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Lepton Flavour Universality tests and determination of Vus using the HFLAV-Tau averages

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We describe the updated tau lepton averages performed by the Heavy Flavour Averaging Group (HFLAV) for the incoming edition of the Heavy Flavour measurements averages, and we use the results to update several Lepton Flavour Universality tests and the computation of $|V_{us}|$ with tau measurements.

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1. HFLAV tau branching fraction fit, tau mass, tau lifetime

For the 2023 HFLAV report, we fit 171 measurements of tau branching fractions and ratios of branching and an estimate of one nuisance parameter to optimize 137 fit parameters and 1 nuisance fit parameter. The fit parameters and the nuisance fit parameter are subjected to 91 constraint equations. The minimized χ^2 is

$$\chi^2 = \sum_{ijkl} (m_i - M_{ik}q_k) \left(V^{-1}\right)_{ij} (m_j - M_{jl}q_l) + \sum_r \frac{(n_r - p_r)^2}{\sigma^2_{n_r}} ,$$

where m_i are the measurements, q_k are the fit parameters, M_{ik} is the model matrix applied to fit parameters to predict measurements, V_{ij} is the measurements covariance, p_r are the nuisance fit parameters, $n_r \pm \sigma_{n_r}$ are the Gaussian constraints on the nuisance fit parameters.

The tau mass and lifetime averages are performed with a minimum χ^2 fit of the existing measurements, which are the same that have been used for the tau mass and lifetime averages in the 2024 edition of the review of Particle Physics [1]. For the measurements that report an asymmetric uncertainty, a symmetric uncertainty, $\sigma^2 = (\sigma^2_- + \sigma^2_+)/2$, has been computed and used in the fit.

2. Tests of lepton universality

The charged-weak-current coupling ratios arising from the tau leptonic branching fractions have been computed with the same procedure documented in the 2021 HFLAV report [2]:

$$\left(\frac{g_\tau}{g_\mu}\right) = 1.0016 \pm 0.0014 , \qquad \left(\frac{g_\tau}{g_e}\right) = 1.0018 \pm 0.0014 , \qquad \left(\frac{g_\mu}{g_e}\right) = 1.0002 \pm 0.0011 . \tag{1}$$

The precision of the third ratio has been improved with respect to the former value of 1.0019±0.0014 by the recent Belle II measurement of $\mathcal{B}_{\tau\mu}/\mathcal{B}_{\tau e} = 0.9675 \pm 0.0007 \pm 0.0036$ [3]. Under the assumption that the muon and electron charged weak couplings are equal, we average $\mathcal{B}_e = \mathcal{B}(\tau \to e\overline{\nu}_e\nu_\tau)$ and its prediction from $\mathcal{B}_\mu = \mathcal{B}(\tau \to \mu\overline{\nu}_\mu\nu_\tau)$,

$$\mathcal{B}_e^{\mathcal{B}_\mu} = \mathcal{B}_\mu \frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)} \frac{R_W^{\tau e}}{R_W^{\tau\mu}} , \tag{2}$$

to obtain $\mathcal{B}_e'$, a measurement of the τ electronic branching fraction that summarizes the experimental information on the electron and the muon couplings from the measurements of B_e and B_μ. We test the universality of the couplings of the lighter leptons with respect to the τ by comparing $\mathcal{B}_e'$ to the predicted τ electronic branching fraction from the measurement of the τ lifetime:

$$\mathcal{B}_e^{\tau_\tau} = \mathcal{B}(\mu \to e\overline{\nu}_e\nu_\mu) \frac{\tau_\tau}{\tau_\mu} \frac{m_\tau^5}{m_\mu^5} \frac{f(m_e^2/m_\tau^2)}{f(m_e^2/m_\mu^2)} \frac{R_W^{\tau e}}{R_W^{\mu e}} \frac{R_\gamma^\tau}{R_\gamma^\mu} . \tag{3}$$

This formula describes a linear relation between $\mathcal{B}_e'$ and the tau lifetime, which is compared with their preliminary HFLAV averages in Figure 1. The uncertainty on the linear prediction depends primarily on the uncertainty on the τ mass, which has been reduced thanks to the recent KEDR [4] and Belle II [5] measurements.

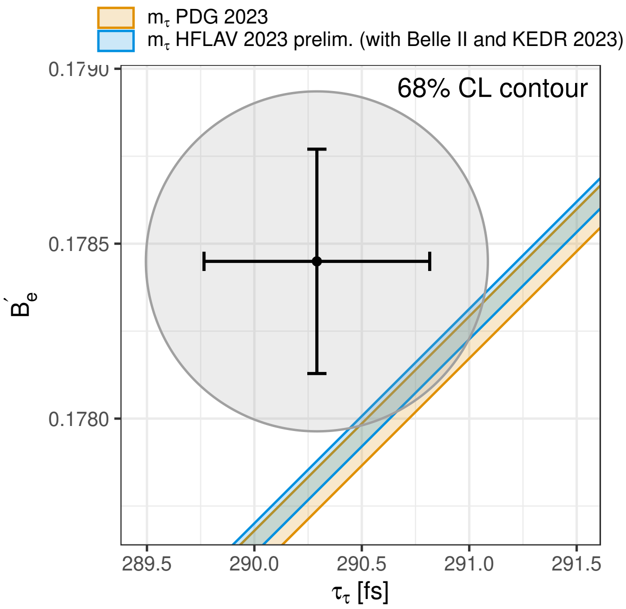

Figure 1: Lepton universality test with the measurements of the tau leptonic branching fractions. The yellow (blue) band corresponds to the tau mass uncertainty before (after) including the recent KEDR and Belle II measurements.

3. Universality-improved value of $\mathcal{B}(\tau \to e\bar{\nu}_e\nu_\tau)$

Following Ref. [6], we compute the experimental value of $\mathcal{B}_e = \mathcal{B}(\tau \to e\bar{\nu}_e\nu_\tau)$ assuming the Standard Model lepton universality by averaging the $\mathcal{B}_e$ fit value $\mathcal{B}_5$, the $\mathcal{B}_e$ determination from the $\mathcal{B}_\mu = \mathcal{B}(\tau \to \mu\bar{\nu}_\mu\nu_\tau)$ fit value $\mathcal{B}_3$ assuming that $g_\mu/g_e = 1$ (Eq. 2), and the $\mathcal{B}_e$ determination from the τ lifetime assuming that $g_\tau/g_\mu = 1$ (Eq. 3). Accounting for correlations, we obtain $\mathcal{B}_e^{\text{uni}} = (17.815 \pm 0.023)\%$.

4. $|V_{us}|$ from tau branching fractions

$|V_{us}|$ is computed using the procedures documented in the 2021 HFLAV report [2] as $|V_{us}|_{\tau\text{-OPE-1}}$, using the sum of exclusive branching fractions to strange hadronic final states, as $|V_{us}|_{\tau K/\pi}$, using the ratio of branching fractions ($|V_{us}|_{\tau K/\pi}$), as $|V_{us}|_{\tau K}$, using the branching fraction $\mathcal{B}(\tau^- \to K^-\nu_\tau)$. For these calculations, and also later in this section, $|V_{ud}| = 0.97384 \pm 0.00026$ is taken from a recent average of measurements of superallowed nuclear β decays, neutron decay measurements [7].

We use a recently reported lattice QCD calculation of the inclusive τ hadronic decay rate [8] to compute

$$|V_{us}|_{\tau\text{-latt-incl}} = \sqrt{\left(\frac{|V_{us}|^2}{R_{us}}\right)_{\text{latt-incl}} \cdot R_{us}} = 0.2189 \pm 0.0019 \,, \tag{4}$$

where $(R_{us}/|V_{us}|^2)_{\text{latt-incl}} = 3.407 \pm 0.022$ [8]. Both $|V_{us}|_{\tau\text{-latt-incl}}$ and $|V_{us}|_{\tau\text{-OPE-1}}$ are not presently including the long-distance isospin-breaking corrections [8].

We recall two additional $|V_{us}|$ determinations, which have not been updated to the present HFLAV averages due to their complexity, $|V_{us}|_{\tau\text{-OPE-2}}$ [9, 10] and $|V_{us}|_{\tau\text{-latt-disp}}$ [10, 11].

Averaging the two $|V_{us}|$ determinations that rely on exclusive τ branching fractions (Eqs. 10, 11), we get $|V_{us}|_{\tau\text{ excl}}$, and averaging the τ inclusive and exclusive $|V_{us}|$ determinations (Eqs. 8, 9, 10, 11), we get $|V_{us}|_\tau$.

We compare all the above $|V_{us}|$ determinations with $|V_{us}|_{\text{uni}}$, predicted from the CKM unitarity relation $(|V_{us}|_{\text{uni}})^2 = 1 - |V_{ud}|^2 - |V_{ub}|^2$, with $|V_{ub}| = 0.00382 \pm 0.00020$ [12], reporting the values, the discrepancy with respect to $|V_{us}|_{\text{uni}}$, and an illustration of the measurement method:

$$
\begin{aligned}
&|V_{us}|_{\text{uni}} &&= 0.2272 \pm 0.0011 && 0.0\sigma && [\sqrt{1 - |V_{ud}|^2 - |V_{ub}|^2} \quad \text{(CKM unitarity)}] \,, && (5)\\
&|V_{us}|_{K\ell 3} &&= 0.2233 \pm 0.0005 && 3.2\sigma && [\mathcal{B}_{K\ell 3}\ [7]] \,, && (6)\\
&|V_{us}|_{K\ell 2} &&= 0.2250 \pm 0.0005 && 1.7\sigma && [\mathcal{B}_{K\ell 2}\ [7]] \,, && (7)\\
&|V_{us}|_{\tau\text{-OPE-1}} &&= 0.2184 \pm 0.0021 && 3.6\sigma && [\mathcal{B}(\tau^- \to X_s^- \nu_\tau)] \,, && (8)\\
&|V_{us}|_{\tau\text{-latt-incl}} &&= 0.2189 \pm 0.0019 && 3.7\sigma && [\mathcal{B}(\tau^- \to X_s^- \nu_\tau)] \,, && (9)\\
&|V_{us}|_{\tau K/\pi} &&= 0.2229 \pm 0.0019 && 2.0\sigma && [\mathcal{B}(\tau^- \to K^- \nu_\tau)/\mathcal{B}(\tau^- \to \pi^- \nu_\tau)] \,, && (10)\\
&|V_{us}|_{\tau K} &&= 0.2224 \pm 0.0017 && 2.3\sigma && [\mathcal{B}(\tau^- \to K^- \nu_\tau)] \,, && (11)\\
&|V_{us}|_{\tau\text{ excl}} &&= 0.2225 \pm 0.0017 && 2.3\sigma && [\text{average of } \tau \text{ exclusive measurements}] \,, && (12)\\
&|V_{us}|_\tau &&= 0.2208 \pm 0.0014 && 3.6\sigma && [\text{average of 4 } |V_{us}|\ \tau \text{ measurements}] \,. && (13)
\end{aligned}
$$

Figure 2 reports the above mentiones $|V_{us}|$ determinations.

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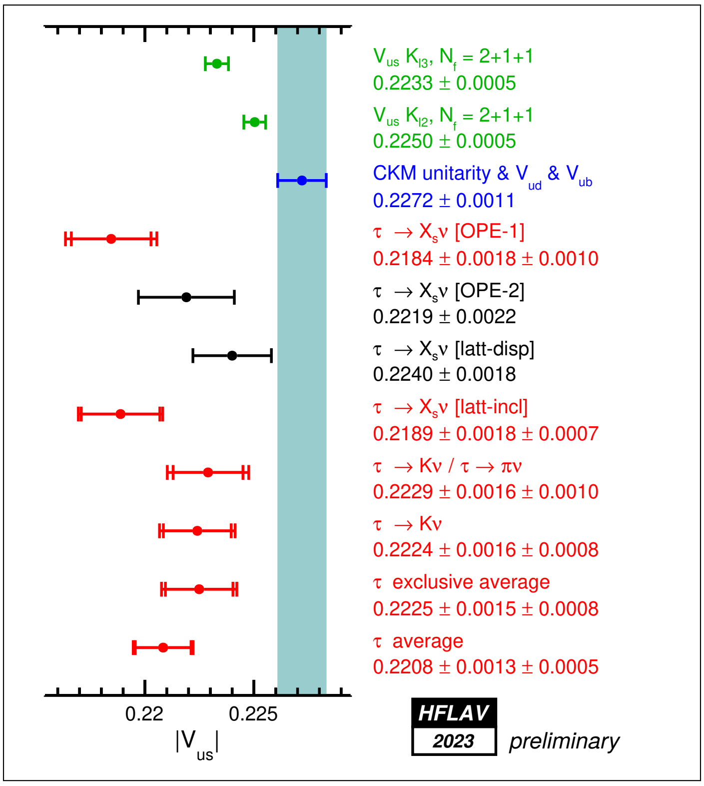

Figure 2: $|V_{us}|$ determinations. The values of $|V_{us}|_{K\ell 3}$, $|V_{us}|_{K\ell 2}$ and the expected $|V_{us}|$ from the CKM matrix unitarity are taked from Ref. [7]. The other reported $|V_{us}|$ values are documented in the text. When two uncertainties are reported, the first one accounts for the uncertainties of the HFLAV-Tau fit results, and the second one accounts for the uncertainties of the theory and the other inputs that are used for the $|V_{us}|$ determinations.

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