PROCEEDINGS OF SCIENCE

# Measure of Quantum Complexity in Neutrino Oscillations

**Khushboo Dixit,**[a,∗] **S. Shajidul Haque**[b,c,e] **and Soebur Razzaque**[a,d,e]

[a] *Centre for Astro-Particle Physics (CAPP) and Department of Physics, University of Johannesburg, PO Box 524, Auckland Park 2006, South Africa*

[b] *High Energy Physics, Cosmology & Astrophysics Theory Group and The Laboratory for Quantum Gravity & Strings, Department of Mathematics & Applied Mathematics, University of Cape Town, Cape Town, South Africa*

[c] *Department of Mathematics and Natural Sciences, Brac University, Dhaka, Bangladesh*

[d] *Department of Physics, The George Washington University, Washington, DC 20052, USA*

[e] *National Institute for Theoretical and Computational Sciences (NITheCS), Private Bag X1, Matieland, South Africa*

*E-mail:* kdixit@uj.ac.za, shajid.haque@uct.ac.za, srazzaque@uj.ac.za

Neutrino flavor oscillations, a fundamental process in particle physics, involve the transition between different neutrino flavors, governed by a complex relationship between mass and flavor eigenstates. Traditionally, probabilistic measures are used to study neutrino oscillations, but recent research suggests that quantum complexity, specifically quantum spread complexity, offers deeper insights, especially regarding charge-parity (CP) violation. Our work shows that quantum complexity can predict CP-violating phases that align with experimental observations from T2K and NOvA. This approach provides a new connection between quantum information theory and particle physics, revealing the potential of complexity in neutrino studies.

*42nd International Conference on High Energy Physics (ICHEP2024)*
*18-24 July 2024*
*Prague, Czech Republic*

∗Speaker

© Copyright owned by the author(s) under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License (CC BY-NC-ND 4.0).

https://pos.sissa.it/

PoS(ICHEP2024)229

## 1. Introduction

Neutrino oscillations involve transitions between different neutrino types as they travel through space. Small differences in mass influence these transitions. Recently, a measure of quantum complexity in the state evolution of the system, called spread complexity [1, 2], has been proposed to analyze these transitions, providing more detailed information than traditional probabilities [3]. Quantum complexity, which refers to the difficulty of constructing a specific quantum state using a set of unitary operations [4–6], has gained interest in the studies of physics behind the horizon of an eternal AdS black hole as well as in characterizing the quantum chaos, quantum phase transition, quantum decoherence and so on. Quantum spread complexity offers insights into important aspects of neutrino behavior.

## 2. Quantum Complexity in Neutrino Oscillations

### 2.1 Quantum spread Complexity

Quantum spread complexity shows the minimum spread of state $|\psi(t)\rangle$ associated with a system in all possible bases in terms of a cost function [1, 2]

$$\chi = c_n |\langle K_n | \psi(t) \rangle|^2.$$

Here $c_n = 0, 1, 2..n$, n is the dimension of the system. The series expansion of unitarity $e^{-iHt}$ provides the time-evolved state as a superposition of infinite, not necessarily orthonarmalized, states $|\psi_n\rangle = H^n |\psi(0)\rangle$. $|K_n\rangle$ are Krylov states, spanning an ordered orthonormal basis, and are obtained by applying the Gram-Schimdt procedure on $|\psi_n\rangle$. Krylov basis minimizes the cost function for a given system defining the complexity in the time evolution of that system.

### 2.2 Spread complexity in neutrino oscillations

- Two-Flavor Scenario: In a simple two-flavor case, Krylov basis is equivalent to the flavor basis. Hence, $\chi_e = P_{e\mu}$ and $\chi_\mu = P_{\mu e}$, *i.e.*, the oscillation probabilities ($P_{\alpha\beta}$) provide equivalent information to complexity measures.

- Three-Flavor Scenario: In the more complex three-flavor case, the Krylov states no longer correspond directly to the flavor states. The spread complexity in this scenario reveals cross-terms that carry additional information beyond probabilities. This is particularly important when analyzing experiments like T2K and NOvA, where only certain beams ($\nu_\mu$) are used.

## 3. Results and Discussions

In three-flavor neutrino oscillations, quantum complexity provides valuable insights. For both experiments, the spread complexity ($\chi_\mu$) is maximized at the specific CP phase ($\delta$), with the T2K best-fit $\delta$ value (-2.14 radians) closely matching the complexity prediction, while the NOvA best-fit (2.58 radians) deviates but still falls within a high-complexity region. NOvA's results suggest further refinement in future models. Moreover, the behavior of complexity also provides information about the mass hierarchy and $\theta_{23}$ octant, which describes the angle associated with neutrino mixing.



PoS(ICHEP2024)229

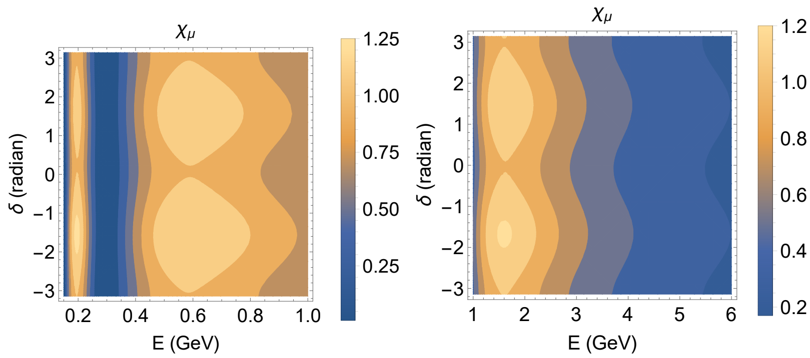


**Figure 1:** $\chi_\mu$ in $E - \delta$ plane corresponding to T2K (left) and NOvA (right) experiments.

Our results suggest that complexity can play a role in determining mass ordering and enhance the understanding of $\theta_{23}$ behavior. The key advantage of quantum complexity is its ability to reveal additional layers of information that are not captured by traditional probability measures. The maximization of complexity in the presence of CP violation suggests a deep connection between these two phenomena, which could be explored further in future experiments.

## 4. Conclusions

Our study demonstrates that quantum complexity, specifically spread complexity, is a powerful tool for understanding neutrino oscillations. In both two- and three-flavor cases, complexity provides predictions that are consistent with experimental data from T2K and NOvA, particularly in the context of CP violation. By applying concepts from quantum information theory to particle physics, we can enhance our understanding of fundamental processes like neutrino oscillations. Complexity thus represents a promising avenue for future research in understanding the neutrino system.

## Acknowledgement

This work was partially supported by grants from the National Research Foundation (NRF) of South Africa through National Institute of Theoretical and Computational Sciences (NITheCS) and the University of Johannesburg Research Council.

## References


[1] V. Balasubramanian et al., Phys. Rev. D **106** (2022) no.4, 046007.

[2] P. Caputa et al., Phys. Rev. B **106** (2022) no.19, 195125.

[3] K. Dixit et al., Eur. Phys. J. C **84** (2024) no.3, 260.

[4] L. Susskind, Fortsch. Phys. **64** (2016), 49-71.

[5] M. A. Nielsen et al., Science **311** (2006) no.5764, 1133-1135.

[6] R. Jefferson and R. C. Myers, JHEP **10** (2017), 107.




PoS(ICHEP2024)229