the closed orbit experiences only constant precession around the main bend field and the local spin rotation of the partial snake. If, however, the particle is also undergoing horizontal betatron oscillations, then the precessional motion is modulated at the betatron frequency as the particle moves with an angle with respect to the closed orbit. For a particle executing horizontal betatron motion in a ring with a single snake, in the presence of coupling, we can model the spin motion in the spinor formalism as

$$\frac{d\Psi}{d\theta} = -\frac{i}{2}\begin{pmatrix} F & -\xi \\ -\xi^* & -F \end{pmatrix}\Psi \qquad (1)$$

with

$$\xi = iX_s\delta(\theta - \theta_s) + \epsilon_x e^{i\nu_x\theta}$$
$$F = G\gamma - (1 + G\gamma)x''(\theta)\rho$$

where $x(\theta)$ is the particle position relative to the reference orbit, X_s is the angle of rotation produced by the snake, δ is the periodic Dirac delta, θ_s the azimuthal position of the snake, ρ is the bend radius of the local reference orbit and ν_x is the horizontal betatron tune. A particle on the closed orbit has $x = 0$, so the precessional term F is then constant turn to turn and the snake only produces the intended imperfection resonances [6]. The term proportional to ϵ_x is a resonant term at the horizontal tune excited by, for example betatron coupling. Under these circumstances it can be shown that the net resonance term is [7]

$$\epsilon = \frac{1}{2\pi}\int \delta_n(\theta - \theta_s)e^{-iG\gamma\theta + iK\theta}\left[1 - \left(\tilde{x}'\frac{(1 + G\gamma)}{2} + \epsilon_x\right)e^{\pm i\nu_x\theta}\right]d\theta \tag{2}$$

The unity term in the square brackets is non-zero when $K - G\gamma = N$ for integer N, and has amplitude $\frac{X_s}{2\pi}$, which is just the imperfection resonance caused by the presence of the partial snake. The other two terms are intrinsic resonances (driven at $K - G\gamma \pm \nu_x = N$) corresponding to the horizontal betatron motion at the snake and the applied coupling resonance respectively. These latter two terms combine linearly and if source terms like skew quadruoles are readily available in the lattice, the coupling resonance can therefore be chosen such that the intrinsic resonance terms sum to zero.

3. Implementation of correction at the AGS

For a typical horizontal tune near 8.72 and the standard ramp rate ($dG\gamma/d\theta = 4.7 \times 10^{-5}$), the horizontal resonance crossings are separated in time by 4-5 ms. Between injection and extraction energy ($G\gamma = 4.5 - 45.5$) eighty-two of these resonances are crossed. Since the phasing of resonances due to betatron coupling relative to those due to the snakes varies with energy, this means that different skew quadrupole settings are required for each resonance every few milliseconds. The relative phasing also depends on the location of each skew quadrupole relative to the snakes. The number, spacing and varied phasing of the resonances suggests a solution involving many distributed skew quadrupoles to better cover the vector space of the resonance driving terms.

A set of 15 skew quadrupoles is chosen for the AGS. The quadrupoles are only 17 cm in physical length to maximize the number of possible installation locations. The magnets pulse with ramp up and down times of 1 ms and hold a flattop for 1.3 ms centered on each resonance crossing time. Pulses go to a maximum of 275 A (for an integrated skew quad gradient of 0.3 T). The resonance strengths from the snakes and the skew quads are calculated using the SPRINT code [8]. The 15 skew quadrupole strengths at each resonance are determined by three constraints: reduction of the model-calculated net spin resonance strength to zero, maintaining the betatron tune shifts from the resultant coupling to <0.005 and minimization of orbit distortion effects. A typical set of corrector functions is shown in Figure 1.

3.1 Proof of principle test

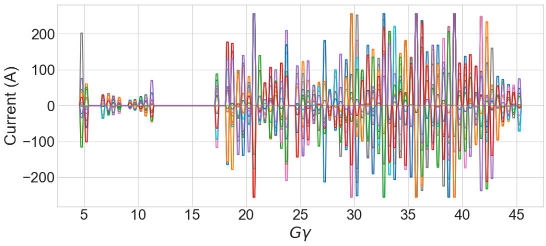
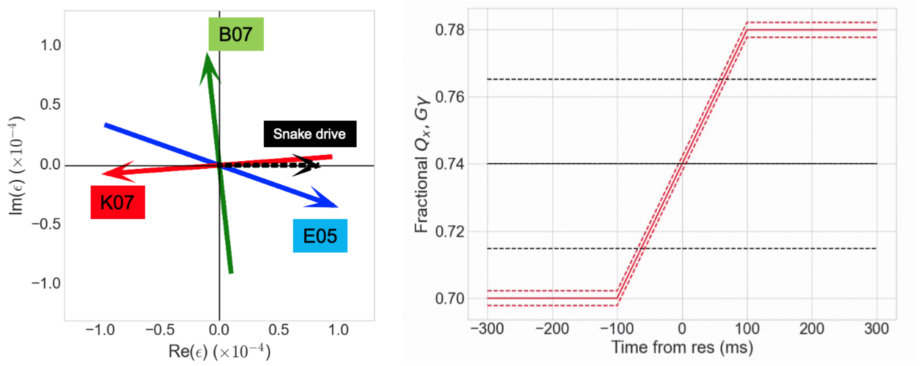


Figure 1: Typical excitation currents for the skew quadrupole correctors. Each trace is a difference magnet, independently powered. The gap around $G\gamma$=15 is near the transition crossing. These were omitted during commissioning.

Individual resonance crossings only result in 0.1-0.5% polarization loss during the ramp, so tuning or measurement of individual resonance coorrection is impractical. To validate the correction mechanism it is necessary to amplify the depolarization from a single resonance by, for example, slowing the crossing rate significantly. This is accomplished in the AGS by accelerating to a fixed energy near a resonance (in this case $G\gamma$ = 45.74). The horizontal tune begins below the resonance condition and ramped slowly across it. The horizontal tune changes from Q_x =8.7 to 8.78 over 200 ms, crossing the resonance condition $G\gamma - Q_x$ = 37 at a rate 25 times slower than crossings during acceleration. The slow crossing results in a relative polarization loss of ≈20% which is practically measurable. Individual skew quadrupoles are then powered one at a time. Three of the fifteen are selected for a scan such that a positive strength would result in strengthening (in phase), weakening (out of phase) or little effect (orthogonal) on the depolarization from the resonance crossing. The experimental design is shown schematically in Figure 2.



(a) Resonance driving terms from the partial snakes (black) and three skew quadrupoles at different location in the AGS. The labels indicate ring locations in the AGS nomenclature. The direction of the arrow represents the direction of increasing positive current, the lengths indicate the full range of available current from the power supplies.

(b) Resonance crossing schematic. Black lines indicate the central energy (solid) and +/-3σ energy spread (dashed). Red indicates the same for the fractional horizontal betatron tune.

Figure 2: Single resonance crossing configuration.

The results of the measurement are shown in Figure 3. The experiment demonstrates that appropriately chosen skew quadrupole excitations can fully correct a depolarizing resonance excited by a partial snake. It also validates the model predictions of the relative phasing of the drive terms.

3.2 Implementation during acceleration

The main impediment to pulsing the skew quadrupoles during acceleration is orbit effects. The baseline horizontal orbit in the AGS has large excursions (e.g. ±2 cm near injection energy, ±1 cm at higher energies). Since the vertical betatron tune is relatively close to an integer (8.985-8.99), powering a skew quadrupole in the presence of such large horizontal orbit excursions can lead to unacceptably large vertical orbit excursions. A beam-based approach is taken to skew quad centering, since beam position monitors are not available adjacent to the skew quad locations. Each skew quadrupole is pulsed individually at each resonance time at the highest current that beam loss from orbit excursions will allow. The resulting vertical difference orbit is caused by an angular deflection, $\theta = g\Delta x/B\rho$, inside the skew quad, where g is the integrated gradient, Δx is the horizontal distance between the beam and the skew quad center and $B\rho$ is the magnetic rigidity of the beam.

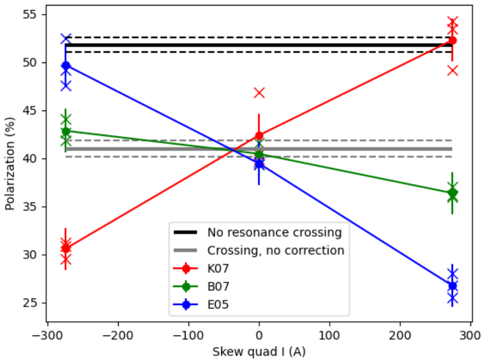
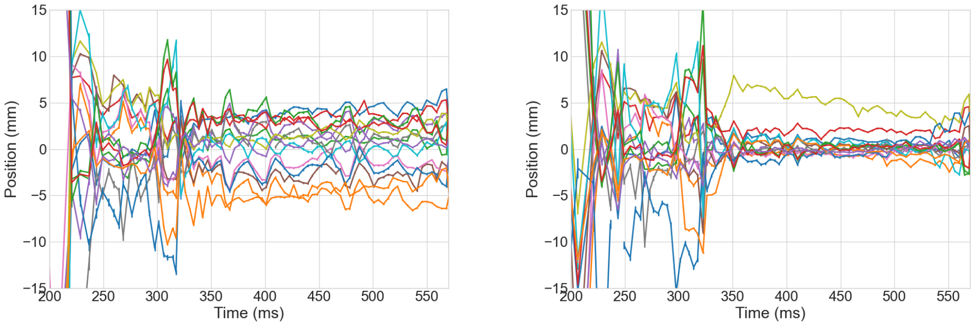


Figure 3: Effect of skew quad excitation on polarization after a single slow resonance crossing. Polarization without the crossing is denoted in black, uncorrected crossing in gray. Three skew quads are powered in turn (indicated by red, green and blue curves) and indicate that an appropriately phased skew quad can fully recover the polarization transmission.

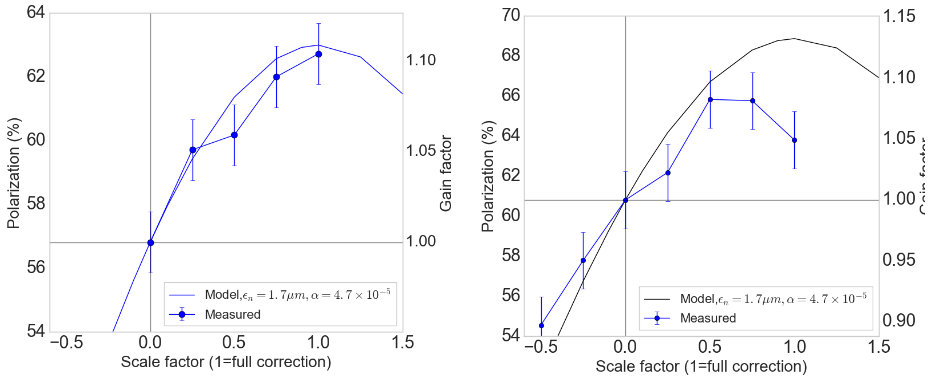
Figure 4 shows the results of beam-based measurements of the horizontal beam positions at the skew quads before and after correction. At low energy (prior to ≈320 ms), the orbit is constrained by the needs of injection and accommodation of the orbit effects of the helical dipoles and therefore requires special care. No attempt was made to center the orbits during that interval during this commissioning phase. After 320 ms (above γ=8), those effects are minimal and centering is accomplished using the standard orbit corrector dipoles. The efficacy of the correction is limited by the corrector strengths. Further improvement in centering is possible with an upgrade of the orbit correction system and improved alignment of the accelerator magnets.



(a) **(b)**

Figure 4: Horizontal orbit position in each skew quadrupole during acceleration. Before (a) and after (b) correction. Position is inferred from vertical difference orbits resulting from skew quadrupole excitation.

With orbit effects minimized, the effects of the correction on the polarization transmission through acceleration can be measured. Figure 5 shows the increase in polarization while the correction currents are gradually introduced from zero (no correction) to full correction. The model prediction of the polarization gain is calculated using SPRINT to calculate the residual resonance strengths and the Froissart-Stora equation to calculate the resulting polarization loss. Figure 5a shows the measurement to model comparison when only high energy (γ>8) resonances are included. The results show good agreement with the model, which assumes full correction of any resonance for which the skew quadrupoles pulse. The agreement with the model is worse when the skew quads are pulsed during the lower energy resonances (Fig. 5b). This suggests that the calculation of the resonance correction is largely accurate for the high energy cases and less so for the low energy resonances. Possible error sources for correction of the low energy resonances are optical errors due to large orbit offsets or model deficiencies (which would change the required skew quad strengths) and betatron coupling, which would add vectorially with the resonance driven by the snakes. Future work will focus on measuring and eliminating or compensating for those effects.



(a) **(b)**

Figure 5: Polarization improvement from increasing correction strength using (a) pulses above $\gamma = 8$ and (b) pulses over the whole ramp. Future work will focus on improving the performance of the low energy pulses.

4. Conclusions

A set of 15 fast-ramping skew quadrupoles has been installed in the AGS in order to compensate for depolarizing resonances driven by the horizontal motion in the snake magnets. A proof of principle experiment was successfully performed at fixed energy showing that the betatron coupling introduced by a set of skew quadrupoles could completely recover the polarization lost due to a snake-driven horizontal resonance. It was further demonstrated that enabling the skew quadrupoles during acceleration the depolarization could be reduced by a factor in good agreement with the accelerator model.

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