PROCEEDINGS OF SCIENCE

Neutrino oscillations and entanglement

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The concept of neutrino oscillations is revisited. Using the Quantum Field Theory approach and applying plane waves for the initial and final states, the processes of neutrino production, propagation, and detection are described through a single Feynman diagram. The updated S-matrix approach ensures energy and momentum are conserved during neutrino oscillations. A master formula is provided to calculate the rate of charged lepton production for neutrinos oscillating over a macroscopic distance L. The process $\pi^+ + n \longrightarrow \mu^+ + e^- + p$ serves to highlight this formalism, and the results obtained manifest the entanglement of the three involved processes: the production, propagation, and absorption of neutrinos.

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Within the standard quantum mechanical (QM) approach [1], neutrino oscillations are broken down into three independent steps: neutrino production at a source **S**, neutrino propagation over a distance L to a detector **D** described by a plane wave, and neutrino interaction with the detector's medium [2]. Taking into account the charged current weak interaction vertices, we have $S \rightarrow S' + \ell_\alpha^+ + \nu_\alpha$, $\nu_\alpha \rightarrow \nu_\beta$, $\nu_\beta + D \rightarrow D' + \ell_\beta^-$. The flavor indices α, β stand for e, μ, τ. Here, $S(D)$ and $S'(D')$ represent initial and final hadrons or nuclei in the vertex **S**(**D**), respectively. In the case of a meson decay, the S' hadron is missing.

Recently, a new formalism for neutrino oscillations has been developed within the framework of Quantum Field Theory (QFT), based on a single Feynman diagram [3]. This innovative approach involves analyzing two consecutive weak charged current processes occurring in the source (**S**) and the detector (**D**). These processes are integrated into a single second-order Feynman diagram represented by the interaction $S + D \rightarrow \ell_\alpha^+ + \ell_\beta^- + S' + D'$. The particles involved are described using plane waves, and the effective vertices in the diagram are separated by a macroscopic distance L.

The developed S-matrix formalism guarantees the conservation of momentum and energy at each vertex and throughout the entire process. In this approach, the fermionic neutrino propagator is utilized to describe the propagation of neutrinos, as opposed to the quantum mechanics formalism, which focuses on the space-time evolution of a plane wave [2]. For the differential rate of the second-order process being analyzed, a master formula that depends on the variable L was derived:

$$\begin{aligned}
d\Gamma^{\alpha\beta}(L) = &\sum_{km} U_{\alpha k} U^*_{\beta k} U_{\alpha m} U^*_{\beta m} \, \frac{e^{i(p_k - p_m)L}}{4\pi L^2} \mathcal{F}^{\alpha\beta}_{km} \times \\
&(2\pi)^7 \, \delta(E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S) \delta(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S) \delta(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_m) \times \\
&\frac{1}{4 E_S E_D} \, \frac{1}{\hat{J}_S \hat{J}_D} \frac{d\mathbf{p}_\alpha}{2E_\alpha \, (2\pi)^3} \, \frac{d\mathbf{p}_\beta}{2E_\beta \, (2\pi)^3} \, \frac{d\mathbf{p}'_S}{2E'_S \, (2\pi)^3} \, \frac{d\mathbf{p}'_D}{2E'_D \, (2\pi)^3},
\end{aligned} \tag{1}$$

where

$$\begin{aligned}
\mathcal{F}^{\alpha\beta}_{km} &= 4\pi \sum_{\text{spin}} \frac{1}{2} \left(T^{\alpha\beta}_k \left(T^{\alpha\beta}_m \right)^* + T^{\alpha\beta}_m \left(T^{\alpha\beta}_k \right)^* \right) \quad \text{and} \\
T^{\alpha\beta}_k &= J^\mu_S(P'_S, P_S) J^\nu_D(P'_D, P_D) \, \overline{u}(P_\beta; \lambda_\beta) \gamma_\nu (1 - \gamma_5) \not{Q}_k \gamma_\mu v(P_\alpha; \lambda_\alpha).
\end{aligned} \tag{2}$$

Here, $J^\mu_S(P'_S, P_S)$ and $J^\nu_D(P'_D, P_D)$ are the hadronic currents associated with the weak interaction at source and in the detector as defined in [3], respectively. The notation for the 4-momenta of hadrons and leptons is $P_{S,D} \equiv (E_{S,D}, \mathbf{p}_{S,D})$, $P'_{S,D} \equiv (E'_{S,D}, \mathbf{p}'_{S,D})$ and $P_{\alpha,\beta} \equiv (E_{\alpha,\beta}, \mathbf{p}_{\alpha,\beta})$. $Q_k \equiv (E_\nu, \mathbf{p}_k)$,= is the 4-momenta of neutrino with mass m_k which energy fulfills the relation $E_\nu = E_S - E'_S - E_\alpha = E_\beta + E'_D - E_D$. The factor $1/(\hat{J}_S \hat{J}_D)$ ($\hat{J} = 2J + 1$) is due to averaging over spin projections of the initial hadrons. $U_{\alpha k}$ ($k = 1, 2, 3$) is the elements of the Pontecorvo-Maki-Nakagawa-Sakata neutrino mixing matrix.

The master formula in Eq. (1) is general and can be adapted, with the appropriate modifications (such as the number of particles produced at the source and detector), for any second-order process involving on-shell intermediate neutrinos regardless of whether charge or neutral currents of neutrinos are considered. It is maintained that it cannot be simplified to the traditional neutrino oscillation probability approach, which separates neutrino production, propagation, and detection. This is illustrated by the process $\pi^+ + n \rightarrow \mu^+ + e^- + p$. With some kinematical assumptions

($E_e \simeq E_\nu$ as $E_{n,p} \simeq m_{n,p} = m_N$), the production rate can be written as [3]

$$\Gamma^{\pi^+ n}_{QFT} = \frac{1}{2\pi^2}\, G_\beta^4 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi}\, E_\nu^2\, \frac{P^{\mathrm{QFT}}_{\mu e}(E_\nu, L)}{4\pi L^2} \left(g_V^2 + 3 g_A^2\right)\, p_e E_e, \tag{3}$$

with

$$\mathcal{P}^{\mathrm{QFT}}_{\mu e}(E_\nu, L) = \frac{1}{2} \sum_{km} U_{ek} U^*_{\mu k}\, U^*_{em} U_{\mu m}\, e^{i(p_m - p_k)L} \left(1 + \frac{p_k p_m}{E_\nu^2}\right). \tag{4}$$

Here, $G_\beta = G_F \cos\theta_C$, where $\cos\theta_C$ is the Cabbibo angle. E_ν (p_ν) and E_e (p_e) are the energies (momenta) of neutrino and electron, respectively. m_π (m_μ) being the mass of pion (muon). The nucleon's vector and axial-vector coupling constants are denoted by g_V and g_A, respectively. f_π is the pion decay constant. $p_{k,m} = \sqrt{E_\nu^2 - m_{k,m}^2}$ is the momentum of a neutrino with mass $m_{k,m}$. The presence of the term $p_k p_m / E_\nu^2$, which appears due to consideration of the neutrino propagator, manifest entanglement of the processes of production and detection of neutrinos unlike it is in the standard QM concept. For $m_{k,m} \ll E_\nu$, $(p_k p_m)/E_\nu^2 \simeq 1 - (m_k^2 + m_m^2)/2E_\nu^2 \simeq 1$ and the oscillation probability is reproduced.

The production rate described in Eqs. (3) and (4) cannot be simplified to align with the conventional approach based on the concept of neutrino oscillation probability. This raises concerns about the validity of disentangling the three processes involved: the production, propagation, and absorption of neutrinos. The two weak processes occurring at the source and the detector are weakly coupled, as indicated by the lepton current trace in Eq. (2). The nature of this coupling should be critically examined for any specific process involving neutrino oscillations, especially in relation to the associated kinematics, which includes both energy and angular correlations of the emitted leptons.

In summary, a new quantum field theory formalism for neutrino oscillations is briefly presented. This approach combines the processes of neutrino emission and detection into a single second-order Feynman diagram. It is shown that these two processes are entangled within this formalism, which contrasts with the quantum mechanics concept of oscillation probability that assumes complete independence between neutrino production and detection. This entanglement is evidenced by the term $p_k p_m / E^2$, which differs from unity.

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References

[1] V.N. Gribov and B. Pontecorvo, Phys. Lett. B **28**, 493 (1969).

[2] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. **59**, 671 (1987).

[3] S. Kovalenko and F. Šimkovic, J. Phys. G **51**, 035202 (2024).