

# Loop Blow-Up Inflation: a novel way to inflate with the Kähler moduli

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The topic of this talk is a new inflationary model in the context of Type IIB string compactifications called *Loop Blow-Up Inflation*, presented in [1]. The original *Blow-Up Inflation* model [2], whose potential was purely non-perturbative, suffered the  $\eta$ -problem, being sensitive to string-loop corrections. If these corrections are introduced in the inflationary potential, they become dominant over the non-perturbative contributions as soon as the inflaton is displaced from its minimum. Therefore, the inflationary potential takes a new inverse-power law behavior. We show that slow-roll inflation is not ultimately spoiled and focus on the post-inflationary history in different scenarios of microscopic Standard Model (SM) realization. Each of them gives precise predictions for the spectral index and the tensor-to-scalar ratio in agreement with CMB data [8]. The model also predicts an amount of dark radiation potentially observable by next-generation CMB experiments.

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## 1. Inflating with Kähler moduli

Upon compactifying Type IIB string theory on a Calabi-Yau (CY) orientifold, one gets a 4d Effective Field Theory (EFT) featuring  $\mathcal{N} = 1$  SUSY. As a side product, one gets a huge number of *moduli*, 4d scalar fields with leading-order vanishing potential. In particular, the *Kähler moduli* sector of the 4d theory exhibits a classically flat potential, which is only lifted by quantum effects. This leading-order flatness makes a Kähler modulus a great candidate for a slow-roll *inflaton* field. A natural example of this is the model we will call *non-perturbative blow-up inflation* [2].

The Large Volume Scenario (LVS) balances  $\mathcal{O}(\alpha'^3)$  corrections to the Kähler potential  $K$  against non-perturbative corrections to the superpotential  $W$  of the  $\mathcal{N} = 1$  EFT to stabilize the moduli. Let us consider a CY with a volume of the form:

$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2} - \tau_\phi^{3/2} \quad \text{with } \tau_b^{3/2} \gg \tau_i^{3/2} \quad i = s, \phi \quad (1)$$

where the  $\tau$ 's are moduli controlling the volume of 4-cycles called *blow-up* modes, as they resolve localized singularities in the CY geometry. Trading in  $\tau_b$  for the overall volume, we get a F-term scalar potential of the form:

$$V_{\text{LVS}} = \tilde{V} \left[ \sum_{i=s,\phi} \left( B_i \frac{\sqrt{\tau_i} e^{-2a_i \tau_i}}{\mathcal{V}} - C_i \frac{\tau_i e^{-a_i \tau_i}}{\mathcal{V}^2} \right) + \frac{3\xi}{4g_s^{3/2} \mathcal{V}^3} + \delta V_{\text{up}} \right] \quad (2)$$

where  $B_i, C_i, \xi$  are constants depending on the details of the compactification and  $\delta V_{\text{up}}$  uplifts the minimum to Minkowski. This ensures that this potential has a non-SUSY minimum with:

$$\langle a_i \tau_i \rangle \sim \xi^{2/3} g_s^{-1}, \quad \langle \mathcal{V} \rangle \sim e^{a_i \tau_i}. \quad (3)$$

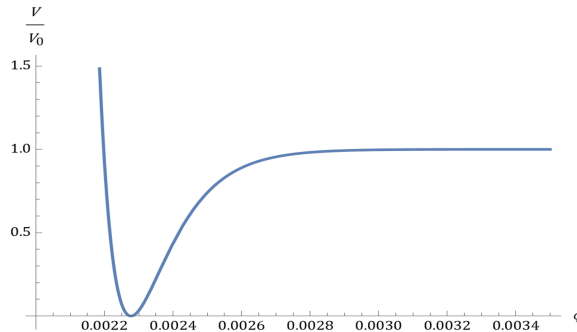
Suppose that  $\tau_\phi$  is displaced from its minimum towards larger field values while keeping the other moduli fixed. The resulting potential is shown in Fig 1. In the inflationary regime,  $\tau_\phi \gg \langle \tau_\phi \rangle$ , we can drop the doubly-exponentially suppressed term:

$$V(\tau_\phi) \simeq V_0 \left[ 1 - C_\phi \mathcal{V} \tau_\phi e^{-a_\phi \tau_\phi} \right] \quad (4)$$

and, upon canonical normalization  $\tau_\phi = (3\mathcal{V}/4)^{2/3} \phi^{4/3}$ , we get:

$$V(\phi) \simeq V_0 \left[ 1 - \tilde{C}_\phi \mathcal{V}^{5/3} \phi^{4/3} \exp[-\tilde{a}_\phi \mathcal{V}^{2/3} \phi^{4/3}] \right] \quad (5)$$

where all the constants due to the canonical normalization have been reabsorbed in  $\tilde{C}_\phi$  and  $\tilde{a}_\phi$ , and  $V_0 \simeq \tilde{V} \mathcal{V}^{-3}$ . In Fig 1 one can see that this potential exhibits an exponentially flat plateau already for small displacements of  $\phi$ , making it a perfect slow-roll potential.



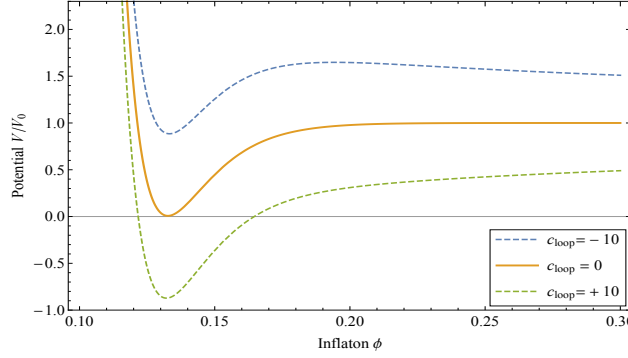
**Figure 1:** Potential of non-perturbative blow-up inflation with  $\mathcal{V} = 10^6$ ,  $a_\phi = 2\pi$ ,  $\tilde{C}_\phi = \left(\frac{3}{4}\right)^{2/3} 8\pi$ .

## 2. String Loops and Loop Blow-Up Inflation

The non-perturbative flatness of this potential makes it very susceptible to radiative corrections in the form of string loop corrections, as already noted in [2]. As of today, we still lack a precise calculation of string-loop corrections to the EFT on a CY background. However, a computation of 1-open-string loop corrections has been performed in a toroidal orientifold background in [3], and later generalized to a conjecture for CY orientifolds in [4]. Moreover, loop corrections have a precise understanding from the EFT point of view as contributions both to the 2-point functions and to the Coleman-Weinberg vacuum potential [5, 6]. For a blow-up mode  $\tau$  the leading corrections to the scalar potential go as:

$$\delta V(\tau) \simeq \frac{c_{\text{loop}}}{\mathcal{V}^3 \sqrt{\tau}} \quad (6)$$

where  $c_{\text{loop}}$  is a loop factor which in principle can depend on the complex structure moduli; at this stage, however, we take it as a constant. Inserting this correction in the potential (2) yields a potential as shown in Fig. 2.



**Figure 2:** Loop Blow-Up potential for different values of  $c_{\text{loop}}$ , setting  $\mathcal{V} \simeq 10^3$ ,  $a_\phi = C_\phi = 1$ . The uplift here is tuned by hand to distinguish the three cases.

Clearly,  $c_{\text{loop}} < 0$  leads to a runaway, so we stick to  $c_{\text{loop}} > 0$ . For  $c_{\text{loop}} \gtrsim 10^{-6}$ , the loop contribution dominates as soon as the inflaton is displaced from its minimum. Then the inflationary potential in terms of  $\phi$  becomes:

$$V(\phi) \simeq V_0 \left[ 1 - \frac{b c_{\text{loop}}}{\mathcal{V}^{1/3} \phi^{2/3}} \right]. \quad (7)$$

where the constant  $b$  contains the canonical normalization terms. This is a completely different inflationary potential form (5) and thus has different inflationary dynamics.

## 3. Inflationary and Post-Inflationary Dynamics

Using (7), we can retrieve the number of e-folds  $N_e$ , the spectral index  $n_s$ , the tensor-to-scalar ratio  $r$  and the amplitude of the scalar perturbations during inflation  $A_s$ :

$$N_e \simeq \frac{9}{16} \frac{\mathcal{V}^{1/3} \phi_*^{8/3}}{b c_{\text{loop}}} \quad (8)$$

$$n_s \simeq 1 - \frac{20}{9} \frac{bc_{\text{loop}}}{\mathcal{V}^{1/3} \phi_*^{8/3}} \quad (9)$$

$$r \simeq \frac{32}{9} \frac{(bc_{\text{loop}})^2}{\mathcal{V}^{2/3} \phi_*^{10/3}} \quad (10)$$

$$A_s \simeq \frac{3V_0}{16\pi^2} \frac{\mathcal{V}^{2/3} \phi_*^{10/3}}{(bc_{\text{loop}})^2} \quad (11)$$

where  $\phi_e$  and  $\phi_*$  are the values of the inflaton field at the end of inflation and at horizon exit respectively. Setting  $c_{\text{loop}} = 1/(16\pi^2)$  and imposing  $A_s = 2 \times 10^{-9}$  [8] we can get a relation between  $\phi_*$ ,  $\mathcal{V}$  and the number of e-folds:

$$\phi_* = 0.06 N_e^{7/22}, \quad \mathcal{V} = 1743 N_e^{5/11} \quad (12)$$

Therefore, to get a precise prediction from the model we need to determine the number of e-folds from post-inflationary dynamics. To do so, we adapt to our specific case a relation from modular cosmology [7]:

$$N_e \simeq 57 + \frac{1}{4} \ln r - \frac{1}{4} N_\phi - \frac{1}{4} N_\chi \quad (13)$$

where  $N_\phi$  and  $N_\chi$  are the number of e-folds of inflaton and volume domination after the end of inflation. These in turn depend on the microscopic realization of the Standard Model (SM) in the compact dimensions, i.e. the brane set-up. Moreover, the last dominating modulus to decay sources an amount of Dark Radiation (DR) depending on its branching ratio towards hidden sector light modes (e.g. volume mode axions). This is quantified in terms of the effective number of extra relativistic species  $\Delta N_{\text{eff}}$ , bounded by [8] to be  $\Delta N_{\text{eff}} \lesssim 0.5$  at 95% CL.

We distinguish 3 scenarios of post-inflationary evolution based on the brane setup determining the decay rate of the moduli:

1. *Scenario I*: SM is realized on D7-branes and the inflaton 4-cycle is wrapped by a D7-brane stack supporting a hidden sector
2. *Scenario II*: SM is realized on D7-branes and the inflaton 4-cycle is *not* wrapped by D7-branes
3. *Scenario III*: The SM is realized on D3-branes at singularity.

In each of these scenarios the post-inflationary history of the Universe has a slightly different play-out, but there are many common features. At the end of inflation, the inflaton field starts oscillating around its minimum, behaving as a cold matter condensate. The duration of this period depends on the Scenario, and lasts until the inflaton decays,  $1 \lesssim N_\phi \lesssim 11$ . The decay products of  $\phi$  dominate the energy density, redshifting as radiation. Meanwhile, the volume mode, whose canonically normalized counterpart we call  $\chi$ , oscillates around its minimum due to a slight misalignment during inflation [9], and redshifts as matter. Therefore, at some point it will become dominant for  $0 \leq N_\chi \lesssim 10.5$ , until its decay.  $N_\chi = 0$  corresponds to Scenario II, where  $\chi$  decays *before* matter-radiation equality. Using (13) we get the number of e-folds of inflation:

$$51.5 \lesssim N_e \lesssim 53 \implies \mathcal{V} \simeq \mathcal{O}(10^4) \quad \text{and} \quad \phi_* \simeq 0.2. \quad (14)$$

Thus we can have a precise prediction for the spectral index and the tensor-to-scalar ratio:

$$n_s \simeq 0.976 \quad \text{and} \quad r \simeq 2 \times 10^{-5}. \quad (15)$$

which are within bounds [8]. One of the most intriguing predictions of this model is the abundance of DR  $\Delta N_{\text{eff}}$ . Although in Scenario I this ends up being negligibly small, in Scenario III it may get to  $\Delta N_{\text{eff}} \simeq 0.36$ , which is well within reach of next-generation searches and experiments.

#### 4. Conclusions and Outlook

In this talk, we have presented a new inflationary model which solves the  $\eta$ -problem of *non-perturbative blow-up inflation*. The inflationary potential is given by perturbative corrections to the potential of a blow-up modulus and inflation is achieved in a region of field-space with full control over the EFT. Moreover, the values for  $n_s$  and  $r$  for this model are within current experimental bounds, and, most importantly, the prediction for  $\Delta N_{\text{eff}}$  is within reach of next-generation experiments.

Since this model combines perturbative and non-perturbative corrections in the same potential, much research still needs to be done. First, the role of the axions in the phenomenology of inflation must be established. Moreover, the robustness of this model under higher loop corrections and higher derivative corrections still needs to be assessed.

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