Overview of Axion-Electrodynamics in Plasmas

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This report is a succinct overview of the classical axion-electrodynamics formalism, outlining the key governing equations and noting the electrostatic wave splitting caused by axion interactions in plasmas.

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1. Introduction

In a context of renewed interest for experimental searches of Weakly Interacting Slim Paritcles (WISP) [1], the interaction between the axion and the electromagnetic field has emerged as a particularly accessible direction. Studies of radiative signatures in astrophysical processes and dedicated experiments have provided tentative constraints on the axion's mass and coupling-to-photons $(m, g_{\gamma\gamma})$ parameter space. However, their sensitivity remains out of reach for probing the QCD axion range, prompting further theoretical investigation into finer electromagnetic processes. One such avenue is the study of axion-plasmon dynamics, where the coupling between the axion field and plasma oscillations may yield distinctive signatures in wave propagation and instabilities.

2. Insights on Classical Axion-Electrodynamics

The full Lagrangian for the axion-photon interaction, effective below the $U(1)_{\mathrm{PQ}}$ symmetry breaking scale, is

$$\mathcal{L} = \underbrace{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{EM}} \underbrace{-A^{\mu}j_{\mu}}_{\text{Source}} \underbrace{-\frac{1}{4}g_{\gamma\gamma}\varphi F^{\mu\nu}F^{*}_{\mu\nu}}_{\text{Axion-Photon coupling}} \underbrace{+\frac{1}{2}\partial_{\mu}\varphi\partial^{\mu}\varphi - \frac{1}{2}m^{2}\varphi^{2}}_{\text{Kinetic terms}} \tag{1}$$

Natural units are used here, $c = \hbar = \epsilon_0 = \mu_0 = 1$, and thereafter. The Euler-Lagrange equations yield with respect to:

– the axion field φ,

$$(\partial^{\mu}\partial_{\mu} + m^2)\varphi = -\frac{g}{2}F \wedge F, \tag{2}$$

a Klein-Gordon equation with the source term $F \wedge F \equiv F^{\mu\nu}F^{*}_{\mu\nu} = 4\mathbf{E}\cdot\mathbf{B}$, particularly relevant to resistive effects in plasmas.

– the photon gauge field A^{μ},

$$\partial_{\mu}\tilde{F}_{\mu\nu} = j_{\mu}, \tag{3}$$

the Axion-Maxwell Equations expressed as the divergence of a dual-rotated EM field $\tilde{F} \equiv F + g\varphi F^{*}$. This rotation equates to the conservation of a (small) net current $j^{\nu}_{ax} = -g(\partial_{\mu}\varphi)F^{*\mu,\nu}$ in vacuum, associated to dual invariance.

The Bianchi Identity (3) yields, including the conserved current, the axial-photon dual wave equation:

$$\partial^{\mu}\partial_{\mu}\tilde{A}^{\nu} - \partial^{\nu}\partial_{\mu}\tilde{A}^{\mu} = \mu_0 j^{\nu} \tag{4}$$

$\tilde{A}^{i} = A^{i} + \chi^{i}$, with gauge offset χ verifying $\partial^{i} \times \chi^{i} = g\varphi E^{i}$, $\partial_0\chi^{i} + \partial^{i}\chi_0 = g\varphi B^{i}$.
The mass term can be eliminated considering the Lorentz Gauge on the axial projection of the photon field,

$$\partial_{\mu}\tilde{A}^{\mu} = 0, \tag{5}$$

but this imposes a mass-term to the wave equation for the standard photon field $\mathbf{A}$. It is therefore not possible to eliminate simultaneously both the photon and the axial-photon mass by a choice of gauge. With this remaining degree of freedom can be associated a quasi-particle state, the

axion-photon, acquiring mass via a non-vanishing derivative term that can also be identified as the axial current.
This axial current, or equivalently the gauge offset χ contributes to the photon Angular Momentum by a finite quantity[1] $\mathbf{J}_\chi$ parallel to the standard one $\mathbf{J}_0 = \int \mathbf{r}\times(\mathbf{E}\times\mathbf{B})\mathrm{d}^3\mathbf{r}$,

$$\mathbf{J}_\chi = g^2\varphi^2\mathbf{J}_0. \tag{6}$$

Reaching across a proper quantization of theory - mentioned hereafter - collinearity would preserve the same eigenvalue for the operator associated to $\mathbf{J}$, and CP invariance can be invoked to argue for the impossibility of mixing between a spin-0 and spin-1 field in electromagnetic vacuum [2].

Raffelt and Stodolsky made the case for mixing in a background magnetic field, further considering birefringence effects due to intense fields as well as in (neutral) media. This presentation makes the case for mixing in plasmas, following recent proposals [3, 4].

3. From Axion-Plasmon Dispersion Relation to Polaritons

A small displacement of an electron from its equilibrium position in a plasma can propagate as Langmuir waves, which correspond to a longitudinal polarization state for the photon field. The frequency of these oscillations is $\omega_{pe} = (\frac{4\pi\alpha n_e}{m_e})^{1/2}$. The dispersion relation for such longitudinal excitations of the photon field (i.e *plasmons*) can be obtained at finite temperature and compared to the axion's dispersion following the wave equations (4),(2):

$$\omega_{pl}^2 = \omega_{pe}^2 + \beta_{T_e}^2 k^2, \text{ plasmon bare dispersion, } \beta_{T_e} = \sqrt{\frac{3T_e}{m_e}} \tag{7}$$

$$\omega_\varphi^2 = m_\varphi^2 + k^2, \text{ axion bare dispersion, } m_\varphi = \sqrt{m^2 + 2g^2B_0^2} \tag{8}$$

The Plasma Frequency ω_{pe} and the axion effective mass m_φ can be fined tuned with Magnetic Field B_0 and Electron Density n_e values, to make the axion and plasmon bare dispersions relations compatible (i.e cross) at a certain wavenumber k.
When that happens, off-diagonal terms of the coupled dispersion relation provoke an avoided-crossing by mixing the axion and plasmon states. This results into the splitting of the longitudinal modes into a pair of Lower and Upper Polariton modes with frequency

$$\omega_{U,L}^2 = \frac{1}{2}\Big(\omega_\varphi^2 + \omega_{pl}^2 \pm \sqrt{(\omega_{pl}^2 - \omega_\varphi^2)^2 + 4\Omega^4}\Big) \tag{9}$$

The Rabi frequency $\Omega = (2g^2 4\pi\alpha B_0^2(n_0/m_e))^{1/4}$ characterizes the typical timescale over which the mixing occurs. A non-perturbative framework (e.g [5]) would be required to effectively quantize the modes in their exact form. It is nonetheless possible to approximate the coupled dispersion relation near the resonance $\omega \simeq \omega_{pe}$, under the Rotating Wave Approximation (RWA). This yields an interaction Hamiltonian

$$\hat{H}_{\gamma\varphi} = \sum_k \omega_{pl}\hat{a}_k^\dagger\hat{a}_k + \sum_k \omega_\varphi\hat{b}_k^\dagger\hat{b}_k + \Omega\sum_k \hat{a}_k^\dagger\hat{b}_k + h.c. \tag{10}$$

[1] In the presentation, an erroneous simplification was made by saying that spin angular momentum vanishes in a given gauge. This argument is revisited here relying on the gauge invariant, total angular momentum

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Mixing terms $\propto \Omega$ can be absorbed in diagonalization with Polariton operators $\hat{L}_k = u_k \hat{a}_k - v_k \hat{b}_k$ and $\hat{U}_k = u_k \hat{b}_k + v_k \hat{a}_k$ (u_v and v_k Hopfield coefficients specified in [3]). In the diagonal basis, the Hamiltonian is written

$$\hat{H}'_{\gamma\varphi} = \tilde{\omega}_L \sum_k \hat{L}_k^\dagger \hat{L}_k + \tilde{\omega_U} \sum_k \hat{U}_k^\dagger \hat{U}_k, \tag{11}$$

with RWA Polaritons frequencies:

$$\tilde{\omega}_{U,L} = \frac{1}{2}\left(\omega_\varphi + \omega_{pl} \pm \sqrt{(\omega_{pl} - \omega_\varphi)^2 + 4\Omega^2}\right). \tag{12}$$

4. Wrap-up

The axion-photon coupling essentially modifies Standard Electromagnetism by rotating the fields in dual-space by a small angle proportional to the axion field - whose propagation follows a Klein-Gordon equation sourced by resistive effects. Forbidden in electromagnetic vacuum, a plasma enables mixing of the axion field with longitudinal electrostatic waves. These oscillations thereby split into a pair of Polariton modes which are eigenstates of the axion-photon interaction Hamiltonian.

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