

## Measurements of $|V_{cb}|$ and $|V_{ub}|$ at Belle and Belle II

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The Belle and Belle II experiments have collected a  $1.1 \text{ ab}^{-1}$  sample of collisions at the  $\Upsilon(4S)$  resonance. These data, with low particle multiplicity and constrained initial state kinematics, are an ideal environment to study semileptonic and leptonic decays of the  $B$  meson. Combined with theoretical inputs, measurements of both inclusive and exclusive decays yield information about the Cabibbo-Kobayashi-Maskawa matrix elements  $V_{cb}$  and  $V_{ub}$ . We review our latest results, which include the first Belle II results with fully leptonic and inclusive semileptonic decays.

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## 1. Introduction

Belle II [1] is a particle detector designed to study 7-on-4 GeV  $e^-e^+$  collisions at the energy of the  $\Upsilon(4S)$  resonance, copiously produced by the SuperKEKB collider located at the KEK laboratory in Japan [2]. The  $\Upsilon(4S)$  decays almost exclusively into B meson pairs, resulting in low backgrounds. Belle [3] was the predecessor of the Belle II experiment and ran from 1999 to 2010. Belle and Belle II detectors are ideally suited to study decays with missing energy; they are arranged hermetically in a cylindrical geometry around the interaction point. The Belle II analyses presented in this document use a sample of  $365 \text{ fb}^{-1}$  of data collected during the 2019-2022 Run 1 period [4]; those at Belle feature the full sample of  $711 \text{ fb}^{-1}$ .

In the Standard Model (SM), the Cabibbo-Kobayashi-Maskawa (CKM) matrix describes quark mixing and accounts for CP-violation in the quark sector. One of the crucial tests of the SM is the precise determination of the magnitude of the CKM matrix elements, such as  $|V_{cb}|$  and  $|V_{ub}|$ , which are extracted almost exclusively from measurements of semileptonic decays of B mesons. The value of  $|V_{ub}|$  is also accessible via purely leptonic decays,  $B^+ \rightarrow \mu^+ \nu_\mu$ ,  $B^+ \rightarrow \tau^+ \nu_\tau$ ; however, such channels are either helicity suppressed, or challenging to access experimentally due to the presence of multiple neutrinos in the final state.

Measurements of  $|V_{cb}|$  and  $|V_{ub}|$  can be grouped into two experimentally and theoretically complementary strategies, known as *exclusive*, focusing on distinct final states, and *inclusive*, considering the sum of all possible final states. The corresponding world averages of  $|V_{xb}|$  from exclusive and inclusive determinations exhibit a disagreement of about  $3\sigma$  [5]. We conducted several measurements with new strategies to investigate these tensions further.

## 2. $|V_{cb}|$ from angular coefficients of $\bar{B} \rightarrow D^* \ell \bar{\nu}$

With the full Belle dataset, the first measurement of the complete set of angular coefficients for exclusive  $\bar{B} \rightarrow D^* \ell \bar{\nu}$  decay is performed [6]. One B is reconstructed using hadronic decays; the other B is reconstructed with the decay chains  $\bar{B}^0 \rightarrow D^{*+} \ell \bar{\nu}$  with  $D^{*+} \rightarrow D^0 \pi^+ / D^+ \pi^0$ , and  $B^- \rightarrow D^{*0} \ell \bar{\nu}$  with  $D^{*0} \rightarrow D^0 \pi^0$ . The non-resonant  $e^-e^+$  interactions are suppressed using a multivariate classifier. The angular coefficients are obtained from data in bins of the hadronic recoil parameter  $w = (m_B^2 + m_{D^*}^2 - q^2) / 2m_B m_{D^*}$  separately for  $\ell = e, \mu$  and  $B = B^0, B^+$  modes. In each  $w$  bin, the signal yields are determined in bins of the decay angles  $\theta_\ell$ ,  $\theta_V$ , and  $\chi$ .  $\theta_\ell$  is the angle between the lepton and the direction opposite to the B meson in the virtual W-boson rest frame,  $\theta_V$  is the angle between the D meson and the direction opposite the B meson in the  $D^*$  rest frame, and  $\chi$  is the angle between the two decay planes spanned by the  $W - \ell$  and  $D^* - D$  systems in the B meson rest frame.

The obtained angular coefficients allow us determine the form factors describing the  $B \rightarrow D^*$  transition and the magnitude of  $V_{cb}$ . Utilizing various sets of recent lattice QCD calculations for the form factors, we find  $|V_{cb}| = (41.0 \pm 0.7) \times 10^3$  based on the Boyd-Grinstein-Lebed (BGL) parameterization [7]. This result is in agreement with the fit of the one-dimensional differential spectra determined from the same dataset [8] and also with the currently most precise determinations from inclusive  $B \rightarrow X_c \ell \nu$  decays [9] [10] [11]. Additionally, potential lepton flavor universality violation as a function of  $w$  is investigated by analyzing the differences in the angular distributions of electrons and muons. No deviation from SM expectations is observed.

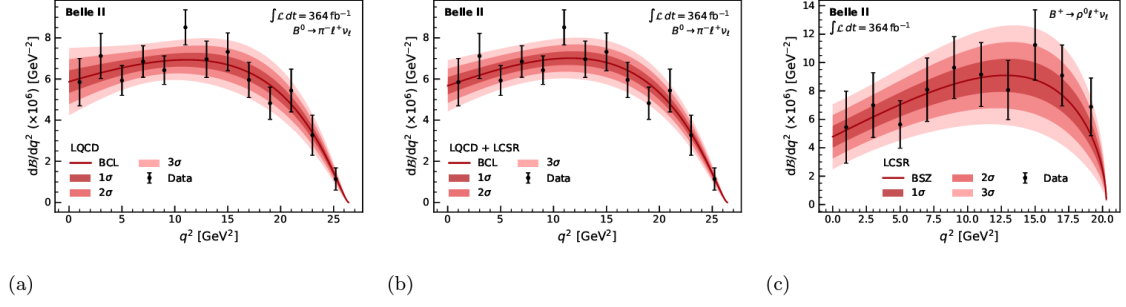
### 3. $|V_{ub}|$ from $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow \rho^0 \ell^+ \nu$

This measurement uses the full Run 1 data sample recorded by the Belle II detector between 2019 and 2022 [12]. The signal decays of  $B^0 \rightarrow \pi^- \ell^+ \nu$  and  $B^+ \rightarrow \rho^0 \ell^+ \nu$  are reconstructed without identifying the partner B meson, and non-resonant  $e^-e^+$  interactions and B background are suppressed using Boosted Decision Trees. The reconstructed events are separated into 13 intervals for the pion mode and 10 for the rho mode of squared momentum transfer  $q^2$ . The signal yields of the two modes are simultaneously extracted from a two-dimensional grid of the energy difference  $\Delta E = E_B^* - E_{beam}^*$  and the beam-constrained mass  $M_{bc} = \sqrt{E_{beam}^{*2} - |\vec{p}_B^*|^2}$  in each  $q^2$  bin, where  $E_{beam}^*$ ,  $E_B^*$ , and  $\vec{p}_B^*$  are the beam energy, reconstructed B energy, and reconstructed B momentum, all determined in the center of mass frame, respectively. With this novel method, cross-feed signals can be properly linked between the two decay modes. The partial branching fractions are determined from the fitted signal yields after efficiency corrections as a function of  $q^2$ . Furthermore, the total branching fraction is computed as the sum of these partial branching fractions, accounting for systematic correlations. As preliminary results, we obtain total branching fractions  $\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu_\ell) = (1.516 \pm 0.042(stat) \pm 0.059(syst)) \times 10^{-4}$  and  $\mathcal{B}(B^+ \rightarrow \rho^0 \ell^+ \nu_\ell) = (1.625 \pm 0.079(stat) \pm 0.180(syst)) \times 10^{-4}$ . These results are consistent with the world averages, and the precision is comparable to previous measurements from Belle and BaBar.

For extracting  $|V_{ub}|$ , the decay form factors of  $B^0 \rightarrow \pi^- \ell^+ \nu$  are parameterized using the Bourrely-Caprini-Lellouch (BCL) model [13], and the Bharucha-Straub-Zwicky (BSZ) parametrization [14] is employed for  $B^+ \rightarrow \rho^0 \ell^+ \nu$ . Using  $\chi^2$  fits to the measured  $q^2$  spectra, with  $\chi^2 = \sum_{i,j=1}^N (\Delta B_i - \Delta \Gamma_i \tau) C_{ij}^{-1} (\Delta B_j \Delta \Gamma_j \tau) + \sum_m \chi_{theory,m}^2$ , and incorporating constraints on non perturbative hadronic contributions from lattice QCD calculations [17], we obtain the preliminary result  $|V_{ub}| = (3.93 \pm 0.09 \pm 0.13 \pm 0.19) \times 10^{-3}$ , where the uncertainties are statistical, systematic, and theoretical, respectively. The preliminary result from the  $B^+ \rightarrow \rho^0 \ell^+ \nu$  decay including the constraints from light-cone sum rule (LCSR) [14] is  $|V_{ub}| = (3.19 \pm 0.12 \pm 0.17 \pm 0.26) \times 10^{-3}$ . The  $|V_{ub}|$  values obtained from the  $B^0 \rightarrow \pi^- \ell^+ \nu$  mode are consistent with previous exclusive measurements. Fig. 1 shows the measured and fitted differential rates of  $B^0 \rightarrow \pi^- \ell^+ \nu$  and  $B^+ \rightarrow \rho^0 \ell^+ \nu$ , as well as the one, two, and three standard-deviation uncertainty bands from the fits. The result from the  $B^+ \rightarrow \rho^0 \ell^+ \nu$  mode is lower but remains consistent with previous experimental determinations from  $B \rightarrow \rho \ell \nu$  decays. In both cases, the precision is limited by theoretical uncertainties.

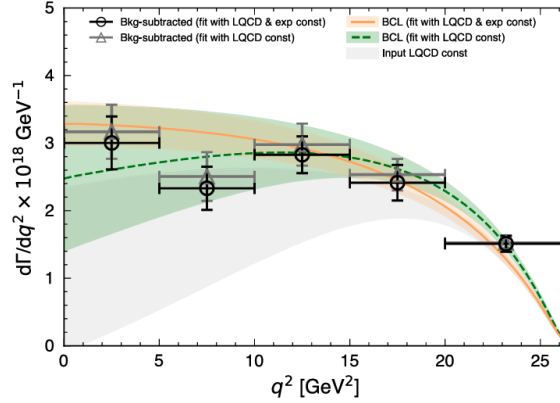
### 4. $|V_{ub}|$ from inclusive and exclusive B decays

The first simultaneous determination of  $|V_{ub}|$  using inclusive and exclusive decays has been performed at Belle [15]. The analysis strategies are inherited from the previous Belle study of  $B \rightarrow X_u \ell \nu$  with hadronic tagging [16]. To distinguish exclusive  $B \rightarrow \pi \ell \nu$  decays from other inclusive  $B \rightarrow X_u \ell \nu$  events and backgrounds, a two-dimensional fit of  $q^2$  and the number of charged pions in the hadronic  $X_u$  system is employed. The  $B \rightarrow \pi \ell \nu$  form factors are parameterized with the BCL expansion [13] and constrained to the LQCD calculations [17] or the combined global fit of previous experimental observations and LQCD [17]. With the nominal setup incorporating the constraints based on the full theoretical and experimental knowledge of the  $B \rightarrow \pi \ell \nu$  form factor shape, we obtain  $|V_{ub}^{excl}| = (3.78 \pm 0.23 \pm 0.16 \pm 0.14) \times 10^{-3}$  and  $|V_{ub}^{incl}| = (3.88 \pm 0.20 \pm$



**Figure 1:** [12] Measured partial branching fractions as a function of  $q^2$  for  $B^0 \rightarrow \pi^- \ell^+ \nu$  (a,b) and  $B^+ \rightarrow \rho^0 \ell^+ \nu$  (c). The fitted differential rates are shown together with the one, two, and three standard-deviation uncertainty bands for fits using constraints on the form factors from (a) LQCD, (b) LQCD and LCSR, and (c) LCSR predictions.

$0.31 \pm 0.09) \times 10^{-3}$  with the uncertainties being the statistical, systematic, and theoretical. The ratio  $|V_{ub}^{excl}|/|V_{ub}^{incl}| = 0.97 \pm 0.12$  is found to be compatible with unity and compatible with the world average in  $1.2\sigma$ . Fig. 2 compares the fitted  $q^2$  spectra of the differential rate of  $B \rightarrow \pi \ell \nu$  for both fit scenarios and the LQCD input. Moreover, the averaged  $|V_{ub}|$  derived from the inclusive and exclusive determinations incorporating LQCD and additional experimental information, is  $(3.84 \pm 0.26) \times 10^{-3}$ . This result is in agreement with the expectation from CKM unitarity [18] of  $|V_{ub}^{CKM}| = (3.64 \pm 0.07) \times 10^{-3}$  within  $0.8\sigma$ .



**Figure 2:** [15] The  $q^2$  spectra of  $B \rightarrow \pi \ell \nu$  obtained from the fit of the combined LQCD and experimental information (orange, solid curve) and from the fit to LQCD only (green, dashed curve) are shown. The data points are the background subtracted postfit distributions, corrected for resolution and efficiency effects, and averaged over both isospin modes. In addition, the LQCD prefit prediction for the  $B \rightarrow \pi \ell \nu$  form factor is shown (gray).

## 5. $|V_{ub}|/|V_{cb}|$ from inclusive decays

The semileptonic inclusive decays  $B \rightarrow X_u \ell \nu$  and  $B \rightarrow X_c \ell \nu$  are analyzed using the full Belle data [19], employing the Belle II software. The  $B \rightarrow X_u \ell \nu$  signal yields are extracted through a two-dimensional fit on  $q^2$  and the charged lepton energy in the B meson rest frame  $p_\ell^B$ . Meanwhile,  $B \rightarrow X_c \ell \nu$  yields are obtained by subtracting contributions from other decays in the total  $B \rightarrow X_u \ell \nu$

sample. This measurement focuses on the partial phase space region with  $p_\ell^B > 1$  GeV, known for cleaner experimental backgrounds in  $B \rightarrow X_u \ell \nu$  decays. The preliminary result for the partial branching fraction ratio is  $\Delta\mathcal{B}(B \rightarrow X_u \ell \nu) / \Delta\mathcal{B}(B \rightarrow X_c \ell \nu) = 1.96(1 \pm 8.4\%(stat) \pm 7.9\%(syst)) \times 10^{-2}$ . This ratio provides insight into the inclusive  $|V_{ub}| / |V_{cb}|$  ratio, incorporating theoretical inputs of partial decay rates for both decays. Employing the theoretical calculations for the partial decay rate of  $B \rightarrow X_u \ell \nu$  from BLNP [20], it is obtained  $|V_{ub}| / |V_{cb}| = 0.0972(1 \pm 4.2\%(stat) \pm 3.9\%(syst) \pm 5.2\%(\Delta\Gamma(B \rightarrow X_u \ell \nu)) \pm 2.0\%(\Delta\Gamma(B \rightarrow X_c \ell \nu)))$ , and employing the GGOU [21] theoretical calculations  $|V_{ub}| / |V_{cb}| = 0.0996(1 \pm 4.2\%(stat) \pm 3.9\%(syst) \pm 2.3\%(\Delta\Gamma(B \rightarrow X_u \ell \nu)) \pm 2.0\%(\Delta\Gamma(B \rightarrow X_c \ell \nu)))$ . The  $|V_{ub}| / |V_{cb}|$  values are consistent with the world averages.

## 6. Summary

Improved measurements of  $|V_{cb}|$  and  $|V_{ub}|$  are essential to increase the constraining power of the Unitarity Triangle fit, and known initial state kinematics and hermetic detectors make Belle and Belle II ideal for these studies. Belle and Belle II are producing many updated and improved measurements of  $|V_{cb}|$  and  $|V_{ub}|$ , with both inclusive and exclusive decays and continued efforts in both experimental and theoretical realms are essential. For instance, some of the experimental uncertainties are expected to be reduced with more collected data at Belle II, and the ongoing developments in theoretical studies are anticipated to further refine experimental simulations. Moreover, beyond these important results, the accumulated knowledge of MC modeling and validated novel approaches will be beneficial for future measurements.

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