PROCEEDINGS OF SCIENCE

Lattice QCD and flavor physics

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The rich structure of flavor physics provides a plethora of possibilities to test or constrain the standard model. This requires both precise experimental measurements as well as theoretical predictions. Determining nonperturbative contributions due to the strong force is the prime task of lattice QCD calculations leading e.g. to determinations of decay constants, form factors or bag parameters. Selecting a few examples we highlight recent progress in lattice QCD determinations in the light and heavy flavor sector and discuss their impact on flavor physics.

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1. Introduction

Lattice quantum chromodynamics (QCD) is an ab initio method to study nonperturbative phenomena of the strong interaction with systematically improvable uncertainties. The key idea is to numerically simulate the QCD Lagrangian. This requires to discretize space-time and restrict the simulation to a finite box typically chosen of dimensions $(L/a)^3 \times T/a$, where the lattice spacing a has a finite value. Moreover, a Wick-rotation to Euclidean time ($t \to i\tau$) is performed such that we can use Feynman's path integral formalism with importance sampling to stochastically calculate observables. For making QCD predictions we further need few experimental inputs (e.g. M_π, f_π, f_K, M_{D_s}, M_{B_s}) to set the lattice scale and fix up/down, strange, charm, and bottom quark masses.

By performing simulations at different values of the lattice spacing and quark masses, phenomenological results are obtained after taking the continuum limit (i.e. $a \to 0$) and inter-/extrapolating to the physical quark masses. The continuum limit also takes care of removing discretization artifacts. Hence simulations can be performed with different choices for the gauge action (e.g. Wilson, Symanzik, Iwasaki, . . .) and fermion action (e.g. Wilson, Kogut-Susskind (staggered), domain-wall or overlap fermion, . . .). Observing agreement between lattice QCD calculations performed by different groups using different discretizations boosts the confidence as have done numerous pre-/postdictions of experimentally verifiable results like the hadron spectrum (see e.g. [1] and references within). 50 years after Wilson's seminal paper [2], high-precision lattice QCD calculations are standard and for state-of-the-art calculations typically simulations at physical up/down quark masses are included. Results reaching the subpercent level of precision are possible, however, such calculations require to account for effects due to quantum electrodynamics (QED) and isospin breaking. Further good control on all systematic effects (e.g. finite volume) is required.

In the following we feature five different highlights to showcase examples of what can be achieved in flavor physics using lattice QCD. First we present the update by the BMW collaboration on the determination of the hadronic vacuum polarization (HVP) relevant for the determination of the muon anomalous moment. Next the determination of the ϱ and K^* resonance at physical up/down quark masses by the RBC/UKQCD collaboration is highlighted, followed by Fermilab/MILC's recent and to-date most precise determination of $|V_{cd}|$. We continue discussing the status of $B \to D^*\ell\nu$ decays before giving an outlook on pioneering work tackling heavy meson lifetimes.

2. Highlights

2.1 $g-2$: updated value for a_μ

In the recent years the anomalous magnetic moment of the muon has been in the spotlight of the particle physics community. Driven by the experimental progress reported from the $g-2$ experiment at Fermilab [3], the theory community has been scrutinizing the different contributions building up the theory predictions of $a_\mu = \frac{g_\mu - 2}{2}$. The contribution due to the hadronic vacuum polarization (HVP) emerged to be most critical in order to understand the discrepancy between the experimental value and the so-called white-paper theory prediction [4]. By now lattice QCD is a major player determining the HVP contribution to a_μ. Although only the BMW collaboration has so far published results covering all contributions over the full phenomenologically relevant range [5], several other collaborations have confirmed the values for an intermediate range of a_μ referred

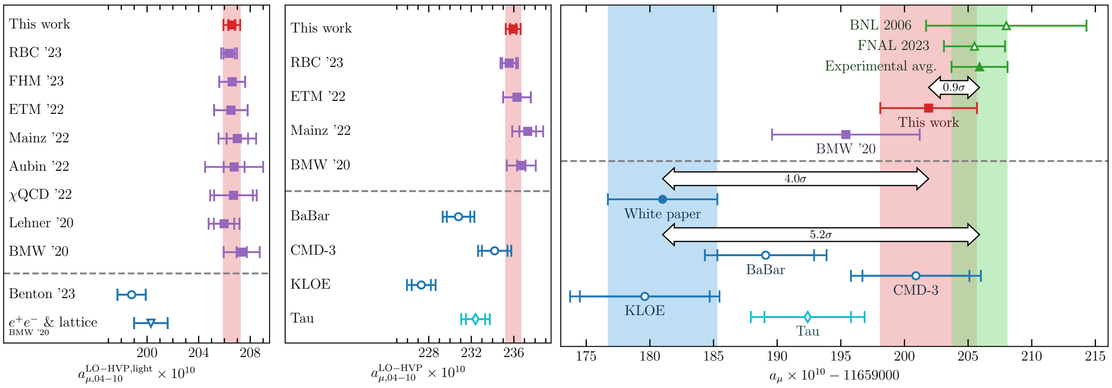

Figure 1: Plots reproduced from Ref. [6] by the BMW collaboration. The left most panel compares different lattice determinations of the light quark contribution to the intermediate window, whereas the central panel compares lattice as well as experimental determinations of the full intermediate window. The excellent agreement boosts confidence in the lattice determination. Adding up all contributions the BMW collaboration compares their value for a_μ to the experimental average as well as data-driven determinations.

to as the "intermediate window" observable. The excellent agreement can be seen in the two panels on the left of Fig. 1. The left most panels shows the 2024 update by the BMW collaboration [6] for the light quark contribution to the intermediate window to other lattice determination, whereas the central panel compares the full intermediate window to other lattice as well as the experimental determinations.

By including new simulations at a physical point ensemble with finer lattice spacing as well as using a data-driven approach for the long distance tail, the BMW collaboration reports a 40% improvement in precision compared to their previous calculation. Their new value is consistent with the experimental value at 0.9 σ as shown in the right most panel of Fig. 1. Instead of the previous discrepancy, $g-2$ is turning into a precision test of the standard model at 0.37 ppm. Updates by other collaborations have been reported at the Lattice conference in Liverpool[1] resulting hopefully soon in additional cross-checks and independent confirmation of the BMW result.

2.2 Physical point calculation of ϱ and K^*

Improving the theoretical understanding of QCD resonances like the ϱ or the K^* is important for various new physics searches but also directly relevant to $g-2$. Determining resonances or multi-hadronic states on the lattice is however complicated because simulations are performed in Euclidean space-time and restricted to a finite box. To calculate mass M and width Γ of vector channel resonances requires to first calculate finite-volume energies and, following ideas by Lüscher, relate them in a second step to partial wave amplitudes. For the ϱ and K^* the RBC/UKQCD collaboration presents the first work at physical pion masses using one lattice spacing [7, 8] and finds

$$M_\rho = 796(5)(50)\text{ MeV}, \qquad \Gamma_\rho = 192(10)(31)\text{ MeV},$$
$$M_{K^*} = 893(2)(54)\text{ MeV}, \qquad \Gamma_{K^*} = 51(2)(11)\text{ MeV}.$$

[1] Lattice 2024, Liverpool, UK: `https://conference.ippp.dur.ac.uk/event/1265/`

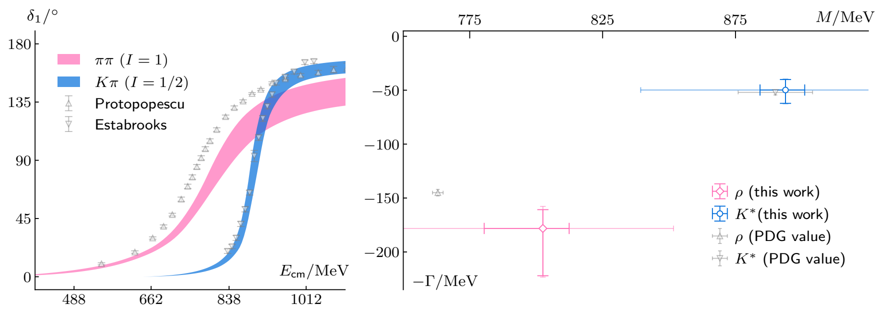

Figure 2: Plots reproduced from Ref. [7] by the RBC/UKQCD collaboration. The left panel shows the results for the scattering phase shifts for $K\pi \to K\pi$ and $\pi\pi \to \pi\pi$, whereas the right panel visualizes the resonance pole positions.

While the width and mass for the K^* agree perfectly with the current PDG value, some tension in the width is observed for the ϱ. The large uncertainties are due to using only one lattice spacing and estimating discretization effects.

2.3 New determination of $|V_{cd}|$

At present the PDG [9] reports a value for the Cabbibo-Kobayashi-Maskawa (CKM) matrix element $|V_{cd}|_{\text{PDG}} = 0.221 \pm 0.004$, which carries an uncertainty of 1.8%. This value is obtained by averaging three determinations:

- Earlier determination based on neutrino scattering data $|V_{cd}|^{\nu}_{\text{PDG}} = 0.230 \pm 0.011$
- Leptonic $D^+ \to \{\mu^+\nu_\mu, \tau^+\nu_\tau\}$ decays using lattice QCD results by Fermilab/MILC and ETMC combined with experimental measurements by BESIII and CLEO: $|V_{cd}|^{f_D}_{\text{PDG}} = 0.2181 \pm 0.0050$
- Semileptonic $D \to \pi\ell\nu$ decays at $q^2 = 0$ with lattice QCD input from ETMC and experimental measurements by BaBar, BESIII, CLEO-c, Belle: $|V_{cd}|^{D\pi(0)}_{PDG} = 0.2330 \pm 0.014$

In 2023 Fermilab/MILC reported a new, most precise determination of $|V_{cd}|$ with an uncertainty of 1.3% by studying semileptonic $D \to \pi\ell\nu$ and $D_s \to K\ell\nu$ decays [10]. Using a state-of-the-art setup and by exploiting full q^2 dependence of the form factors, they find

$$|V_{cd}|^{D\pi}_{\text{Fermilab/MILC}} = 0.2238 \pm 0.0029 \quad \text{(with BaBar, BESIII, CLEO-c, Belle data)},$$
$$|V_{cd}|^{D_sK}_{\text{Fermilab/MILC}} = 0.258 \pm 0.015 \quad \text{(with BESIII data)}.$$

Figure 3 presents a comparison of Fermilab/MILC's new values compared to other determinations in the literature as well as a prediction assuming CKM unitarity. All-in-all there is excellent agreement, however the uncertainty of the determination based on $D_s \to K\ell\nu$ semileptonic decays suffers from the large experimental uncertainty.

2.4 Updates on $B \to D^*\ell\nu$

Semileptonic decays are the method of choice to extract the CKM matrix element $|V_{cb}|$. Since many years, however, there has been a long standing tension between the inclusive and exclusive determination of $2-3\sigma$. For exclusive determinations $B \to D^*\ell\nu$ is the experimentally preferred

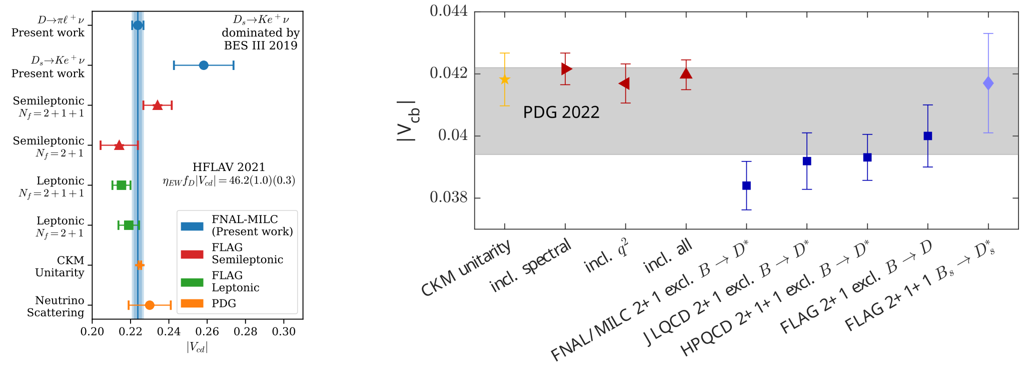

Figure 3: Left: Plot reproduced from Ref. [10] by the Fermilab/MILC collaboration comparing their new determinations of $|V_{cd}|$ to other determinations in the literature. Right: Plot reproduced from Ref. [11] comparing inclusive $|V_{cb}|$ determination (red triangles) to exclusive $B \to D^*\ell\nu$ determination (blue squares).

channel and measurements have been performed by BaBar, Belle, Belle II, and LHCb. To extract $|V_{cb}|$ we need to combine the measured differential branching fractions with theoretical form factor predictions. By now three lattice collaborations (Fermilab/MILC [12], JLQCD [13], HPQCD [14]) have reported form factors with full error budget over the full range in q^2. While all result in compatible determinations of $|V_{cb}^{\text{excl}}|$, the tension with inclusive determinations persists as we show in the right panel of Fig. 3. Moreover, there is also some tension in the slope of the different lattice form factors as well as in comparison to the experimental results.

While the tension between exclusive and inclusive determinations is persisting, the fact that three groups have reported results for $B \to D^*\ell\nu$ form factors opens new possibilities to scrutinize the findings. In the future, further improvements e.g. directly covering a larger range in q^2 and/or reducing uncertainties are expected. Furthermore, tremendous progress towards calculating inclusive decays on the lattice has been reported at the Lattice 2024.

2.5 First steps to determine heavy meson lifetimes on the lattice

The determination of heavy meson lifetimes on the lattice is still an outstanding task. Using the heavy quark expansion (HQE), lifetimes are described by $\Delta Q = 0$ four-quark operators. While some diagrams are similar to $\Delta Q = 2$ meson mixing diagrams, the calculation of lifetimes requires in addition to evaluate quark line disconnected contributions and eye diagrams which are computationally more demanding and typically noisy. A further complication arises from the fact that $\Delta Q = 0$ operators mix under renormalization.

Black et al. tackle the latter issue by using gradient flow in combination with the short-flow-time expansion to renormalize the operators [15, 16]. In the left panel of Fig. 4 the method is demonstrated for the established case of short-distance meson mixing, whereas on the right first results for two connected operators contributing to the $\Delta Q = 0$ calculation are shown. To simplify the pioneering work the calculation is performed considering "neutral" charm-strange mesons.

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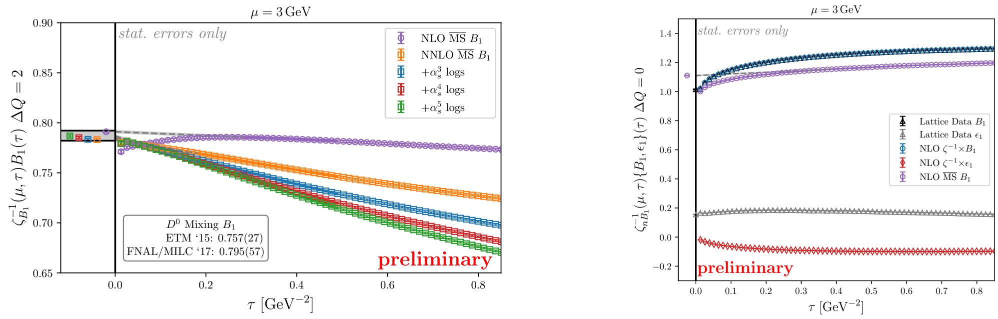

Figure 4: Plots reproduced from Ref. [15] by Black et al. Left panel: Validating the use of gradient flow + short-flow-time expansion to renormalize operators for $\Delta Q = 2$ neutral meson mixing. Right panel: First results showing two of the four connected operators to determine $\Delta Q = 0$ lifetime operators.

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