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Determination of $\mathcal{CP}$-violating HZZ interaction with polarised beams at the ILC

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We study possible CP-violation effects of the Higgs to Z-boson coupling at a future e^+e^- collider, e.g. at the International Linear Collider (ILC). We find that the azimuthal angular distribution of the muon pair, produced by $e^+e^- \to HZ \to H\mu^+\mu^-$, can be sensitive to such a CP-violation effect when we apply initial transversely-polarized beams. Based on the angular distribution, we construct a CP-sensitive asymmetry numerically and obtain this asymmetry by `Whizard` simulation. By comparing the SM prediction with the 2σ range for this asymmetry, we estimate the limit of a CP-odd coupling in HZZ interaction.

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1. Introduction

In order to explain the matter anti-matter asymmetry in the universe [1], the Baryogenesis has been proposed [2], which requires the CP-violation. In particular, a possible CP-violation source can be introduced in the Higgs sector, which is generated by the Two-Higgs-Doublet Model (2HDM) with spontaneous CP violation [3]. In this model, the CP-even 125 GeV Higgs boson gets CP-odd admixture and the Higgs to fermions couplings are CP-violating. So far, such a Baryogenesis and CP-violation test has been exploited by [4] via Htt interaction at LHC , and via $H\tau\tau$ at Higgs factories [5]. On the other hand, the CP-violating Higgs to fermions couplings can contribute a CP-violating Higgs to gauge bosons couplings at one-loop level. Therefore, the HVV interaction can be possibly CP-violated as well.

Currently, the LHC experiments, ATLAS [6] and CMS [7, 8] perform the measurement for the HVV coupling via $H \to 4\ell$ decay, and yield a limit of CP-odd HVV coupling, which can be interpreted as $(\tilde{c}_{ZZ})_{\text{CMS}} \sim [-1.63, 4.66]$ and $(\tilde{c}_{ZZ})_{\text{ATLAS}} \sim [-1.2, 1.75]$. On the other hand, one can probe directly the CP-violating HVV couplings at e^+e^- colliders (CEPC [9] and ILC), since a CP-violating HVV coupling can lead to deviations in the Higgs strahlung process at e^+e^- colliders. In addition, initial polarization of both beams can be applied at the ILC [10], where particularly the transverse polarization can be used to construct new CP sensitive observables [10, 11].

In this work, which is based on [12], we will focus on the Higgs strahlung process at the ILC with a center of mass energy of 250 GeV and apply transversely polarized electron-positron beams. In such a case, we construct the CP sensitives observable to probe the CP-odd HZZ coupling. Eventually, we try to determine the limit of the CP-odd HZZ coupling and compare with the other studies.

2. Theoretical framework

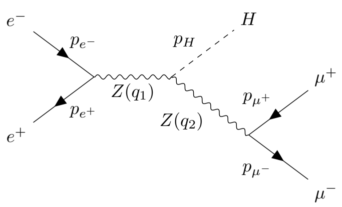
The Higgs to Z boson interaction can be expressed by the following effective lagrangian [13]:

$$\mathcal{L}_{\text{EFF}} = c_{\text{SM}}\, Z_\mu Z^\mu H - \frac{c_{HZZ}}{v} Z_{\mu\nu} Z^{\mu\nu} H - \frac{\tilde{c}_{HZZ}}{v} Z_{\mu\nu} \tilde{Z}^{\mu\nu} H, \tag{1}$$

where $\tilde{c}_{HZZ}$ denotes the loop-induced CP-odd coupling, and can be parameterised by the mixing angle $\tilde{c}_{HZZ} \propto \sin\xi_{CP}$. Concerning the polarization of the initial electron and positron beams, one defines a projection operator, which is given by:

$$\frac{1}{2}(1 - \boldsymbol{P}\cdot\boldsymbol{\sigma}) = \frac{1}{2}(\delta_{\lambda\lambda'} - P^a \sigma^a_{\lambda\lambda'}) = \frac{1}{2}\begin{pmatrix} 1 - P_3 & P_1 - iP_2 \\ P_1 + iP_2 & 1 + P_3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 - f\cos\theta_P & f\sin\theta_P e^{-i\phi_P} \\ f\sin\theta_P e^{i\phi_P} & 1 + f\cos\theta_P \end{pmatrix}, \tag{2}$$

where f is the polarization fraction. The Higgs strahlung $e^+e^- \to ZH$ is the dominant Higgs production process at $e^+\, e^-$ collider. By taking the polarization of the initial beams into account, the spin-density matrix of the Higgs strahlung $\rho_{\lambda_r\lambda_u}$ can be derived, see the following diagram,



where the λ_r, λ_u are the spin indices of the initial electron and positron. Eventually, we can obtain the spin-density matrix by applying the Bouchiat-Michel formula [14, 15], and the spin density matrix is given by

$$\begin{aligned}\rho^{ii'}(e^+e^- \to ZH) =&\frac{1}{2}(\delta_{\lambda_r\lambda_r'} + P_-^m\sigma^m_{\lambda_r\lambda_r'})\frac{1}{2}(\delta_{\lambda_u\lambda_u'} + P_+^n\sigma^n_{\lambda_u\lambda_u'})M^i_{\lambda_r\lambda_u}M^{*i'}_{\lambda_r'\lambda_u'}\\ =&(1-P_-^3P_+^3)A^{ii'} + (P_-^3 - P_+^3)B^{ii'} + \sum_{mn}^{1,2} P_-^m P_+^n C^{ii'}_{mn}.\end{aligned} \tag{3}$$

Furthermore, we introduced the CP-odd operator for the HZZ interaction of Eq. (1), and obtain the following form of the total amplitude squared:

$$\begin{aligned}|\mathcal{M}|^2 =& (1-P_-^3P_+^3)(\cos^2\xi_{CP}\,\mathcal{A}_{\text{CP-even}} + \sin 2\xi_{CP}\,\mathcal{A}_{\text{CP-odd}} + \sin^2\xi_{CP}\,\widetilde{\mathcal{A}}_{\text{CP-even}})\\ &+ (P_-^3 - P_+^3)(\cos^2\xi_{CP}\,\mathcal{B}_{\text{CP-even}} + \sin 2\xi_{CP}\,\mathcal{B}_{\text{CP-odd}} + \sin^2\xi_{CP}\,\widetilde{\mathcal{B}}_{\text{CP-even}})\\ &+ \sum_{mn}^{1,2} P_-^m P_+^n \left(\cos^2\xi_{CP}\,C^{mn}_{\text{CP-even}} + \sin 2\xi_{CP}\,C^{mn}_{\text{CP-odd}} + \sin^2\xi_{CP}\,\widetilde{C}^{mn}_{\text{CP-even}}\right).\end{aligned} \tag{4}$$

Note, that the CP-invariant parts with $\cos^2\xi_{CP}$ and $\sin^2\xi_{CP}$ are both CP-conserving, while the CP-mixing terms with $\sin 2\xi_{CP}$ violate the CP symmetry. Particularly, the CP-violating terms $\mathcal{A}_{\text{CP-odd}}$, $\mathcal{B}_{\text{CP-odd}}$ and $C^{mn}_{\text{CP-odd}}$ can be extracted by the triple-products, which are given by

$$\mathcal{A}_{\text{CP-odd}}, \mathcal{B}_{\text{CP-odd}} \propto \epsilon_{\mu\nu\alpha\beta}[p^\mu_{e^-}p^\nu_{e^+}p^\alpha_{\mu^+}p^\beta_{\mu^-}] \propto (\vec{p}_{\mu^+}\times\vec{p}_{\mu^-})\cdot\vec{p}_{e^-}, \tag{5}$$

$$C^{mn}_{\text{CP-odd}} \propto \epsilon_{\mu\nu\rho\sigma}[(p_{e^-}+p_{e^+})^\mu p^\nu_{\mu^+}p^\rho_{\mu^-}s^\sigma_{e^-}] \propto (\vec{p}_{\mu^+}\times\vec{p}_{\mu^-})\cdot\vec{s}_{e^-}. \tag{6}$$

The triple-product is the azimuthal-angle difference between the $\mu^+\mu^-$ plane and the spin of the electron. Therefore, defining the orientation of the azimuthal plane by fixing the direction of electron transverse polarization, the $C^{\text{CP-mix}}_{mn}$ depends directly on the azimuthal angle of the final state μ^-.

Regarding the triple products in the CP-odd amplitude terms, one can define two CP-odd observables

$$O^T_{CP} = \cos_{\theta_H}\sin 2\phi_{\mu^-}, \quad O^{UL}_{CP} = \cos\theta_\mu \sin\Delta\phi. \tag{7}$$

Hence, we can define the asymmetries by the CP-odd observables:

$$\mathcal{A}_{CP} = \frac{1}{\sigma_{\text{tot}}}\int \text{sgn}(O_{CP})d\sigma = \frac{N(O_{CP}<0) - N(O_{CP}>0)}{N(O_{CP}<0) + N(O_{CP}>0)}, \tag{8}$$

where N denotes the corresponding number of events. Since the SM is CP conserving for the neutral current, the SM background for this asymmetry is negligible. However, the number of events fluctuates statistically leading to the uncertainty of this asymmetry, which is based on a binomial distribution and given by $\Delta\mathcal{A}_{CP} = \sqrt{\frac{1-\mathcal{A}^2_{CP}}{N_{\text{tot}}}}$.

3. Determination of CP-violation at ILC

In order to study the ability of measuring CP-properties, we assume that the total cross-section is fixed to the SM value, while the CP properties are varied by the CP-mixing angle. In such a scenario, the dependence of the asymmetries $\mathcal{A}^T_{CP}$ and $\mathcal{A}^{UL}_{CP}$ are presented in Figs. 1, and both asymmetries linearly depend on the $\sin 2\xi_{CP}$. For the left panel, the asymmetry $\mathcal{A}^T_{CP}$, constructed by transverse polarisation, can be enhanced by a larger polarisation degree. Hence, the larger transverse polarisation degree would be helpful for the determination of the CP properties via $\mathcal{A}^T_{CP}$. In contrast, the right panel demonstrates that the size of $\mathcal{A}^T_{CP}$ is basically independent on the polarisation degree. However, the longitudinal polarisation can enhance the total cross-section and suppress the statistical uncertainty. Consequently, the determination precision of the $\sin 2\xi_{CP}$ can be improved as well by increasing the longitudinal polarisation degree.

By comparing the asymmetry values with the SM 2σ upper bounds, one can determine the limit of the CP-mixing angle $\sin 2\xi_{CP}$ and obtain the numbers in the Tab. 1. As we observed, the determination precision of using $\mathcal{A}^{UL}_{CP}$ are both much better than using $\mathcal{A}^T_{CP}$ individually, since the $\mathcal{A}^T_{CP}$ highly depends on the polarisation degree, and the positron beams can not be strongly polarised at 250 GeV. However, one can actually measure both $\mathcal{A}^T_{CP}$ and $\mathcal{A}^{UL}_{CP}$ simultaneously, when the initial beams are transversely polarised. Therefore, we can combine the two observables for this case and the sensitivity to CP-mixing angle can be improved as well. The combined results are also presented in the Tab. 1, which shows that the combined observables can lead to the similar precision as using the longitudinal polarisation.

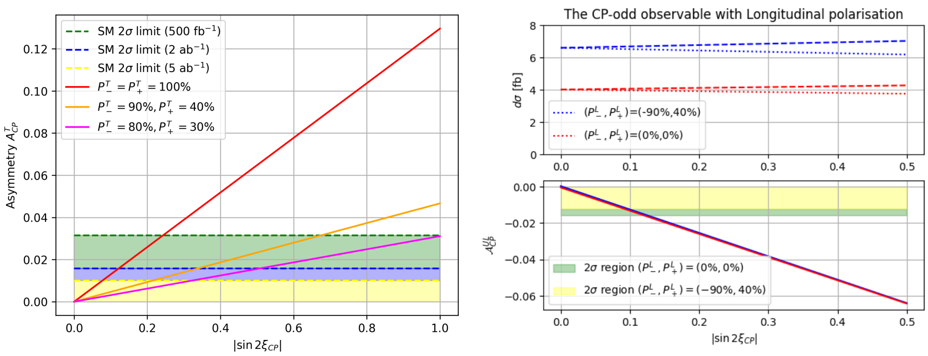


Figure 1: The analytical results of the asymmetries from Eq. (8) with the statistical uncertainties of the asymmetries, where the total cross section is fixed for varying $|\sin 2\xi_{CP}|$. Left panel: The asymmetry $\mathcal{A}^T$ constructed by O^T_{CP}. Right panel: The asymmetry $\mathcal{A}^{UL}$ constructed by O^{UL}_{CP}. The red solid line corresponds to the completely polarized beams $(P^T_-, P^T_+) = (100\%, 100\%)$, while the orange line and magenta line demonstrate the asymmetries with $(P^T_-, P^T_+) = (90\%, 40\%)$ polarized beams and $(P^T_-, P^T_+) = (80\%, 30\%)$ polarized beams, respectively. The blue and green region are the region below the 2σ limit of the SM CP-conserving case for 500 fb^{-1} and 2000 fb^{-1}, respectively.

Furthermore, we can explore another scenario, which fixes the SM contribution and the total cross-section is varied by the varying CP-odd coupling. In this scenario, we can compare the ability of determining the CP-odd coupling with the other experiments and analysis. In Tab. 2, we present the CP-odd coupling determination limits, where the number of ILC 250 GeV using polarisation is given by the case with transverse polarisation, since the combined results using transverse polarisation have roughly the same sensitivity to the CP-violation as the results using only longitudinal polarisation with the same polarisation degree. We can found that the result of using polarisation can improve the sensitivity to the CP-odd coupling comparing with the unpolarisation study at CEPC.

(P_-, P_+) Observables	$\mathcal{L}$ [ab^{-1}]	$\sin 2\xi_{CP}$ limit $\mathcal{A}^T_{CP}$	Combine $\mathcal{A}^T_{CP}$ & $\mathcal{A}^{UL}_{CP}$	$\mathcal{A}^{UL}_{CP}$
Transverse polarisation				
$(80\%, 30\%)$	2.0	[-0.50, 0.53]	[-0.113, 0.125]	
$(80\%, 30\%)$	5.0	[-0.36, 0.36]	[-0.068, 0.079]	
$(90\%, 40\%)$	2.0	[-0.33, 0.34]	[-0.118, 0.110]	
$(90\%, 40\%)$	5.0	[-0.23, 0.22]	[-0.066, 0.077]	
$(100\%, 100\%)$	5.0	[-0.082, 0.069]	[-0.056, 0.051]	
Longitudinal polarisation				
$(-80\%, 30\%)$	2.0			[-0.119,0.082]
$(-80\%, 30\%)$	5.0			[-0.066,0.063]
$(-90\%, 40\%)$	2.0			[-0.085,0.106]
$(-90\%, 40\%)$	5.0			[-0.059,0.062]
$(-100\%, 100\%)$	5.0			[-0.047,0.053]

Table 1: The summary table fo ar 2σ limit of the CP-mixing angle $\sin 2\xi_{CP}$ with center-of-mass energy 250 GeV. The column of $\mathcal{A}^T_{CP}$ show, the determination results only when using the observable $\mathcal{A}^T_{CP}$ with transverse polarisation, while the column of $\mathcal{A}^{UL}_{CP}$ corresponds to the results with using longitudinal polarisation. Note that the column of "Combine $\mathcal{A}^T_{CP}$ & $\mathcal{A}^{UL}_{CP}$" still uses the experimental set up of transverse polarisation but measures the two observables.

Experiments	ATLAS[6]	CMS[16]	HL-LHC[17]	CEPC[9]	CLIC[18]	ILC [19]	ILC
Processes	$H \to 4\ell$	$H \to 4\ell$	$H \to 4\ell$	HZ	W-fusion	Z-fusion	$HZ,\ Z \to \mu^+\mu^-$
$\sqrt{s}$ [GeV]	13000	13000	14000	240	3000	1000	250
Luminosity [fb^{-1}]	139	137	3000	5600	5000	8000	5000
$(\lvert P_-\rvert, \lvert P_+\rvert)$							$(90\%, 40\%)$
$\tilde{c}_{HZZ}$ ($\times 10^{-2}$) 68% C.L. (1σ)limit	[-5.1, 16.6]	[-7.2, 15.2]	[-4.5, 4.5]	[-0.8, 0.8]	[-1.6, 1.6]	[-0.4, 1.6]	[-0.4, 0.7]
$\tilde{c}_{HZZ}$ ($\times 10^{-2}$) 95% C.L. (2σ)limit	[-16.4, 24.0]	[-22.4, 63.9]	[-9.1, 9.1]	[-1.6, 1.6]	[-3.3, 3.3]	[-1.4, 2.7]	[-1.1, 1.0]

Table 2: Summary of the limits of $\tilde{c}_{HZZ}$ at 95% C.L., where the results are obtained from both current LHC measurements and future colliders analysis, including HL-LHC, CEPC, ILC and CLIC.

4. Conclusion

In this work, we investigate the CP-violation effects in the process $e^+e^- \to HZ \to H\mu^+\mu^-$ with initial transversely polarized beams. Based on the azimuthal angular distribution of the final state muons, we constructed two CP sensitive observables and exploited their dependence on the CP-violation parameters. We can measure the two observables simultaneously when the initial beams are only transversely polarised, and the two observables can be combined to improve the sensitivity to CP properties. On the other hand, if the beams are longitudinally polarised, the statistical uncertainty can be suppressed by enhanced total cross-section and the sensitivity can be improved as well. Eventually we determine the CP-odd coupling $|\tilde{c}_{HZZ}| \lesssim -0.01$ at 95% C.L. for $(P^T_{e^-}, P^T_{e^+}) = (90\%, 40\%)$ polarisation and 5000 fb^{-1} at ILC 250 GeV.

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