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Trinification from a complete E_6 GUT model

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E_6 Grand Unified Theories (GUTs) introduce novel symmetry-breaking patterns compared to the more common SU(5) and SO(10) GUT. We explore how $\mathrm{SU}(3)^3$ (trinification) or $\mathrm{SU}(6)\times\mathrm{SU}(2)$ symmetries can explicitly arise from E_6 at an intermediate breaking stage.

The representation **650** of E_6 emerges as the lowest-dimensional candidate for breaking into one of the novel intermediate symmetries. Demanding subsequent breaking to the Standard Model group and a realistic Yukawa sector, we argue that the minimal "realistic" model of this type has the scalar sector $\mathbf{650}\oplus\mathbf{27}\oplus\mathbf{351'}$. Perturbativity curbs the construction of larger alternatives, so this model seems to be unique in its class. Assuming minimal tuning in scalar masses, three intermediate scenarios are consistent with unification: trinification $\mathrm{SU}(3)_C\times\mathrm{SU}(3)_L\times\mathrm{SU}(3)_R$ with either LR (left-right) or CR (color-right) parity, and $\mathrm{SU}(6)_{CR}\times\mathrm{SU}(2)_L$.

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1. Introduction and motivation

Grand Unified Theories (GUTs) [1] are a conceptually intriguing approach to extending the gauge interactions of the Standard Model (SM), whereby the SM gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \equiv 3_C\, 2_L\, 1_Y$ is embedded in a simple Lie group G. Since the SM is a chiral theory, the unified group G should admit complex representations, which leads to the possibilities $\mathrm{SU}(n)$ for $n \geq 5$, $\mathrm{SO}(4n+2)$ for $n \geq 2$, and the exceptional case E_6 [2]. The minimal cases form a subgroup chain

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset \mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6. \tag{1}$$

The unified group G breaks to the SM group in one or more steps. The $\mathrm{SU}(5)$-case does not admit an intermediate step, since $3_C\, 2_L\, 1_Y$ is its maximal subgroup, while $\mathrm{SO}(10)$ allows for a breaking chain through one of the maximal subgroups $\mathrm{SU}(5) \times \mathrm{U}(1)$ or Pati-Salam $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. The larger E_6, however, offers novel possibilities for the intermediate stage, such as the trinification group $\mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{SU}(3)$ [3] or the group $\mathrm{SU}(6) \times \mathrm{SU}(2)$. These novel possibilities for an intermediate breaking stage in a GUT is what we focus on in this work.

We argue there is essentially a unique realistic model (in a certain class), in which the novel intermediate symmetries can be realized. This model has the scalar sector $\mathbf{650} \oplus \mathbf{27} \oplus \mathbf{351'}$. We organize the manuscript as follows: we discuss the model building possibilities and limitations in sec. 2, present the aforementioned model in sec. 3, and conclude in sec. 4.

Note: this report for the proceedings provides a conceptual overview and a summary of results of our papers [4] and [5]; technical details can be found therein.

2. Model building considerations and limitations

Our goal is to build a realistic E_6 GUT model breaking through a $\mathrm{SU}(3)^3$ or $\mathrm{SU}(6) \times \mathrm{SU}(2)$ intermediate symmetry. The non-trivial irreducible representations (irreps) of E_6 are listed in table 1. Although building blocks with even higher dimensionality exist, they are problematic for perturbativity — a limitation we shall imminently discuss.

irrep **R**	**27**	**78**	**351**	**351′**	**650**	**1728**	**2430**
$l(\mathbf{R})$	3	12	75	84	150	480	810
$\mathbb{R}/\mathbb{C}$	$\mathbb{C}$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{C}$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{R}$

Table 1: The lowest-dimensional irreps **R** of the group E_6 [6], their Dynkin indices l, and whether they are real (self-conjugate) or complex. We normalize l in the usual way: for **5** of $\mathrm{SU}(5) \subset \mathrm{E}_6$ we have $l(\mathbf{5}) = 1/2$.

Considerations for a realistic model necessarily include the following:

1. If a scalar representation is to spontaneously break E_6 to an intermediate symmetry H, it must contain a singlet under H, which can acquire a vacuum expectation value (VEV) by minimizing the scalar potential. For $H = \mathrm{SU}(3)^3$ or $\mathrm{SU}(6) \times \mathrm{SU}(2)$, the only representations with H-singlets from table 1 are the **650** and **2430**. The minimal choice is **650**.

2. A realistic Yukawa sector should reproduce the pattern of fermion masses and mixing-angles in the SM. The most economical choice for representing fermions is the smallest non-trivial

irrep **27**, which already contains the **16** of SO(10) (all SM fermions of one family plus a right-handed neutrino), as well as some vector-like exotics. The minimal choice for the fermion sector is thus $3 \times \mathbf{27}_F$. The scalar representations that contain the SM Higgs and acquire electroweak (EW) VEVs couple to two such fermions in a renormalizable Yukawa operator. The scalar possibilities are thus given by the decomposition of the tensor product

$$\mathbf{27} \otimes \mathbf{27} = \overline{\mathbf{27}}_s \oplus \mathbf{351'}_s \oplus \mathbf{351}_a, \tag{2}$$

where subscripts s and a denote whether an irrep comes from a symmetric or anti-symmetric combination, respectively. Borrowing a result from the SO(10) renormalizable Yukawa sector where two symmetric Yukawa matrices are necessary [7], the minimal realistic choice seems to be the presence of scalars $\mathbf{27} \oplus \mathbf{351'}$. Incidentally, these must also be involved in breaking the intermediate H to the SM group, for which they indeed prove sufficient.

An important limitation on model building, however, are considerations of perturbativity. Since the irreps of E_6 are large, the RGEs can lead to a Landau pole for gauge couplings at a scale Λ almost immediately above the GUT scale M_U. The theory would then have to be saved either by new physics or non-perturbative dynamics at Λ, which presumably induces non-renormalizable operators with effects suppressed by powers of M_U/Λ. The results of a perturbative computation can thus hardly be trusted if $M_U/\Lambda \gtrsim 10^{-1}$.

The acute limitations in the use of scalar representations in E_6 GUT are revealed by the simple analysis that follows. Above M_U, the RGE for the unified gauge coupling $\alpha := g^2/4\pi$ is

$$\tfrac{d}{dt}\alpha^{-1} = -\tfrac{1}{2\pi}\Big(a + b\ (\tfrac{\alpha}{4\pi}) + c\ (\tfrac{\alpha}{4\pi})^2 + \dots\Big), \tag{3}$$

where a, b and c are the 1-, 2-, and 3-loop coefficients, respectively. For an E_6 Yang-Mills theory with 3 generations of fermions in the $\mathbf{27}_F$, the 1-loop coefficient is computed to be

$$a = -38 + N_{27} + 2\,N_{78} + 25\,N_{351} + 28\,N_{351'} + 25\,N_{650} + 160\,N_{1728} + 135\,N_{2430}, \tag{4}$$

where N_X is the number of copies of the scalar representation $\mathbf{X}$. For $\alpha^{-1}(M_U) \simeq 40$, the value $M_U/\Lambda \simeq 10^{-1}$ is reached for $a \simeq 109$. This roughly demands $N_X = 0$ for irreps $\mathbf{X} = \mathbf{1728}$ or larger.

The limitations are even more severe once RGE contributions beyond 1-loop are taken into account. The minimal model following the enumerated guidelines has the scalar sector $\mathbf{650} \oplus \mathbf{27} \oplus \mathbf{351'}$ and RGE coefficients $a = 16$, $b = 11956$ and $c = 560730$. This results for $\alpha^{-1}(M_U) \simeq 40$ in $M_U/\Lambda \simeq 10^{-1.4}$, where the 2-loop effect is larger than the sporadically small 1-loop contribution.

Since the minimal model is already precipitously close to the perturbativity bound, the only feasible alterations would be to add copies of **27** or **78**, which however does not impact the 1st stage breaking or qualitatively change the workings of the Yukawa sector. In this sense, the minimal model is essentially unique in its class.

3. The E_6 GUT model 650 ⊕ 27 ⊕ 351'

The model under consideration is a non-supersymmetric E_6 gauge theory with the following field content:

$$\text{fermions:}\quad 3 \times \mathbf{27}_F, \qquad\qquad \text{scalars:}\quad \mathbf{650} \oplus \mathbf{27} \oplus \mathbf{351'}. \tag{5}$$

We present the salient features and viable scenarios realized in this model below.

3.1 Spontaneous symmetry breaking

To obtain an intermediate stage with $H = \mathrm{SU}(3)^3$ or $\mathrm{SU}(6) \times \mathrm{SU}(2)$, and assuming 2-stage symmetry breaking, the breaking pattern must be

$$\mathrm{E}_6 \xrightarrow[M_U]{\langle \mathbf{650} \rangle} H \xrightarrow[M_I]{\langle \mathbf{27}, \mathbf{351'} \rangle} \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y, \tag{6}$$

where the irreps responsible are written above the arrows, and the associated scales below.

The renormalizable scalar potential can be split into a **650**-only part V_1, a part V_2 not involving that irrep, and a mixed part V_{mix}:

$$V(\mathbf{650}, \mathbf{27}, \mathbf{351'}) = V_1(\mathbf{650}) + V_2(\mathbf{27}, \mathbf{351'}) + V_{\mathrm{mix}}(\mathbf{650}, \mathbf{27}, \mathbf{351'}), \tag{7}$$

$$V_1(\mathbf{650}) = -M^2\, \mathbf{650}^2 + \sum_i m_i\, \mathbf{650}^3 + \sum_j \lambda_j\, \mathbf{650}^4. \tag{8}$$

The 1st-stage breaking proceeds via **650** acquiring a VEV by minimizing V_1, which we wrote schematically in eq. (8): the number of independent m- and λ-type invariants is 2 and 5, respectively. Our detailed analysis of the potential V_1 from [4] shows that the lowest minimum can have a symmetry H from any of the following (depending on numerical values for m_i and λ_j):

$$\mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{SU}(3), \quad \mathrm{SU}(6) \times \mathrm{SU}(2), \quad \mathrm{SO}(10) \times \mathrm{U}(1), \quad \mathrm{F}_4, \quad \mathrm{SU}(3) \times \mathrm{G}_2, \tag{9}$$

in accordance with Michel's conjecture on breaking to maximal little groups when using a single irrep. Only the first three options in eq. (9) are viable, however, since only those contain the SM group. Further discussion on H-vacua is postponed until sec. 3.3.

The irreps **27** and **351′** contain 2 and 5 complex SM-singlets, respectively. Some of the H-irreps containing these singlets must survive down to the intermediate scale M_I in order to trigger the 2nd symmetry-breaking stage, cf. eq. (6).

3.2 Yukawa sector

The renormalizable Yukawa sector schematically consists of the terms

$$\mathcal{L} \supset \mathbf{Y}_{27}\, \mathbf{27}_F^2 \cdot \mathbf{27} + \mathbf{Y}_{351'}\, \mathbf{27}_F^2 \cdot \mathbf{351'}^* + h.c., \tag{10}$$

where $\mathbf{Y}_{27}$ and $\mathbf{Y}_{351'}$ are symmetric 3×3 matrices, and fermion family indices are suppressed.

The fermionic representation $\mathbf{27}_F$ contains all the SM fermions of one family, along with right-handed neutrinos $\nu^c, n \sim (1,1,1)$, as well as vector-like pairs of down quarks $d' \oplus d'^c$ and lepton doublets $L' \oplus L'^c$. The Yukawa sector in all its generality has been analyzed in [5]; we show here only a special case, when spinorial VEVs of the standard $\mathrm{SO}(10) \subset \mathrm{E}_6$ vanish, i.e., when spinorial parity $\mathbb{Z}_2^\psi$ is preserved. Under such a scenario, the exotics in the **10** of SO(10) do not mix with the standard fermions in the **16**, and the SM mass matrices at the GUT scale take the simple form

$$\mathbf{m}_u = -\mathbf{Y}_{27}\delta_7 + \tfrac{1}{\sqrt{15}}\mathbf{Y}_{351'}\delta_8, \tag{11}$$

$$\mathbf{m}_d = \ \ \mathbf{Y}_{27}\,\delta_1^* - \tfrac{1}{\sqrt{15}}\mathbf{Y}_{351'}\delta_2^*, \tag{12}$$

$$\mathbf{m}_e = -\mathbf{Y}_{27}\delta_1^* - \tfrac{1}{2}\sqrt{\tfrac{3}{5}}\mathbf{Y}_{351'}\delta_2^* + \tfrac{1}{2}\mathbf{Y}_{351'}\delta_3^*, \tag{13}$$

$$\mathbf{m}_\nu = -(\mathbf{Y}_{27}\delta_7 + \tfrac{1}{2}\sqrt{\tfrac{3}{5}}\mathbf{Y}_{351'}\delta_8 - \tfrac{1}{2}\mathbf{Y}_{351'}\delta_9)\ (\mathbf{Y}_{351'}W_3^*)^{-1}\ (\mathbf{Y}_{27}\delta_7 + \tfrac{1}{2}\sqrt{\tfrac{3}{5}}\mathbf{Y}_{351'}\delta_8 - \tfrac{1}{2}\mathbf{Y}_{351'}\delta_9)^T, \tag{14}$$

where δ_i are the EW VEVs acquired by SM-doublets $(1, 2, +1/2)$ that contain the SM Higgs, while W_3 is a specific SM-singlet VEV in $\mathbf{351'}$ and is of scale M_I.

The form of expressions (11)–(14) is reminiscent of the $\mathbf{10} \oplus \mathbf{126}$ Yukawa sector in $SO(10)$ [7], except that the E_6 case has 2 more parameters and thus implies an even better fit.

3.3 Unification of gauge couplings in minimally tuned scenarios

We return now to the topic of H-vacua from sec. 3.1 and analyze their viability.

Each of the first three SM-containing subgroups H in eq. (9) may be embedded into E_6 in inequivalent ways vis-à-vis the SM. The possible H-vacua are listed in tab. 2, where we abbreviated $SU(n) \equiv n$ and the subscripts denote the location of well-known subgroups (C,L,R for *color*, *left*, *right*). For trinification $3_C \times 3_L \times 3_R$, there is only one embedding, but the solution in [4] exhibits three degenerate vacua, each preserving one of the LR-, CL- and CR-parities, cf. tab. 2. There are three embeddings for 6×2 depending on where 2_L and $U(1)_Y$ is embedded (hypercharge commutes with $2_{R'}$ but not 2_R). Finally, there is a standard and flipped (denoted by primes) $SO(10) \times U(1)$ embedding, depending on whether the abelian factor doesn't or does contain part of the hypercharge, respectively.

H-vacuum	H-irreps of scalars at M_I under ESH + $\mathbb{Z}_2^\psi$	unifies?
$3_C \times 3_L \times 3_R \times \mathbb{Z}_2^{LR}$	$2 \times (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6})$	yes
$3_C \times 3_L \times 3_R \times \mathbb{Z}_2^{CL}$	$2 \times (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6}) \oplus 2 \times (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{6}}, \mathbf{1}, \bar{\mathbf{6}})$	—
$3_C \times 3_L \times 3_R \times \mathbb{Z}_2^{CR}$	$2 \times (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6}) \oplus 2 \times (\mathbf{3}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{6}, \mathbf{6}, \mathbf{1})$	yes
$6_{CL} \times 2_{R'}$	—	—
$6_{CL} \times 2_R$	$(\mathbf{15}, \mathbf{1}) \oplus (\overline{\mathbf{21}}, \mathbf{3}) \oplus (\bar{\mathbf{6}}, \mathbf{2}) \oplus (\mathbf{84}, \mathbf{2})$	—
$6_{CR} \times 2_L$	$(\mathbf{15}, \mathbf{1}) \oplus (\overline{\mathbf{105}'}, \mathbf{1}) \oplus (\bar{\mathbf{6}}, \mathbf{2}) \oplus (\mathbf{84}, \mathbf{2})$	yes
$SO(10) \times U(1)$	—	—
$SO(10)' \times U(1)'$	$(\mathbf{16}, +1) \oplus (\mathbf{126}, +2) \oplus (\mathbf{10}, -2)$	—

Table 2: The possible H-vacua of the model that contain the SM, the scalar content of the intermediate H-theory assuming ESH + $\mathbb{Z}_2^\psi$ (see main text), and whether gauge coupling unification works in such a case.

Each of these cases is then further assessed for viability under gauge coupling unification. The cases $6_{CL} \times 2_{R'}$ and $SO(10) \times U(1)$ already unify the SM couplings in their first factor, but this is incompatible with bottom-up RGE running in the SM, so the two cases are discarded. In the other cases, however, we identify the scalar contents at M_I assuming the extended survival hypothesis (ESH) and spinorial parity $\mathbb{Z}_2^\psi$, see [5] for analysis. The ESH says that the intermediate theory only has H-components that are necessary for 2nd-stage breaking and a viable Higgs in the Yukawa sector, i.e., it assumes minimal tuning in a viable theory. Spinorial parity is a choice of the vacuum, motivated by simplicity and the presence of a ψ-odd scalar dark matter candidate (doublet DM).

Fixing the intermediate theory determines the bottom-up RG evolution of gauge couplings and we can asses which scenarios unify successfully at some M_U. We only state here the results [5]: there are three viable cases, as indicated in the last column of tab. 2. Other cases fail to unify because the couplings above M_I in the intermediate theory diverge.

4. Conclusions

We set out to build a complete E_6 GUT model which breaks into a novel intermediate stage of trinification $SU(3)^3$ or $SU(6) \times SU(2)$. By complete we mean that both symmetry breaking and Yukawa sector can realistically recreate the SM at low energies.

The minimal scalar sector was determined to be $\mathbf{650} \oplus \mathbf{27} \oplus \mathbf{351'}$, and given perturbativity limitations only additions of **27** or **78** are possible. Once this model was identified, we analyzed which intermediate scenarios within it could be viable. Assuming minimal tuning via the extended survival hypothesis and a choice of spinorial parity for the vacuum, the 3 viable cases compatible with unification constraints are trinification $SU(3)^3$ with LR or CR parity, and $SU(6)_{CR} \times SU(2)_L$.

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