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A minimalistic perspective on neutrino CP-violation and Leptogenesis: Modular Invariance

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We studied a model for leptons based on the smallest modular finite group $\Gamma_2 \cong S_3$ that accounts for both the hints of large low-energy CP-violation in the lepton sector and the matter-antimatter asymmetry of the Universe, generated by two heavy right-handed neutrinos through leptogenesis. These same states are also employed in a Minimal seesaw mechanism to generate light neutrino masses. The remaining particle content is the same as the Standard Model (SM), with the only addition being a single modulus τ, whose vacuum expectation value is responsible for both the modular and CP-symmetry breakings. This is enough to achieve an excellent fit to low energy neutrino observables and to the required baryon asymmetry η_B. Various predictions for the neutrino sector observables are also provided.

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1. Introduction

In the recent past, substantial effort went into the understanding of lepton masses and mixing. Non-Abelian discrete symmetries, see e.g. [1], provided a suitable framework to this purpose. Such symmetries act linearly on the fields, which are supposed to belong to irreducible representations (irreps) of the group. The spurion fields that break the flavour symmetry are called flavons, and their specific vacuum expectation value (VEV) in flavour space shapes the mass matrices. The approach has been quite successful at reproducing (at leading order), approximate forms of the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix $U_{\rm PMNS}$, which can then be made compatible with experiments through small perturbative corrections. Due to these necessary corrections, typical drawbacks of traditional flavour models are related to the increased number of free parameters, and to the complicated scalar sector needed to correctly align the flavons in the flavour space. In [2] a new promising direction was suggested, a "bottom-up" approach based on modular invariance: the Yukawa couplings of the Standard Model (SM) become modular forms of level N, and functions of a complex scalar field τ, (called *modulus*), which acquires a VEV at some high-energy scale. In some of its minimal realizations, no flavons other than the τ are needed, and the VEV of the modulus is the only source of flavour symmetry breaking. A direct consequence of modular invariance is the remarkably limited number of free parameters. In our work [3], we presented a modular model based on modular S_3, capable of explaining the low-energy lepton observables and CP-violation as well as to account for the correct amount of the baryon asymmetry of the Universe, η_B.

2. Modular flavor symmetry at level 2

The modular group $\overline{\Gamma} = {\rm SL}(2,\mathbb{Z})/\{\pm\mathbf{1}\}$ acts on the modulus τ, restricted to the upper-half complex plane, through the transformation $\gamma : \tau \to \gamma(\tau)$,

$$\gamma(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad a, b, c, d \in \mathbb{Z} \quad , \quad ad - bc = 1 \, , \tag{1}$$

which is generated by S and T defined as: $S : \tau \to -\dfrac{1}{\tau}$ and $T : \tau \to \tau{+}1$ satisfying $S^2 = (ST)^3 = 1$. These generators are represented in ${\rm SL}(2,\mathbb{Z})$ by the 2×2 matrices $S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, and $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. A modular form is a holomorphic function of τ which, under $\overline{\Gamma}$, transforms as:

$$f(\gamma(\tau)) = (c\tau + d)^k f(\tau) \, , \tag{2}$$

where k is a positive integer called "*weight*". While the group $\overline{\Gamma}$ is infinite and non-compact, compact and finite groups can be constructed from its infinite normal subgroups $\Gamma(N)$ for $N = 1, 2, 3...$, where the natural number N is called the *level*. In [2] Feruglio has shown that it is always possible to find a basis where modular forms of a given level transform in unitary representations of the finite groups $\Gamma_N \equiv \overline{\Gamma}/\overline{\Gamma}(N)$ which, for $N \leq 5$, are isomorphic to the non-Abelian discrete groups S_3, A_4, S_4, A_5. For our work, we chose the smallest modular finite group S_3, corresponding to the level $N = 2$. The value of the modulus can be restricted to a *fundamental* domain defined as $\mathcal{D} = \left\{\tau \in \mathbb{C} \, : \, {\rm Im}\,\tau > 0 \, , \ |{\rm Re}\,\tau| \leq \frac{1}{2} \, , \ |\tau| \geq 1\right\}$. Values outside $\mathcal{D}$ are redundant and can be

	E_1^c	E_2^c	E_3^c	D_ℓ	ℓ_3	$H_{d,u}$	N^c
$SU(2)_L \times U(1)_Y$	$(\mathbf{1}, +1)$	$(\mathbf{1}, +1)$	$(\mathbf{1}, +1)$	$(\mathbf{2}, -1/2)$	$(\mathbf{2}, -1/2)$	$(\mathbf{2}, \mp 1/2)$	$(\mathbf{1}, 0)$
$\Gamma_2 \cong S_3$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{1'}$	$\mathbf{2}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{2}$
k_I	4	0	−2	2	2	0	2

Table 1: *Chiral supermultiplets, transformation properties under $\Gamma_2 \cong S_3$ and modular charges.*

mapped inside, through the appropriate modular transformation (1). For $\Gamma_2 \cong S_3$, the modular forms of lowest weight $Y_1(\tau), Y_2(\tau)$ are intrinsically hierarchical for $\tau \in \mathcal{D}$. This allows us to define the appropriate ratio: $\frac{Y_2(\tau)}{Y_1(\tau)} \equiv \zeta = |\zeta|\, e^{i\,g}$, where $g = g(\mathrm{Re}\,\tau)$ is a real function depending on $\mathrm{Re}\,\tau$. The absolute value $|\zeta|$ satisfies $|\zeta| \lesssim 1$ for $\tau \in \mathcal{D}$ and is suppressed by $e^{-\pi \mathrm{Im}\,\tau}$. Thus, we used $Y_2(\tau)/Y_1(\tau)$ as an expansion parameter. The vacuum expectation value (VEV) of the modulus τ is the only source of flavor symmetry breaking: no additional flavons are needed.

2.1 gCP symmetry

To reduce the number of model free parameters, we assumed a generalized CP symmetry (gCP). This was made consistent with modular symmetry in [4] which has shown that this translates to the reality condition of the superpotential couplings. Given this assumption, the only source of CP violation in gCP models is the VEV of the complex modulus. Since τ appears in the expansions of the modular forms through powers of $e^{2\pi i \tau}$, the CP violation is controlled entirely by $\mathrm{Re}\,\tau$ for $\tau \in \mathcal{D}$. This means that the only complex phase in the mass matrices is represented by powers of $e^{2\pi i\, \mathrm{Re}\,\tau}$. Apart from $\mathrm{Re}\,\tau = 0$, the other CP-conserving points consist in the boundary of $\mathcal{D}$.

3. The model

For the charged-leptons sector we simply employ the transformation properties under Γ_2 and a charge assignment compatible with *naturally* hierarchical charged leptons, adopted from the construction illustrated in [5]. In table 1 we summarize the chiral supermultiplets of our model, the transformation properties under $\Gamma_2 \cong S_3$ and the related modular charges. The resulting mass matrix (in the right-left basis) after electroweak symmetry breaking reads[1]:

$$M_\ell = \begin{pmatrix} \alpha (Y_{\mathbf{2}}^{(3)})_1 & \alpha (Y_{\mathbf{2}}^{(3)})_2 & \alpha_D Y_{\mathbf{1'}}^{(3)} \\ \beta Y_2 & -\beta Y_1 & 0 \\ 0 & 0 & \gamma \end{pmatrix}_{\mathrm{RL}} v_d\,, \tag{3}$$

where v_d is the VEV of H_d and the modular forms of weight 6. The hierarchy $(m_\tau, m_\mu, m_e) \sim m_\tau(1, |Y_1|, |Y_1|^3)$ naturally arises considering that $|Y_1| \approx 7/100$, as can be seen with an expansion of (3) in ζ. The mass spectrum is completely determined by five dimensionless real parameters, $\mathrm{Re}\,\tau$, $\mathrm{Im}\,\tau$, α_D/α, β/α and γ/α and one global scale $v_d \alpha$.

3.1 Neutrino sector

We adopted the Minimal seesaw scenario consisting in the introduction of two SM singlets right-handed Majorana fields N_1^c, N_2^c. As listed in table 1, these are assigned to an S_3 doublet

[1] In the parentheses notation $(\ldots)_{1,2}$ we denote the two components of the corresponding doublet $Y_{\mathbf{2}}^{(a)}$.

$N^c \sim \mathbf{2}$. The Dirac mass matrix reads:

$$M_D = g v_u \begin{pmatrix} -(Y_2^2 - Y_1^2) + \frac{g''}{g}(Y_1^2 + Y_2^2) & 2Y_1Y_2 & \frac{g'}{g}(2Y_1Y_2) \\ 2Y_1Y_2 & (Y_2^2 - Y_1^2) + \frac{g''}{g}(Y_1^2 + Y_2^2) & -\frac{g'}{g}(Y_2^2 - Y_1^2) \end{pmatrix}_{\mathrm{RL}}, \quad (4)$$

where g, g', g'', λ are dimensionless free parameters and Λ is the Majorana right-handed mass scale. As mentioned in section 2.1, all these couplings (as well as the ones from the charged-leptons sector) are real due to the imposed gCP symmetry. The only complex parameter is the modulus τ. Here v_u denotes the VEV of the neutral scalar component of H_u. The Majorana mass matrix of right-handed neutrinos is given by:

$$\mathcal{M}_R = \Lambda \begin{pmatrix} -(Y_2^2 - Y_1^2) + \lambda(Y_1^2 + Y_2^2) & 2Y_1Y_2 \\ 2Y_1Y_2 & (Y_2^2 - Y_1^2) + \lambda(Y_1^2 + Y_2^2) \end{pmatrix}_{\mathrm{RR}}. \quad (5)$$

The mass matrix of the light Majorana neutrinos is then obtained from the well known formula of type-I seesaw: $m_\nu = -M_D^T \mathcal{M}_R^{-1} M_D$. The matrix m_ν has rank 2, i.e. in the limit of exact SUSY our model predicts that the lightest neutrino is massless. The mass-scale of light neutrinos is completely determined by the global parameter $g^2 v_u^2/\Lambda$ and the low-energy neutrino mixing and mass-splittings will be mainly dictated by five parameters: $\mathrm{Re}\,\tau, \mathrm{Im}\,\tau, g'/g, g''/g, \lambda$.

4. Numerical analysis and results

To verify that our model is able to reproduce the seven experimental dimensionless observables consisting in $\{\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}\}$, the mass ratios $\{m_e/m_\mu, m_\mu/m_\tau, \Delta m^2_{\mathrm{sol}}/\Delta m^2_{\mathrm{atm}}\}$ and the Jarlskog invariant J_{CP}, we performed a χ^2 analysis for which we used the Gaussian approximation. In doing so, the experimental data was taken from NuFIT 5.2 [6]. Given the CP symmetry of the model which consists in the invariance under $\tau \to -\tau^*$ (a reflection across the imaginary axis), we obtain two sets of points distinguished only by $\pm\mathrm{Re}\,\tau$ with the CP violating (CPV) phases given by $\{\pm\delta_{\mathrm{CP}}, \pm\alpha_1, \pm\alpha_2\}$. We reported both sets in figure 1. The fit is excellent, with $\chi^2_{\mathrm{min}} = 0.98$, and we now discuss the predictions of the model. Inverted Ordering of the neutrino mass eigenstates is strongly disfavoured compared to the Normal Ordering (**NO**); equally relevant is the fact that, as anticipated, the lightest neutrino mass is exactly vanishing. This allows us to easily understand the numerical results on the Majorana effective mass of neutrinoless double-beta decay and the effective neutrino mass of tritium decay. In the first case, we obtain $|m_{\beta\beta}| \sim \mathcal{O}(1)$ meV, while in the latter case $m_\beta^{\mathrm{eff}} \sim \mathcal{O}(10)$ meV. The non-vanishing neutrino mass eigenstates lie in the region of tens of meV, implying $\sum_i m_i \sim 0.06$ eV, perfectly compatible with the most recent upper bound of 0.115 eV (95 % C.L.) from [7]. In figure 1 some correlations between observables (both fitted and predicted ones) and free parameters are displayed. All points in the plots have been selected by our algorithm and satisfy the relation $\sqrt{\chi^2} \leq 5$.

5. Leptogenesis

The mechanism of leptogenesis provides an elegant explanation for the matter-antimatter asymmetry of the Universe, attributing it to a lepton number asymmetry generated by the decays

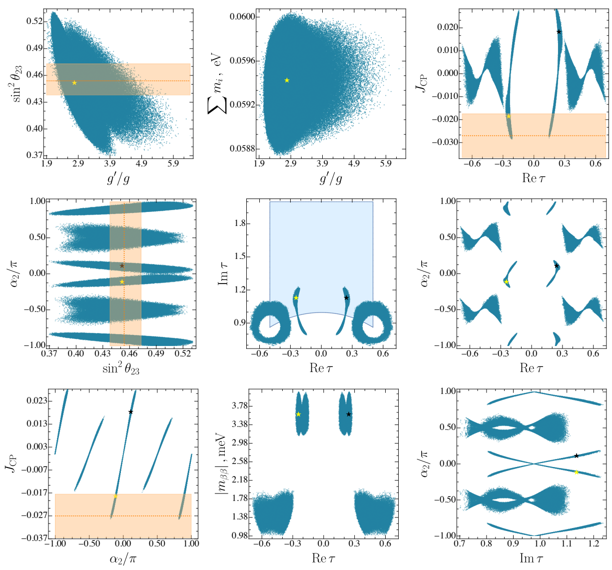

Figure 1: *Correlations between observables and free parameters. All the plotted points were accepted by the algorithm used for the fit and satisfy $\sqrt{\chi^2} \leq 5$. The yellow point corresponds to the minimum χ^2, while the black one is the CP-transformed value of the modulus, which produces the inversion of all the CPV phases in the mass matrices. The orange lines and bands correspond to the experimental best fit and 1 σ ranges.*

of heavy right-handed neutrinos.The baryon asymmetry is defined by the parameter η_B, as $\eta_B \equiv \left.\frac{n_B - n_{\bar{B}}}{n_\gamma}\right|_0$, where $n_B, n_{\bar{B}}$ and n_γ refer to the number densities of the baryons, antibaryons and photons, while the subscript “0" means “at present time". We explored the so-called N_1 dominated scenario (N1DS), even though our model involves one additional sterile state N_2. Then, for the best fit points that satisfy $\sqrt{\chi^2} \leq 2$ we solved numerically the semi-classical Boltzmann Equations [8] for the baryon asymmetry evolution, and we presented a realization that provides the value of the baryon-to-photon ratio η_B inside the 3σ experimental allowed region. With the heavy sterile neutrino masses at $O(10^{12})$ GeV, the model provides $\eta_B \lesssim 10^{-10}$, slightly below the observed value. However, since only the ratio $g^2 v_u^2/\Lambda$ is constrained by the masses of the light neutrinos, the mass scale of the Majorana neutrinos can be regarded as a free parameter of the model. We can safely rescale $\Lambda \to r\,\Lambda$ and $g \to \sqrt{r}\,g$ without altering the results of the low-energy fit. Notice that varying the mass-scale of the right-handed neutrinos have relevant consequences. For $r \lesssim 10^{-1}$ the mass-scale of the sterile neutrinos is lowered down to $O(10^{11})$, below which the *flavored regime* sets in; on the other hand, if $r \gtrsim 10^2$ the *resonant regime* occurs. We confined the analysis to the range $r \in \left[10^{-1}, 10^2\right]$, focusing on the unflavored scenario that involves solely the lightest sterile neutrino. The best fit realization of our model provides a successful leptogenesis if $\Lambda \to \bar{r}\Lambda$, where $\bar{r} = 19.9$.

6. Conclusions

In the present work, we have addressed the problem of finding a modular invariant minimalistic construction based on the smallest modular finite group $\Gamma_2 \cong S_3$ that was compatible with low energy neutrino oscillation data and, with the same model parameters, able to accommodate the measured value of the baryon asymmetry of the Universe. The only source of CP violation in our model is dictated by Re τ. Given that in the presence of a mild hierarchical heavy mass spectrum, only the contribution from the lightest sterile neutrino to the leptogenesis is commonly considered, we solved the appropriate Boltzmann Equations in the N_1-dominated scenario and shown that a mild rescaling of the Majorana right-handed mass scale Λ (not fixed by the fit) is enough to guarantee the required η_B value. Given the minimality of our $\Gamma_2 \cong S_3$ construction and the recent results in the quark sector [9], we consider this feature a relevant result in the model building based on modular invariant theories.

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