

## Inverse seesaw with flavour and CP symmetries

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We study charged lepton flavour violation (cLFV) in a scenario in which light neutrino masses are generated via the inverse seesaw mechanism with 3 + 3 heavy sterile states. Lepton mixing is predicted with the help of flavour symmetries combined with CP. The impact of current and prospective bounds of different cLFV processes on the parameter space of this scenario (choice of symmetry and its residuals, mass scales  $\mu_0$  and  $M_0$  and the free angle  $\theta_R$ ) is analysed.

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## 1. Introduction

The Standard Model (SM) is very successful. Nevertheless, several phenomena are not explained within the SM, such as the replication of fermion generations, the fermion masses, quark and lepton mixing and their striking difference, the baryon asymmetry of the Universe, and Dark Matter. Beyond SM (BSM) theories can have a rich phenomenology, because processes that are highly suppressed or even forbidden in the SM can be in the reach of current/near-future experiments, e.g.  $\mu \rightarrow e \gamma$ . On the one hand, both flavour and CP violation need to be kept well under control in these extensions. On the other hand, possible correlations among different signals can give further information about the flavour sector. In the following, we consider a framework in which the SM is augmented by 3 + 3 heavy sterile states  $N_i$ ,  $i = 1, 2, 3$ , and  $S_j$ ,  $j = 1, 2, 3$ , and light neutrino masses arise from the inverse seesaw mechanism [1]. A flavour symmetry  $G_f$ , belonging to the series of groups  $\Delta(3n^2)$  and  $\Delta(6n^2)$  [2], and a CP symmetry are imposed that are non-trivially broken in the charged lepton sector (to the residual group  $G_e = Z_3$ ) and among the neutral states (to  $G_\nu = Z_2 \times CP$ ).

## 2. Scenario and results

The Lagrangian, relevant for the masses of the neutral states, is of the form

$$- (y_D)_{\alpha i} \bar{L}_\alpha^c H N_i^c - (M_{NS})_{ij} \bar{N}_i S_j - \frac{1}{2} (\mu_S)_{kl} \bar{S}_k^c S_l + \text{h.c.} , \quad (1)$$

where  $L_\alpha$  are left-handed lepton doublets,  $H$  the Higgs field,  $y_D$  the Dirac neutrino Yukawa matrix,  $M_{NS}$  the matrix connecting the gauge singlets  $N_i$  and  $S_j$  and  $\mu_S$  the Majorana mass matrix of the singlets  $S_i$ . Defining the Dirac neutrino mass matrix  $m_D$  as  $m_D = y_D \langle H \rangle$  with  $\langle H \rangle \approx 174$  GeV, the masses of all neutral states originate from the nine-by-nine mass matrix

$$\mathcal{M}_{\text{Maj}} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_{NS} \\ 0 & M_{NS}^T & \mu_S \end{pmatrix} , \quad (2)$$

given in the basis  $(\nu_{\alpha L}, N_i^c, S_j)$ . For  $|\mu_S| \ll |m_D| \ll |M_{NS}|$  the light neutrino mass matrix reads at leading order [3]

$$m_\nu = m_D \left( M_{NS}^{-1} \right)^T \mu_S M_{NS}^{-1} m_D^T , \quad (3)$$

while the heavy sterile states form pseudo-Dirac pairs. The upper-left three-by-three submatrix  $\tilde{U}_\nu$  of the (unitary) mixing matrix determines lepton mixing

$$\tilde{U}_{\text{PMNS}} = U_\ell^\dagger \tilde{U}_\nu = \tilde{U}_\nu = (1 - \eta) U_0 \quad \text{with} \quad \eta = \frac{1}{2} m_D^* \left( M_{NS}^{-1} \right)^\dagger M_{NS}^{-1} m_D^T , \quad (4)$$

since  $U_\ell = 1$  in the analysed scenario.<sup>1</sup>

Here, we present results for option 2 of the scenario [4] in which  $N_i$  and  $S_j$  both transform as irreducible, real (and unfaithful) representation  $\mathbf{3}'$ , while  $L_\alpha$  form an irreducible, (in general) complex and faithful irrep, called  $\mathbf{3}$ , of  $G_f$ . In this way, both  $M_{NS}$  and  $\mu_S$  are non-zero in the limit in which  $G_f$  (and CP) are unbroken and have a trivial structure, whereas all flavour information is encoded in  $y_D$  that preserves the residual group  $G_\nu$ ,

$$M_{NS} = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \mu_S = \mu_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{with} \quad M_0, \mu_0 > 0 \quad (5)$$

<sup>1</sup>The choice of the transformation properties of  $L_\alpha$  and the right-handed charged leptons under  $G_f$ , the residual group  $G_e$  and the basis ensure that the charged lepton mass matrix is diagonal and has three independent entries.

and

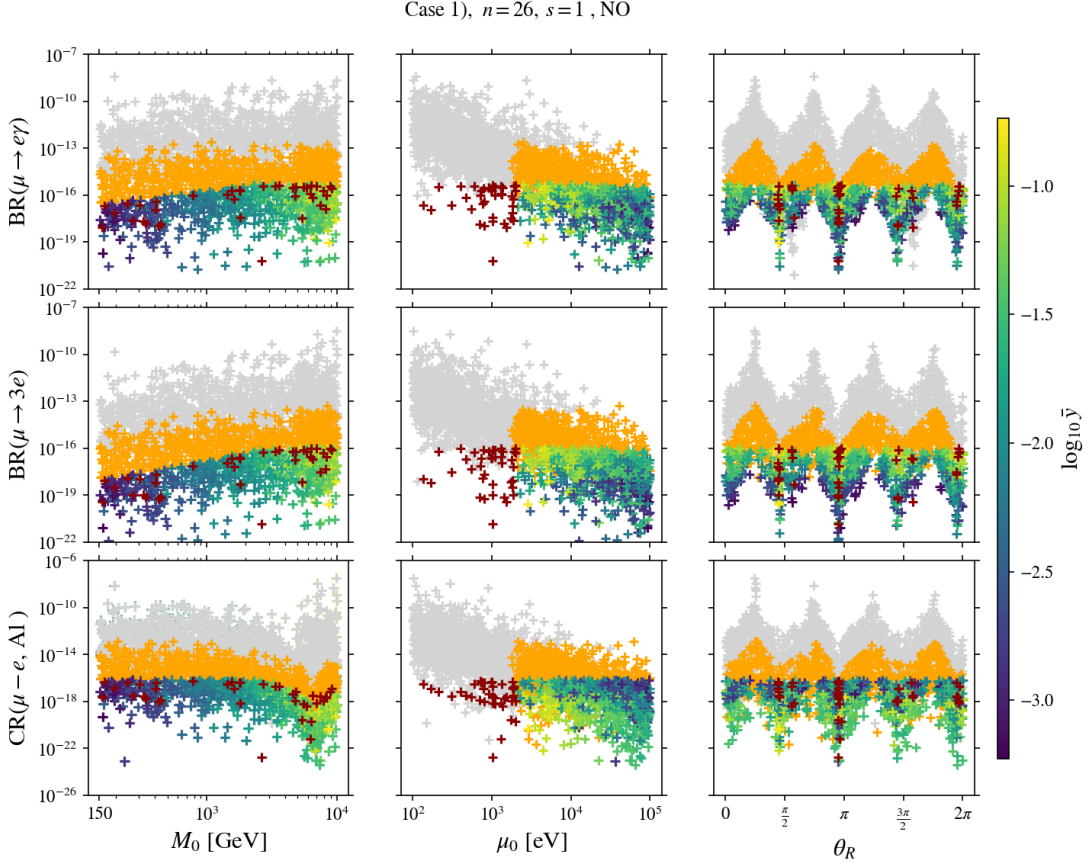
$$y_D = \Omega(\mathbf{3})^* R_{ij}(\theta_L) \text{diag}(y_1, y_2, y_3) P_{kl}^{ij} R_{kl}(-\theta_R) \Omega(\mathbf{3}')^T. \quad (6)$$

The matrices  $\Omega(\mathbf{3})$  and  $\Omega(\mathbf{3}')$  are determined by the CP transformation, and the planes of the rotation matrices  $R_{ij}(\theta_L)$  and  $R_{kl}(\theta_R)$  are given by the plane of the two degenerate eigenvalues of the  $Z_2$  generators  $Z(\mathbf{3})$  and  $Z(\mathbf{3}')$  in the basis transformed by  $\Omega(\mathbf{3})$  and  $\Omega(\mathbf{3}')$ , respectively. If these two planes do not coincide, the permutation matrix  $P_{kl}^{ij}$  is necessary. The form of these matrices depends on the mixing pattern which we consider, i.e. Case 1) through Case 3 b.1), see [5–7]. The parameters of the neutral sector are: two scales  $M_0$  and  $\mu_0$ , three real couplings  $y_1, y_2, y_3$  as well as two real angles  $\theta_L$  and  $\theta_R$ . Their role is the following:  $M_0$  sets the mass of the three pseudo-Dirac pairs and its scanned range is  $150 \text{ GeV} \leq M_0 \leq 10 \text{ TeV}$ ,  $\mu_0$  breaks lepton number and its size is small,  $100 \text{ eV} \leq \mu_0 \leq 100 \text{ keV}$ , the couplings  $y_f$  are adjusted to reproduce light neutrino masses (their ordering, NO/IO, and the lightest neutrino mass  $m_0$ ) and in the scan they lie in the interval  $4 \cdot 10^{-5} \lesssim y_f \lesssim 1.2$ , the angle  $\theta_L$  is fitted such that the measured lepton mixing angles are accommodated best, and  $\theta_R$  is a free parameter that varies in the range  $[0, 2\pi]$ . We have studied lepton mixing and cLFV for all four cases, Case 1) through Case 3 b.1), analytically and numerically, see [4].

Among the analytical results for cLFV processes we highlight that (i) the branching ratios (BRs)  $\text{BR}(l_\beta \rightarrow l_\alpha \gamma)$  and  $\text{BR}(l_\beta \rightarrow 3 l_\alpha)$  as well as the conversion rate (CR)  $\text{CR}(\mu - e, \text{N})$  are mostly proportional to  $|\eta_{\alpha\beta}|^2$ , (ii) tri-lepton decays and  $\mu - e$  conversion in nuclei are dominated by  $Z$  penguin contributions, especially for larger  $x_0$ , where  $x_0 = \left(\frac{M_0}{M_W}\right)^2$ , and (iii) a strong suppression of  $\text{CR}(\mu - e, \text{N})$  can be observed for a certain value of  $x_0$  depending on the nucleus N, e.g. for aluminium  $x_0 \approx 6470$  corresponding to  $M_0 \approx 6.5 \text{ TeV}$ , see also [8].

For concreteness, we show results for Case 1) and the choices  $n = 26$  ( $G_f$ ) and  $s = 1$  (CP), assuming light neutrino masses with normal ordering (NO) and  $m_0 = 0.03 \text{ eV}$ , see Fig. 1. We display the BRs of  $\mu \rightarrow e \gamma$  and  $\mu \rightarrow 3 e$  as well as the  $\mu - e$  conversion rate in aluminium,  $\text{CR}(\mu - e, \text{Al})$ . The colour-coding of the points indicates for grey points that at least one of the prospective bounds on  $\text{BR}(\mu \rightarrow e \gamma)$  [9],  $\text{BR}(\mu \rightarrow 3 e)$  [10] or  $\text{CR}(\mu - e, \text{Al})$  [11, 12] is violated as well as the current limit on at least one of the elements of the matrix  $\eta$  [13], while the orange points correspond to the situation in which at least one of the prospective bounds is violated, but all current constraints on the magnitude of the matrix elements  $\eta_{\alpha\beta}$  are fulfilled and red points represent data points that are excluded by the current limits on  $\eta$ , but not by the future bounds on the three studied  $\mu - e$  transitions. Furthermore, the different colours which can be read off from the colour bar signal the average value of the Yukawa coupling,  $\bar{y} = \frac{1}{3} (y_1 + y_2 + y_3)$ , for each viable data point that satisfies both the expected limits on the mentioned cLFV processes and the current bounds on  $\eta_{\alpha\beta}$ . As we can see, the parameter  $\mu_0$  needs to be larger than about 2 keV, the mentioned cancellation in  $\text{CR}(\mu - e, \text{Al})$  is clearly visible, and the shown quantities depend on  $\theta_R$  in such a way that for  $\cos 2\theta_R \approx 0$  these are enhanced, whereas for  $|\cos 2\theta_R|$  large a suppression occurs. We also notice that both the prospective bounds on  $\text{BR}(\mu \rightarrow 3 e)$  and for  $\mu - e$  conversion in aluminium can be reached, while  $\text{BR}(\mu \rightarrow e \gamma)$  remains about two orders of magnitude smaller than the future limit. So, the latter only poses a mild constraint on the scanned parameter space, while  $\text{BR}(\mu \rightarrow 3 e)$  can reduce it and the largest potential have  $\mu - e$  conversion experiments. Results for light neutrino masses with inverted ordering (IO) and smaller  $m_0$  are also given in [4]. For Case 2),  $n = 14$ ,  $s = 1$ ,  $t = 2$  (meaning  $u = 0$ ) and light neutrino masses with NO and  $m_0 = 0.03 \text{ eV}$ , we instead find no dependence on  $\theta_R$  for the cLFV observables. In addition, the BR of  $\mu \rightarrow e \gamma$  is larger than  $10^{-19}$  in the scanned parameter space and  $\text{BR}(\mu \rightarrow 3 e) \gtrsim 10^{-21}$  as well as  $\text{CR}(\mu - e, \text{Al}) \gtrsim 5 \cdot 10^{-22}$  outside the region of the cancellation. For the same case and values of  $n$  and  $s$  and light neutrino mass spectrum, but  $t = 1$  (corresponding to  $u = 1$ ), both BRs and the CR depend on  $\theta_R$  and are enhanced for  $\sin 2\theta_R \approx 0$  and suppressed for  $|\sin 2\theta_R|$  large. Comparing different choices of  $s$  and  $t$ , being equivalent to different CP symmetries, similar results are obtained.

The cLFV tau decays  $\tau \rightarrow \mu \gamma$ ,  $\tau \rightarrow e \gamma$ ,  $\tau \rightarrow 3 \mu$  and  $\tau \rightarrow 3 e$  have also been studied, but their BRs are generally much below current and future bounds.



**Figure 1:** Results for different cLFV observables for Case 1); for details see text and [4].

Results for option 1, in which only  $\mu_S$  has a non-trivial flavour structure, while  $y_D$  and  $M_{NS}$  preserve  $G_f$  and CP, can be found in [14].

### 3. Conclusions

Flavour and CP symmetries can be the key to understand fermion mixing and also fermion masses. The inverse seesaw mechanism is an interesting way to generate neutrino masses with potentially rich phenomenology. Different realisations of the residual group  $G_\nu$  among the neutral states lead to distinct signals; here we have presented results for option 2. Signals of cLFV processes ( $\mu - e$  transitions) can be sizeable, while effects on lepton mixing are small, but more general than for option 1, see [14].

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