PROCEEDINGS OF SCIENCE

Modular neutrinos

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In recent years, the (string-inspired) paradigm of modular invariance has been applied to the SM flavour puzzle with interesting results. Taking a bottom-up approach, one relies on very simple scalar sectors. Indeed, couplings and mass matrices can be functions of a single complex VEV, which is the only source of flavour symmetry breaking and of CP violation. Such setups can be used to predict neutrino masses, neutrino mixing angles and all (Dirac and Majorana) CP violating phases in the lepton sector. They may also shed light on the origin of fermion mass hierarchies.

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1. Modular flavour symmetries

The knowledge of the mixing patterns of fundamental fermions and the observation of hierarchies in their mass spectrum presents us with a curious set of puzzles. Why is neutrino mixing so different from quark mixing? Is there a dynamical reason explaining why the electron is much lighter than the muon, which in turn is $\sim$ 20 times lighter than the tau? Is the same mechanism responsible for the $O(30)$ factor between the atmospheric and solar mass-squared differences sourcing neutrino oscillations? What is the origin of CP violation in the quark sector, and is this source of CP violation shared with the lepton sector?

Taking the point of view that an organizing principle is at work, flavour symmetries arise as a natural candidate for such a principle. Traditionally, one assumes invariance of the action under a discrete non-Abelian symmetry connecting the different flavours. If one takes the choice of symmetry group as a starting point, it may later turn out to be difficult to arrange for its breaking while maintaining predictivity. One may have to deal with cumbersome scalar potentials, and introduce driving fields and shaping symmetries to guarantee some desired alignment. A more recent avenue in flavour model building considers instead modular flavour symmetries [1] (see [2, 3] for recent reviews), which in their minimal incarnation require a rather minimal symmetry-breaking sector. From a bottom-up perspective, the VEV of a complex scalar spurion — the modulus τ, with $\operatorname{Im}\tau > 0$ — can act as the sole source of both flavour symmetry breaking and CP violation. A detailed review of the modular invariance SUSY framework is beyond the present scope. Nevertheless, note that:

- Under modular transformations γ, the modulus τ transforms as

$$\tau \to \frac{a\tau + b}{c\tau + d}, \qquad \text{with } \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

  where γ is an element of the modular group Γ, generated by S ($\tau \to -1/\tau$) and T ($\tau \to \tau + 1$), with $(ST)^3 = S^4 = \mathbb{1}$ and $S^2T = TS^2$. Here, $a, b, c, d \in \mathbb{Z}$ satisfy $ad - cb = 1$.

- Comparing with the traditional flavour symmetry case, the transformation of (super)fields now includes an extra automorphy factor,

$$\psi \to (c\tau + d)^{-k}\rho_{\mathbf{r}}(\gamma)\psi\,,$$

  where k (or $-k$ in some conventions) is the so-called (modular) weight of ψ. From the bottom-up, one is free to select the representations $\mathbf{r}$ and the 'charges' k.

- The unitary representation matrix $\rho_{\mathbf{r}}$ is effectively a representation of a finite non-Abelian group Γ'_N, as one assumes a subgroup $\Gamma(N)$ of the full modular group (with $N = 2, 3, \ldots$ being the so-called level) to be 'quotiented out'. Thus, ρ is only sensitive to a finite number of cosets, i.e. the elements of $\Gamma'_N = \Gamma/\Gamma(N)$. Some of these small groups are isomorphic to those used in traditional flavour model building, like S_3 and T'.

- While modular symmetry does not constrain the non-holomorphic Kähler potential, it significantly restricts the allowed terms in the superpotential W. To be able to write modular-invariant terms, one makes use of modular forms $Y^{(k_Y)}_{\mathbf{r}_Y}(\tau)$. These are functions of the modulus transforming like superfields, $Y \to (c\tau + d)^{k_Y}\rho_{\mathbf{r}_Y}(\gamma)\,Y$. The predictive power of the setup is in part connected to the fact that, for a given k_Y, only a finite set of forms is available.

- Any value of the spurion τ breaks the full modular group Γ. To fit a minimal (no-flavons) modular-invariant model to data from the bottom-up, it is enough to scan τ in the fundamental domain $\mathcal{D}$, which loosely corresponds to $|\tau| \geq 1$ and $-1/2 \leq \operatorname{Re}\tau \leq 1/2$.
- One may impose a generalized CP (gCP) symmetry on the theory. Then, the VEV of τ can also be the only source of CP violation (CPV). Under gCP, the modulus transforms as $\tau \to -\overline{\tau}$ [4, 5]. Moreover, there is a basis in which $\psi \to \overline{\psi}$ and $Y(\tau) \to Y^*(\tau)$ under gCP [5]. Special values of τ preserve the CP symmetry. Within $\mathcal{D}$, these are $|\tau| = 1$ and $\operatorname{Re}\tau = 0, \pm 1/2$.
- At special 'symmetric points' in $\mathcal{D}$, some residual symmetry may also be preserved, e.g. a $\mathbb{Z}_3 \subset \Gamma$ symmetry for $\tau \simeq e^{2\pi i/3} \equiv \omega$. Thus, many entries of the mass matrix may vanish in the limit $\tau \to \omega$. For a small but non-zero deviation ϵ from the symmetric point, these zeroes are lifted by powers ϵ^n, which can lead to hierarchical fermion mass spectra [6].[1] In particular, a non-fine-tuned fit of neutrino data with charged-lepton mass hierarchies originating from this mechanism has been found in Ref. [6] for $\tau \simeq \omega$. While stabilizing τ at a small but non-zero distance from a symmetric point may seem *ad hoc*, simple modular-invariant potentials for τ can have non-fine-tuned minima in the requisite close vicinity of ω [7].

2. Neutrino mixing and CP violating phases

We recall here an example of a viable lepton flavour model [8]. Three gauge singlets N^c are added to the MSSM, with the following assignments $\psi \sim (\mathbf{r}, k)$ under the modular S_4 group ($N = 4$, with $\rho(S^2) = \mathbb{1}$): $L \sim (\mathbf{3}, 2)$, $E^c \sim (\mathbf{1}', 0) \oplus (\mathbf{1}, 2) \oplus (\mathbf{1}', 2)$, $N^c \sim (\mathbf{3}', 0)$, resulting in the superpotential

$$
\begin{aligned}
W &= \alpha \left(E_1^c L\, Y_{\mathbf{3}'}^{(2)}\right)_{\mathbf{1}} H_d + \beta \left(E_2^c L\, Y_{\mathbf{3}}^{(4)}\right)_{\mathbf{1}} H_d + \gamma \left(E_3^c L\, Y_{\mathbf{3}'}^{(4)}\right)_{\mathbf{1}} H_d \\
&\quad + g \left(N^c L\, Y_{\mathbf{2}}^{(2)}\right)_{\mathbf{1}} H_u + g' \left(N^c L\, Y_{\mathbf{3}'}^{(2)}\right)_{\mathbf{1}} H_u + \Lambda \left(N^c N^c\right)_{\mathbf{1}} ,
\end{aligned}
$$

where all parameters except g' can be taken to be real without loss of generality. This form of W corresponds (in a right-left convention) to the charged-lepton Yukawa matrix

$$
Y_e = \alpha \begin{pmatrix} Y_1 & Y_3 & Y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{Y=Y_{\mathbf{3}'}^{(2)}} + \beta \begin{pmatrix} 0 & 0 & 0 \\ Y_1 & Y_3 & Y_2 \\ 0 & 0 & 0 \end{pmatrix}_{Y=Y_{\mathbf{3}}^{(4)}} + \gamma \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ Y_1 & Y_3 & Y_2 \end{pmatrix}_{Y=Y_{\mathbf{3}'}^{(4)}} ,
$$

and, respectively, to the neutrino Dirac-type Yukawa matrix and N^c Majorana mass matrix

$$
Y_D = g \begin{pmatrix} 0 & Y_1 & Y_2 \\ Y_1 & Y_2 & 0 \\ Y_2 & 0 & Y_1 \end{pmatrix}_{Y=Y_{\mathbf{2}}^{(2)}} + g' \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix}_{Y=Y_{\mathbf{3}'}^{(2)}} , \qquad M_N = \Lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} .
$$

By varying $\tau \in \mathcal{D}$, as well as $\alpha, \beta, \gamma, g'/g$, and g^2/Λ, a successful fit of the data on lepton masses and mixing is possible both for $\arg g' \neq 0$ and $\arg g' = 0$. This means that the variant model where a gCP symmetry is imposed is also viable [5]. This variant fit predicts a lower atmospheric octant, $\sin^2\theta_{23} \sim 0.49$ and the sum of neutrino masses $\sum_i m_i \sim 0.08$ eV. A crucial aspect of modular models is that *all Dirac and Majorana CPV phases are determined*, since the full mass and Yukawa

[1]For each entry of the mass matrix, the exponents n are determined by the group Γ'_N, the field irrep and weight assignments $(\mathbf{r}, k)$, and the residual symmetry at the symmetric point.

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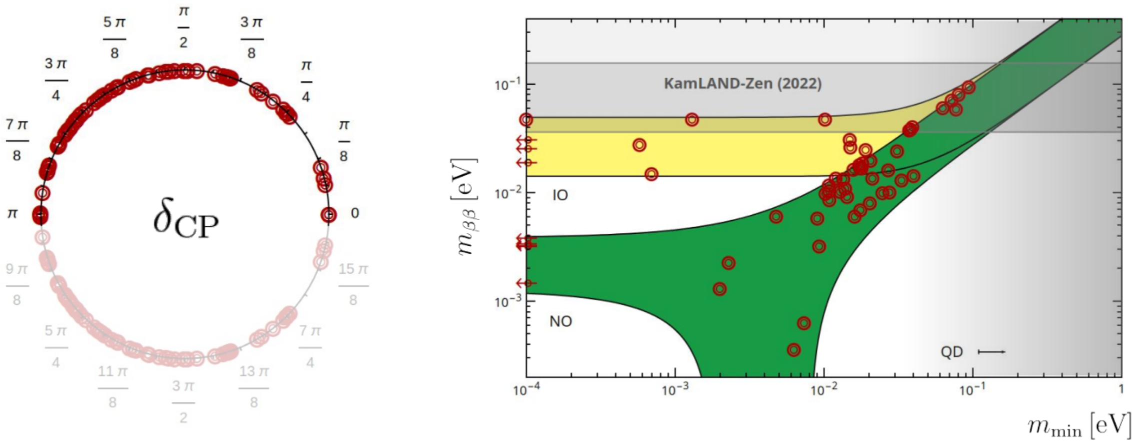

Figure 1: Predictions of modular models for the CPV Dirac phase (left) and the effective Majorana mass $m_{\beta\beta}$ as a function of the lightest neutrino mass (right). These figures were produced by surveying lepton-only modular models in Table 14 of Ref. [3]. Only *best-fit points* are shown (3σ-allowed ranges are larger).

matrices are constrained. One finds $\delta \sim 1.6\pi$, $\alpha_{21} \sim 0.3\pi$, and $\alpha_{31} \sim 1.3\pi$ for the gCP variant where only τ sources CPV, implying an effective Majorana mass $m_{\beta\beta} \sim 0.012$ eV in neutrinoless double-beta decay. An overview of predictions for CPV in modular models of the lepton sector alone is given in Figure 1. No universal prediction for the yet-unknown Dirac phase $\delta_{(\mathrm{CP})}$ seems to arise. The conclusion is similar for $m_{\beta\beta}$, with some hints of clustering around $m_{\beta\beta} \sim 0.01$ eV.

Parting words. Modular symmetry can offer a predictive framework for the understanding of the flavour puzzle. One can generate fermion mass hierarchies and CP violation via a single complex VEV (obtaining both, however, is not without its challenges, see e.g. [9]). One expects future efforts to also i) connect bottom-up and top-down results, ii) explore consistent non-SUSY alternatives, iii) unravel additional modular phenomenology, and iv) pursue hints of universality [10].

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