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Amplitude-Based IR-Improvement in Precision LHC/FCC Physics

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We present recent results based on the IR-improvement of unintegrable singularities in the infrared regime via amplitude-based resummation in $QED \times QCD \subset SU(2)_L \times U_1 \times SU(3)_c$. In the context of precision LHC/FCC physics, we focus on specific examples, such as the removal of QED contamination in PDF's evolved from data at $Q_0^2 \sim 2$ GeV^2 and used in the evaluating precision observables in $pp \to Z + X \to \ell\bar{\ell} + X'$, in which we discuss new results and new issues.

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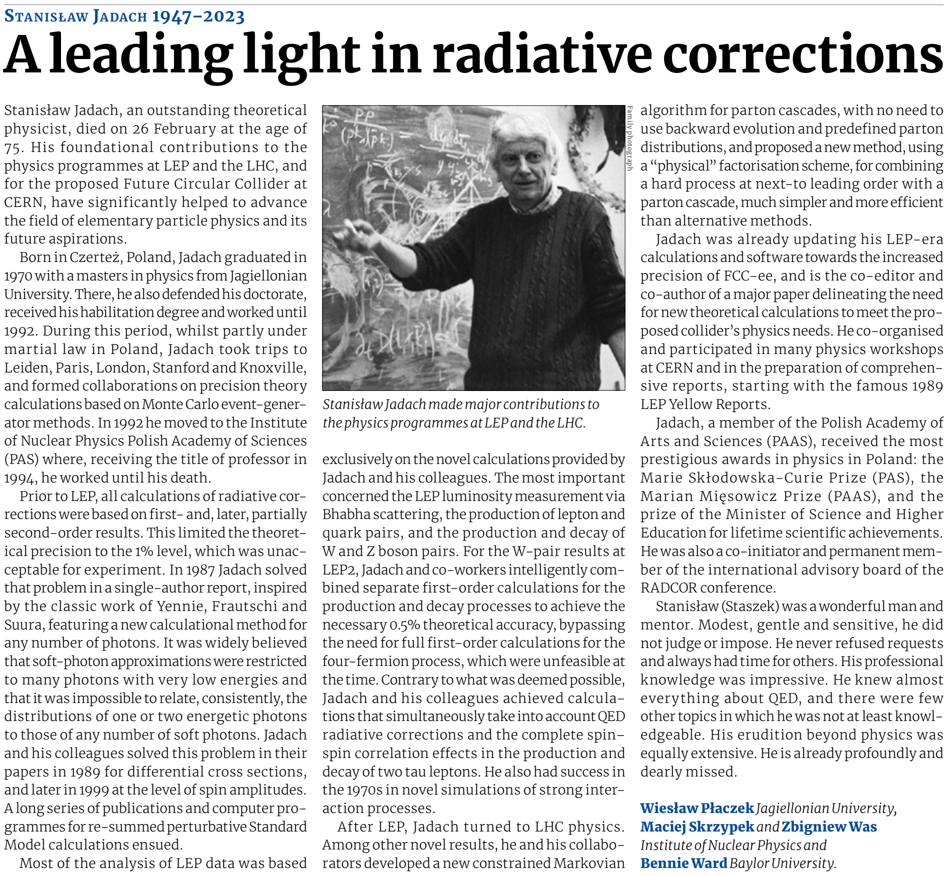
1. In Memory of Prof. Stanislaw Jadach

Sadly, on February 26, 2023 my (BFLW) close friend and collaborator, Prof. Stanislaw Jadach, passed away suddenly.Here in Fig. 1 we reproduce his *CERN Courier*[*] obituary. He was a pioneering researcher in the theory and MC simulation of higher order corrections in quantum field theory. As this is the first Rochester Conference since his passing, we lift up his special contributions to our field which helped to keep our field alive. We all miss him dearly. This contribution is dedicated in memoriam to him.



STANISŁAW JADACH 1947–2023

A leading light in radiative corrections

Stanisław Jadach, an outstanding theoretical physicist, died on 26 February at the age of 75. His foundational contributions to the physics programmes at LEP and the LHC, and for the proposed Future Circular Collider at CERN, have significantly helped to advance the field of elementary particle physics and its future aspirations.

Born in Czerteż, Poland, Jadach graduated in 1970 with a masters in physics from Jagiellonian University. There, he also defended his doctorate, received his habilitation degree and worked until 1992. During this period, whilst partly under martial law in Poland, Jadach took trips to Leiden, Paris, London, Stanford and Knoxville, and formed collaborations on precision theory calculations based on Monte Carlo event-generator methods. In 1992 he moved to the Institute of Nuclear Physics Polish Academy of Sciences (PAS) where, receiving the title of professor in 1994, he worked until his death.

Prior to LEP, all calculations of radiative corrections were based on first- and, later, partially second-order results. This limited the theoretical precision to the 1% level, which was unacceptable for experiment. In 1987 Jadach solved that problem in a single-author report, inspired by the classic work of Yennie, Frautschi and Suura, featuring a new calculational method for any number of photons. It was widely believed that soft-photon approximations were restricted to many photons with very low energies and that it was impossible to relate, consistently, the distributions of one or two energetic photons to those of any number of soft photons. Jadach and his colleagues solved this problem in their papers in 1989 for differential cross sections, and later in 1999 at the level of spin amplitudes. A long series of publications and computer programmes for re-summed perturbative Standard Model calculations ensued.

Most of the analysis of LEP data was based exclusively on the novel calculations provided by Jadach and his colleagues. The most important concerned the LEP luminosity measurement via Bhabha scattering, the production of lepton and quark pairs, and the production and decay of W and Z boson pairs. For the W-pair results at LEP2, Jadach and co-workers intelligently combined separate first-order calculations for the production and decay processes to achieve the necessary 0.5% theoretical accuracy, bypassing the need for full first-order calculations for the four-fermion process, which were unfeasible at the time. Contrary to what was deemed possible, Jadach and his colleagues achieved calculations that simultaneously take into account QED radiative corrections and the complete spin-spin correlation effects in the production and decay of two tau leptons. He also had success in the 1970s in novel simulations of strong interaction processes.

After LEP, Jadach turned to LHC physics. Among other novel results, he and his collaborators developed a new constrained Markovian algorithm for parton cascades, with no need to use backward evolution and predefined parton distributions, and proposed a new method, using a "physical" factorisation scheme, for combining a hard process at next-to leading order with a parton cascade, much simpler and more efficient than alternative methods.

Jadach was already updating his LEP-era calculations and software towards the increased precision of FCC-ee, and is the co-editor and co-author of a major paper delineating the need for new theoretical calculations to meet the proposed collider's physics needs. He co-organised and participated in many physics workshops at CERN and in the preparation of comprehensive reports, starting with the famous 1989 LEP Yellow Reports.

Jadach, a member of the Polish Academy of Arts and Sciences (PAAS), received the most prestigious awards in physics in Poland: the Marie Skłodowska-Curie Prize (PAS), the Marian Mięsowicz Prize (PAAS), and the prize of the Minister of Science and Higher Education for lifetime scientific achievements. He was also a co-initiator and permanent member of the international advisory board of the RADCOR conference.

Stanisław (Staszek) was a wonderful man and mentor. Modest, gentle and sensitive, he did not judge or impose. He never refused requests and always had time for others. His professional knowledge was impressive. He knew almost everything about QED, and there were few other topics in which he was not at least knowledgeable. His erudition beyond physics was equally extensive. He is already profoundly and dearly missed.

Wiesław Płaczek *Jagiellonian University,* **Maciej Skrzypek** *and* **Zbigniew Was** *Institute of Nuclear Physics and* **Bennie Ward** *Baylor University.*

Family photograph

Stanisław Jadach made major contributions to the physics programmes at LEP and the LHC.

Figure 1: *CERN Courier* obituary for the late Prof. Stanislaw Jadach.

2. Recapitulation of YFS Exact Amplitude-Based Resummation

It is still true that our YFS [1] exact amplitude-based resummation theory, especially its CEEX [2] realization, is generally familiar in our field. We present in this section a brief recapitulation of the theory accordingly before turning to some new results based on it in the next section.

The theory is exhibited by the following master formula:

$$d\bar{\sigma}_{\rm res} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}}$$

[*] *CERN Courier*, May/June 2023 issue, p.59.

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$$\prod_{j_2=1}^{m} \frac{d^3k'_{j_2}}{k'_{j_2}} \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2})+D_{\rm QCED}}$$
$$\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3p_2}{p_2^0}\frac{d^3q_2}{q_2^0}, \tag{1}$$

where the *new*[†] (YFS-style) residuals $\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$ have n hard gluons and m hard photons. The new residuals and the infrared functions $\mathrm{SUM_{IR}(QCED)}$ and $D_{\rm QCED}$ are defined in Ref. [4, 5]. As explained in Ref. [4, 5], parton shower/ME matching engenders the replacements $\tilde{\bar{\beta}}_{n,m} \rightarrow \hat{\tilde{\bar{\beta}}}_{n,m}$, which allow us to connect with MC@NLO [6, 7], via the basic formula

$$d\sigma = \sum_{i,j}\int dx_1dx_2F_i(x_1)F_j(x_2)d\hat{\sigma}_{\rm res}(x_1x_2s). \tag{2}$$

Eq.(1) has been used to obtain new results in precision LHC and FCC physics. One of us (BFLW) has extended (See Ref. [8] and references therein.) eq.(1) to general relativity as an approach to quantum gravity. New results are accompanied with new perspectives in each of our applications, as we illustrate in the next Section.

3. New Perspectives for Precision Collider Physics: LHC, FCC, CEPC, CPPC, ILC, CLIC

We have a new perspective on the expectations for precision physics for the Standard Theory EW interactions at HL-LHC due to the realization of eq.(1) in the MC event generator $\mathcal{KK}$MC-hh [9] by four of us (SJ, BFLW, ZAW, SAY). We illustrate this here with the plots in Fig. 2 in the ATLAS analysis [10] of $Z\gamma$ production at 8 TeV. The Powheg-Pythia8-Photos [11–16],

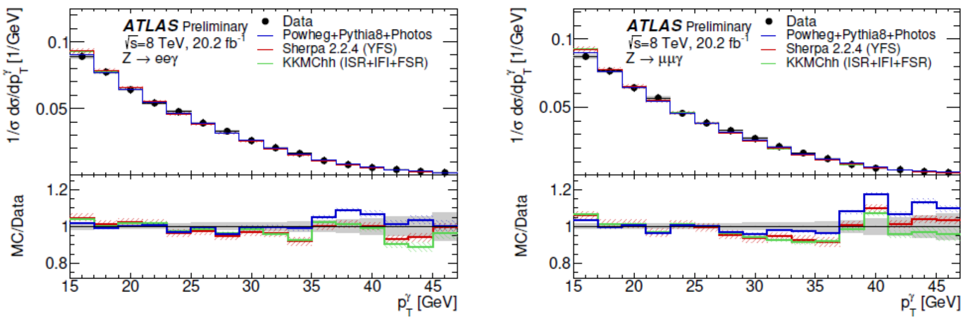

Figure 2: ATLAS analysis of Z/γ production at 8 TeV.

Sherpa2.2.4(YFS) [17, 18], and $\mathcal{KK}$MC-hh predictions for the γp_T spectrum are compared with the ATLAS data. At this point, with the level of the uncertainties in the data, all three predictions are in reasonable agreement with the data. With 100 times the featured statistics, at HL-LHC a precision test against the theories will obtain.

In a new perspective toward the important issue of the effect of QED contamination in non-QED PDFs [19–21] we use Negative ISR (NISR) evolution to address the size of this contamination

[†]The *non-Abelian* nature of QCD requires a new treatment of the corresponding part of the IR limit [3] so that we usually include in $\mathrm{SUM_{IR}(QCED)}$ only the leading term from the QCD exponent in Ref. [3] – the remainder is included in the residuals $\tilde{\bar{\beta}}_{n,m}$.

directly. Using a standard notation for PDFs and cross sections, we have the cross section representation

$$\begin{aligned}\sigma(s) = &\frac{3}{4}\pi\sigma_0(s) \sum_{q=u,d,s,c,b} \int d\hat{x}\; dzdrdt \int dx_q dx_{\bar{q}}\; \delta(\hat{x} - x_q x_{\bar{q}} zt)\\ &\times f_q^{h_1}(s\hat{x}, x_q) f_{\bar{q}}^{h_2}(s\hat{x}, x_{\bar{q}})\; \rho_I^{(0)}(\gamma_{Iq}(s\hat{x}/m_q^2), z)\; \rho_I^{(2)}(-\gamma_{Iq}(Q_0^2/m_q^2), t)\\ &\times \sigma_{q\bar{q}}^{Born}(s\hat{x}z)\; \langle W_{MC}\rangle,\end{aligned} \tag{3}$$

which includes an extra convolution with the well known second order exponentiated ISR "radiator function" $\rho_I^{(2)}$ with the negative evolution time argument $-\gamma_{Iq}(Q_0^2/m_q^2)$ defined in Ref. [19]. This removes the QED below Q_0. This is exhibited in Fig. 3 from Ref. [20] for the P_{T_γ} for the photon

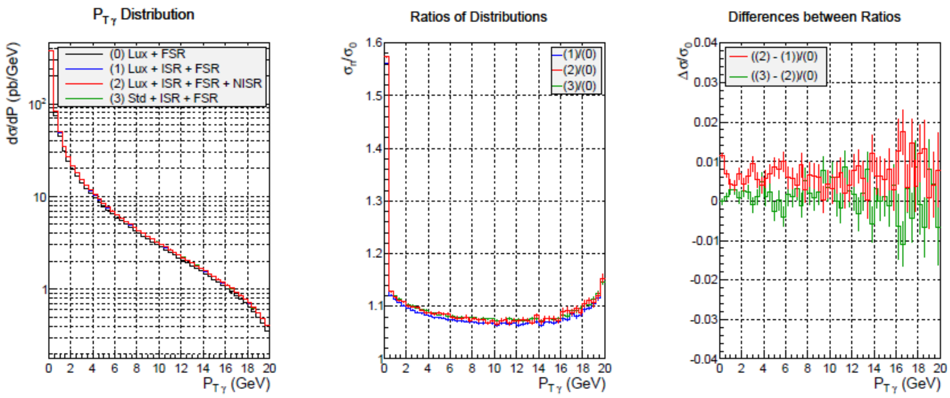

Figure 3: For events with at least one photon, the distribution for P_{T_γ} of the photon for which it is greatest for events with each lepton having $p_{T\ell} > 25GeV, \eta_\ell < 2.5$ calculated with (0) FSR only (black). (1) FSR + ISR (blue). and (2) FSR + ISR with NISR (red) for NNPDF3.1-LuxQED NLO PDFs. For comparison, (3) shows FSR + ISR with ordinary NNPDF3.1 NLO PDFs (green). The center graph shows ISR on/off ratios (1)/(0) (blue),(2)/(0) (red) and (3)/(0) (green). The right-hand graph shows the fractional differences ((1)-(2))/(0) in red and ((2)- (3))/(0) in green.

for which it is the largest in $Z\gamma^*$ production and decay to lepton pairs at the LHC at 8 TeV for cuts as described in the figure. In agreement with arguments in Ref. [22], the results in the figure show that the effect of QED contamination in non-QED PDFs is below the errors on the PDFs.

In view of the planned EW/Higgs factories, five of us (SJ, WP, MS, BFLW, SAY) have discussed in Refs. [23–26] the new perspectives for the BHLUMI [27] luminosity theory error. In Fig. 4 [25], to illustrate this new perspective, we show the current purview for the FCC-ee at M_Z and for the proposed higher energy colliders. We note the improvements at M_Z shown in Fig. 4(a) to 0.007% and that the use of the results in Ref. [28] together with lattice methods [29–31] opens the possibility that item (c) in Fig 4(a) could be reduced by a factor of 6[‡].

4. Improving the Collinear Limit in YFS Theory

The higher precision requirements for HL-LHC/FCC physics motivate improving the collinear limit of YFS theory. For, it is known [32] that, for the process $e^+(p_2)\; e^-(p_1) \rightarrow \bar{f}(p_4)\; e^-(p_3)$,

[‡]The formula to be studied is $\Delta\alpha_{had}(t) = \Delta\alpha_{had}(-Q_0^2)|_{lat} + [\Delta\alpha_{had}(t) - \Delta\alpha_{had}(-Q_0^2)]|_{pQCDAdler}$ with *lat* denoting the methods of Refs. [29–31] and *pQCDAdler* denoting the methods of Ref. [28].

Forecast study for FCCee$_{M_Z}$			
Type of correction / Error	Published [1]	Strict	Redone
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	0.06×10^{-4}	0.06×10^{-4}
(b') Photonic $\mathcal{O}(\alpha^2 L^0)$		0.17×10^{-4}	0.17×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	0.6×10^{-4}	0.6×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.4×10^{-4}	0.27×10^{-4}
(e) Z and s-channel γ exch.	0.1×10^{-4}	0.1×10^{-4}	0.1×10^{-4}
(f) Up-down interference	0.1×10^{-4}	0.08×10^{-4}	0.08×10^{-4}
Total	1.0×10^{-4}	0.76×10^{-4}	0.70×10^{-4}

(a)

Forecast			
Type of correction / Error	ILC$_{500}$	ILC$_{1000}$	CLIC$_{3000}$
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.13×10^{-4}	0.15×10^{-4}	0.20×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.27×10^{-4}	0.37×10^{-4}	0.63×10^{-4}
(c) Vacuum polariz.	1.1×10^{-4}	1.1×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.4×10^{-4}	0.5×10^{-4}	0.7×10^{-4}
(e) Z and s-channel γ exch.	$1.0 \times 10^{-4(*)}$	2.4×10^{-4}	16×10^{-4}
(f) Up-down interference	$< 0.1 \times 10^{-4}$	$< 0.1 \times 10^{-4}$	0.1×10^{-4}
Total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}

(b)

Figure 4: Current purview on luminosity theory errors: (a), FCC-ee at M_Z; (b), proposed higher energy colliders

in the usual YFS theory the s-channel virtual infrared function B resums (exponentiates) the non-infrared term $\frac{1}{2}Q_e^2\frac{\alpha}{\pi}L$ whereas the methods in Ref. [33] show that the term $\frac{3}{2}Q_e^2\frac{\alpha}{\pi}L$ exponentiates – see also Refs. [34–37] for recent developments in the attendant collinear factorization approach. Here, we use an obvious notation where the respective big log is $L = \ln(s/m_e^2)$ when $s = (p_1 + p_2)^2$ is the center-of-mass energy squared. In Ref. [32], three of us (SJ, BFLW, ZAW) have shown that the suggested collinear enhancement of YFS theory which exponentiates the term $\frac{3}{2}Q_e^2\frac{\alpha}{\pi}L$ does exist.

Specifically, we find that the virtual infrared function B in the s-channel and the corresponding real infrared function $\tilde{B}$ can be extended to

$$
\begin{aligned}
B_{CL} &\equiv B + \mathbf{\Delta B} \\
&= \int \frac{d^4k}{k^2}\frac{i}{(2\pi)^3}\left[\left(\frac{2p-k}{2kp-k^2} - \frac{2q+k}{2kq+k^2}\right)^2 - \frac{\mathbf{4pk-4qk}}{\mathbf{(2pk-k^2)(2qk+k^2)}}\right],
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\tilde{B}_{CL} &\equiv \tilde{B} + \mathbf{\Delta\tilde{B}} \\
&= \frac{-1}{8\pi^2}\int\frac{d^3k}{k_0}\left\{\left(\frac{p_1}{kp_1} - \frac{p_2}{kp_2}\right)^2 + \frac{\mathbf{1}}{\mathbf{kp_1}}\left(\mathbf{2} - \frac{\mathbf{kp_2}}{\mathbf{p_1p_2}}\right) + \frac{\mathbf{1}}{\mathbf{kp_2}}\left(\mathbf{2} - \frac{\mathbf{kp_1}}{\mathbf{p_1p_2}}\right)\right\},
\end{aligned} \tag{5}
$$

where the extensions are indicated in boldface in an obvious notation. These extensions leave the YFS infrared algebra unaffected while the B_{CL} does exponentiate the entire $\frac{3}{2}Q_e^2\frac{\alpha}{\pi}L$ term and the $\tilde{B}_{CL}$ does carry the respective collinear big log of the exact result in Ref. [38] in the soft regime.

In the CEEX case, the corresponding collinear extension of the soft eikonal amplitude factor defined in Ref. [39] for the photon polarization σ and e^- helicity σ' is given by

$$
\begin{aligned}
\mathfrak{s}_{CL,\sigma}(k) = \sqrt{2}Q_e e\Bigg[&-\sqrt{\frac{p_1\zeta}{k\zeta}}\frac{< k\sigma|\hat{p}_1 - \sigma >}{2p_1k} + \delta_{\sigma'-\sigma}\sqrt{\frac{\mathbf{k\zeta}}{\mathbf{p_1\zeta}}}\frac{\mathbf{< k\sigma|\hat{p}_1\sigma' >}}{\mathbf{2p_1k}} \\
&+\sqrt{\frac{p_2\zeta}{k\zeta}}\frac{< k\sigma|\hat{p}_2 - \sigma >}{2p_2k} + \delta_{\sigma'\sigma}\sqrt{\frac{\mathbf{k\zeta}}{\mathbf{p_2\zeta}}}\frac{\mathbf{< \hat{p}_2\sigma'|k - \sigma >}}{\mathbf{2p_2k}}\Bigg],
\end{aligned} \tag{6}
$$

where from Ref. [39] $\zeta \equiv (1,1,0,0)$ for our choice for the respective auxiliary vector in our Global Positioning of Spin (GPS) [40] spinor conventions with the consequent definition $\hat{p} = p - \zeta m^2/(2\zeta p)$ for any four vector p with $p^2 = m^2$. The collinear extension terms are again indicated in boldface.

These extended infrared functions are expected to give in general a higher precision for a given level of exactness [41].

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