PROCEEDINGS OF SCIENCE

Probing cosmic censorship in Reissner-Nordström de Sitter black holes

A. Chrysostomou,[a,b,*] **A. S. Cornell,**[a] **A. Deandrea,**[a,b] **H. Noshad**[a] **and S. C. Park**[c]

[a] *Department of Physics, University of Johannesburg, PO Box 524, Auckland Park 2006, South Africa*
[b] *Université Claude Bernard Lyon 1, IP2I, UMR 5822, CNRS/IN2P3, 4 rue Enrico Fermi, 69622 Villeurbanne Cedex, France*
[c] *Department of Physics and IPAP, Yonsei University, Seoul 03722, Republic of Korea*

E-mail: chrysostomou@ip2i.in2p3.fr, acornell@uj.ac.za, deandrea@ipnl.in2p3.fr, hnoshad@uj.ac.za, sc.park@yonsei.ac.kr

From black hole quasinormal frequencies (QNFs), we can extract characterising information about their perturbed source. For the Reissner-Nordström de Sitter (RNdS) black hole, the damping of the QNF spectrum relays information about whether these fields evolve past the Cauchy horizon, in violation of Strong Cosmic Censorship. Here, we examine the QNF spectrum corresponding to a massive scalar test field carrying an electric charge, oscillating in the outer region of a RNdS black hole. Our analysis provides insight into QNF behaviour, particularly in regions approaching the extremal conditions of the black hole. Our semi-classical analysis suggests that Strong Cosmic Censorship may be violated for black holes that are in close proximity to extremality.

42nd International Conference on High Energy physics - ICHEP2024
17-24 July, 2024
Prague, Czech Republic

*Speaker

Introduction

A black hole is distinguished from other compact bodies by two features: its event horizon and the (curvature) singularity it encloses. Within General Relativity (GR) [1, 2], the event horizon is the boundary of the black hole from within which nothing can escape; assuming a causal and asymptotically-flat space-time $(\mathcal{M}, g)$, it is formally defined as a null hypersurface comprised of future inextendible null geodesics without caustics. Assuming $(\mathcal{M}, g)$ is a 4D time-orientable Lorentzian manifold, the singularity is then denoted by a future-directed future-inextendible time-like curve $C \subset \mathcal{M}$. The "Weak Cosmic Censorship" (WCC) conjecture stipulates that there are no naked singularities on $(\mathcal{M}, g)$ [3]. The "Strong Cosmic Censorship" (SCC) conjecture is the requirement that for generic initial data, the "maximal future Cauchy development" (MFCD) (see Theorem 10.2.2. of Ref. [2]) is locally inextendible as a suitably regular Lorentzian manifold [4]. Arguments in favour of and in opposition to SCC have become dominated by the "initial-value approach" to GR, where SCC preservation is posed as an initial-value problem that seeks to prove for the most stringent possible criteria that the MFCD is inextendible as a solution to the vacuum Einstein equations [5].

Here, we shall demonstrate how WCC preservation affects the mass-charge phase space of the Reissner-Nordström de Sitter (RNdS) black hole and how the preservation of SCC can be verified through the computation of black hole quasinormal frequencies (QNFs). For our consideration of SCC preservation, the scalar field Φ serves as a convenient proxy for the full non-linear Einstein field equations; to interrogate the influence of field parameters for the problem at hand, we shall consider a charged and massive Φ, with mass parameter μ and electromagnetic charge q.

The RNdS black hole

In the mostly-plus GR signature, the spherically-symmetric black hole metric is given by

$$ds^2 = -f(r)dt^2 \ + \ f(r)^{-1}dr^2 \ + \ r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \ , \tag{1}$$

written in terms of the usual Schwarzschild coordinates (t, r, θ, ϕ), with $t \in (-\infty, +\infty)$, $\theta \in (0, \pi)$, and $\phi \in (0, 2\pi)$. Under the appropriate parametrisations (e.g. Refs. [6, 7]), we can express the metric function of the RNdS black hole in terms of its mass M, charge Q, and de Sitter radius L_{dS},

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2}{L_{dS}^2} \ , \tag{2}$$

in Planck units. From the four real roots of Eq. (2), we can identify three Killing horizons: the Cauchy horizon r_-, the event horizon r_+, and the cosmological horizon r_c, where

$$0 < r_- \leq r_+ \leq r_c \leq L_{dS} < \infty \ . \tag{3}$$

The fourth (and unphysical) root is given by $r_0 = -(r_- + r_+ + r_c)$.

In order to delineate the RNdS (M, Q) phase space, we begin with the polynomial,

$$\Pi(r) \equiv -r^2 f(r) = -r^2 + 2Mr - Q^2 + L_{dS}^2 r^4 \ . \tag{4}$$

We then determine the discriminant thereof,

$$\Delta \equiv -16L_{dS}^{-2}\left[27M^4L_{dS}^{-2} - M^2(1 + 36Q^2L_{dS}^{-2}) + (Q + 4Q^3L_{dS}^{-2})^2\right] \ , \tag{5}$$

set $L_{dS}^{-2} = \Lambda/3 = 1$, and plot $\Delta = 0$. This yields the "sharkfin" (Fig. 1), containing all valid black hole solutions. In asymptotically-flat space-time, $Q/M \leq 1$ to preserve WCC; here, $Q/M \leq 1.0607$.

PoS(ICHEP2024)782

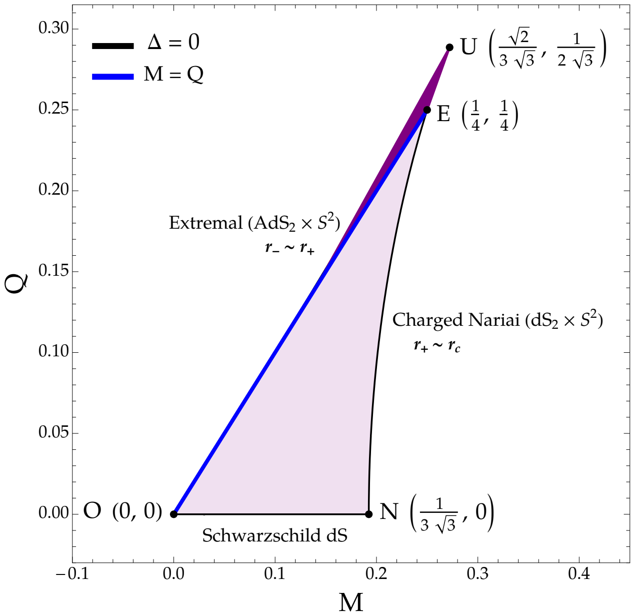

Figure 1: *A 2D projection of the parameter space for $H^2 = \Lambda/3 = 1$ for the 4D RNdS black hole. Dark (light) shading corresponds to cold (warm) black holes; lines OE and NU represent thermal equilibrium.*

Background on Strong Cosmic Censorship in charged black holes

The Cauchy horizon of the Reissner-Nordström (RN) space-times presents a challenge for the preservation of SCC: the MFCD can be smoothly future-extended across the space-like Cauchy horizon towards the inner time-like singularity, where the unknown boundary conditions thereof modify the evolution of the MFCD non-uniquely. Since determinism is a prerequisite for a well-defined theory, this non-unique evolution of the MFCD threatens the integrity of GR. However, in recognising the "blue-shift instability" of the Cauchy horizon, Penrose found that the MFCD indeed becomes inextendible past $r = r_-$, thereby rescuing SCC and restoring determinism in GR [8]. To understand the premise of this "blue-shift mechanism", consider two observers external to the RN black hole: a time-like observer A remaining in the exterior region and a time-like observer B falling into the black hole. Observer A will reach future infinity at infinite proper time; observer B will reach the Cauchy horizon at finite proper time. If observer A sends periodic (in A's time) signals to observer B, observer B will perceive them as incoming with greater frequency/energy. By this logic, the frequency of an oscillating scalar field Φ entering the black hole will increase infinitely as it approaches $r = r_-$, leading to an infinite "blue-shifting" of Φ, such that the scalar field is rendered inextendible past the Cauchy horizon. Formally, Penrose's proposal was that for generic asymptotically-flat initial data within a Kerr background[1], the MFCD is inextendible as a continuous Lorentzian metric, as the infinite amplification of energy at $r = r_-$ corresponds to a curvature divergence [9]. This was a claim for the most stringent (i.e. the C^0-) formulation of SCC.

However, Penrose's formulation does not take into account the global black hole metric. It was recently argued by Dafermos *et al.* [10, 11] that the well-known stability of the metric exterior in the wake of some space-time perturbation [12, 13] implies that the incoming scalar field experiences an inverse polynomial decay in time along the event horizon. Decay in the exterior is therefore in competition with the blue-shift mechanism at the Cauchy horizon. The consequence of this argument is that for smooth localised initial data in sub-extremal Kerr – and, equivalently, RN –

[1]This result is applicable to RN black holes due to the similar structure of their interior space-time.

black hole space-times, the local energy of Φ "blows up" at the Cauchy horizon. Here, the local energy is the integral of the energy as measured by a local observer; both Φ and its first weak derivative are locally square-integrable. Formally, Φ is then inextendible in H^1_{loc} across the Cauchy horizon. Thus, the slightly weaker "Christodoulou version" of SCC [14] holds in the case of the RN black hole [15]. Recall that this "blowing-up" of the energy is a requirement for Christodoulou's formulation of SCC. As such, this was perceived as a possible means of "saving" SCC by weakening the C^0 requirement, such that the MFCD of the metric need only be inextendible as a continuous Lorentzian manifold whose Christoffel symbols are locally square-integrable. We can consider this loosely as a relaxation of the C^0-formulation of SCC, where the metric cannot be interpreted even as a weak solution of the Einstein equations past the Cauchy horizon.

In the case of the RNdS black hole, however, the decay of Φ differs: while generic initial data decays inverse-polynomially on the event horizon in asymptotically-flat space-time, the decay becomes exponential in asymptotically-de Sitter space-time [16]. Intuitively, we understand this to be a consequence of $\Lambda > 0$, which naturally leads to cosmological expansion, and therefore results in a "red-shifting" effect. Φ at the exterior then experiences exponential damping. In other words, the asymptotics of Φ demonstrate quasinormal mode (QNM) behaviour [17]. In Ref. [18], it was proven that for a non-degenerate RNdS black hole of dimension $d \geq 4$, there exists some $\beta > 0$ dependent only on the black hole parameters such that the exponential decay for massive and neutral scalar fields is governed by the expression,

$$|\Phi| \leq C e^{-\beta t} \ , \quad \beta \equiv -\frac{\Im m\{\omega^{n=0}\}}{|\kappa_-|} \ , \tag{6}$$

for small mass $\mu > 0$ and surface gravity $\kappa_- = f'(r)/2$ evaluated at the Cauchy horizon $r = r_-$. Here, Φ is a linear scalar perturbation; $C \geq 0$ is a constant and $\Im m\{\omega^{n=0}\}$ is the least-damped QNF. As such, Φ becomes continuous up to the Cauchy horizon; the derivatives of Φ lie in the Sobolev space $H^{(\beta+\frac{1}{2})-\epsilon}$ $\forall \epsilon > 0$. To satisfy the stronger Christodoulou formulation of SCC (Φ must be inextendible across the Cauchy horizon in H^1_{loc}), we require that $\beta < 1/2$ [18, 19].

Note that since the RNdS black hole is static and spherically-symmetric, the QNF eigenvalue problem simplifies to an ordinary differential equation in the radial Schwarzschild coordinate,

$$\frac{d^2\varphi(r_*)}{dr_*^2} + \left[\left(\omega - \frac{qQ}{r}\right)^2 - f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r} + \mu^2\right]\right]\varphi(r_*) = 0 \ , \tag{7}$$

where ℓ is the angular momentum number and qQ/r represents the coupling between scalar field charge and the electrostatic four-potential of the black hole $A_t = -Q/r$. The QNM boundary conditions are then imposed,

$$\varphi(r_*) \sim \begin{cases} e^{-i\left(\omega - \frac{qQ}{r_+}\right)r_*} \ , & r \to r_+ \quad (r_* \to -\infty) \ , \\ e^{+i\left(\omega - \frac{qQ}{r_c}\right)r_*} \ , & r \to r_c \quad (r_* \to +\infty) \ , \end{cases} \tag{8}$$

with radiation purely ingoing at the horizon and purely outgoing at the de Sitter horizon [20]. The tortoise coordinate $r_* = \int dr/f(r)$ serves as a bijection from (r_+, r_c) to $(-\infty, +\infty)$.

In this work, we use a semi-classical WKB-based technique described in Ref. [21]. The QNF is expressed as a series expansion $\omega = \sum_{k=-1} \omega_k L^{-k}$, where $L = \sqrt{\ell(\ell+1)}$. The series expansion is then inserted into

$$\omega = \sqrt{V(r_*^{max}) - 2iU} \ , \qquad U \equiv U(V^{(2)}, V^{(3)}, V^{(4)}, V^{(5)}, V^{(6)}) \ . \tag{9}$$

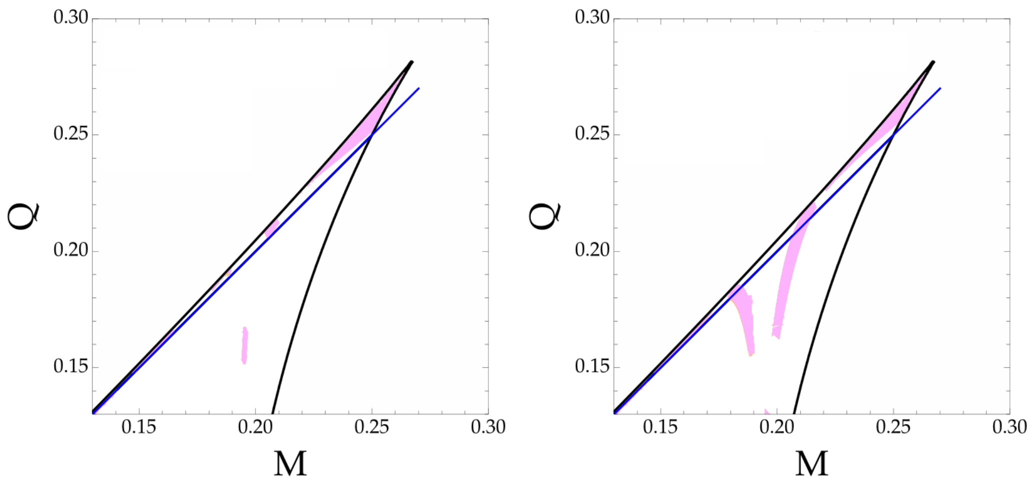

Figure 2: *We shade the parameter space of Fig. 1 in which both ω_+ and ω_c violate the condition for SCC preservation for $\ell = 1$, comparing $\mu = 1$ and $q = 0.1$ (left) against $\mu = 0.1$ and $q = 1$ (right).*

The objective of the method is to solve iteratively for the ω_k coefficients for increasing orders of k. $V(r_*^{max})$ corresponds to the peak of the barrier potential, located at

$$r_*^{max} \approx r_0 + r_1 L^{-2} + \dots , \qquad V(r_*^{max}) \approx V_0 + V_1 L^{-2} + \dots , \tag{10}$$

where subscripts refer to terms in a series expansion around the peak. The numbered superscripts in Eq. (9) refer to derivatives V^j, taken with respect to the tortoise coordinate, such that

$$V^j = \frac{d^j V(r_*^{max})}{dr^j} = f(r)\frac{d}{dr}\left[f(r)\frac{d}{dr}\left[\dots \left[f(r)\frac{dV(r)}{dr}\right] \dots \right]\right]_{r\to r_*^{max}} . \tag{11}$$

This method is most reliable in the large-ℓ regime, and for small values of Q and q. Beyond these limitations, the method allows us to maintain the black hole and scalar field parameters as free variables. Note that upon introducing a non-zero field charge, the QNM reflection symmetry is broken, leaving us with two distinct sets of QNF solutions, here referred to as ω_+ and ω_c.

Evidence of Strong Cosmic Censorship violations

As discussed for the RNdS case, to ensure the damping of the perturbations in the exterior does not suppress the amplifications within the black hole interior, $\beta < 1/2$ in Eq. (6). By computing $\mathcal{I}m\{\omega\}$, we explore the RNdS (M, Q) parameter space in order to determine for which space-time and scalar field parameters we find evidence for $\beta > 1/2$ for $\ell = 1$. The SCC is largely preserved within the sharkfin; we only find evidence of its violation for $M, Q > 0.15$. In particular, we consistently find evidence that SCC is violated within the shaded OEU region of the sharkfin and on certain points on the OU line, particularly near $M \sim Q \sim 0.089$ for near-zero mass and charge. For $\ell = 1$ and $q = 0.1$, we find that $\mu = 0.1$ and $\mu = 1$ violate identical regions, yielding Fig. 2. When $q = 1$, however, a larger region of the parameter space is violated, extending from the extremal $r_- \sim r_+$ regime.

Conclusion

Where singularities exist, the WCC conjecture requires the existence and stability of event horizons. The SCC conjecture, on the other hand, requires the absence or instability of Cauchy horizons. Through our semi-classical analysis, we found that the condition for the preservation of SCC was violated near $M \sim Q \sim 0.089$. For non-zero μ, black holes "colder" than the cosmological horizon were associated with SCC violations. Our results motivate for follow-up studies using more precise computational methods for $\mathcal{I}m\{\omega\}$ (e.g. pseudospectral methods; time-domain analyses, etc.) in this regime.

References

[1] S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge Monographs on Mathematical Physics, Cambridge University Press, Cambridge (1973)

[2] R.M. Wald, *General Relativity*, Chicago Univ. Pr., Chicago, USA (1984)

[3] R. Penrose, *Gravitational collapse: the role of General Relativity*, *Riv. Nuovo Cim.* **1** (1969) 252

[4] M. Dafermos, *Black holes without spacelike singularities*, *Commun. Math. Phys.* **332** (2014) 729 [1201.1797]

[5] K. Landsman, *Penrose's 1965 singularity theorem: from geodesic incompleteness to cosmic censorship*, *Gen. Rel. Grav.* **54** (2022) 115 [2205.01680]

[6] R. Bousso, *Charged Nariai black holes with a dilaton*, *Phys. Rev. D* **55** (1997) 3614 [gr-qc/9608053]

[7] I. Antoniadis and K. Benakli, *Weak Gravity Conjecture in de Sitter space-time*, *Fortschritte der Physik* **68** (2020) 2000054 [2006.12512]

[8] M. Simpson and R. Penrose, *Internal instability in a Reissner-Nordström black hole*, *Int. J. Theor. Phys.* **7** (1973) 183

[9] R. Penrose, *The question of cosmic censorship*, *Journal of Astrophysics and Astronomy* **20** (1999) 233

[10] M. Dafermos and J. Luk, *The interior of dynamical vacuum black holes I: the C^0-stability of the Kerr Cauchy horizon*, 1710.01722

[11] M. Dafermos, I. Rodnianski and Y. Shlapentokh-Rothman, *A scattering theory for the wave equation on Kerr black hole exteriors*, 1412.8379

[12] S.A. Teukolsky, *Perturbations of a rotating black hole. 1. Fundamental equations for gravitational electromagnetic and neutrino field perturbations*, *Astrophys. J.* **185** (1973) 635

[13] W.H. Press and S.A. Teukolsky, *Perturbations of a rotating black hole. II. Dynamical stability of the Kerr metric*, *Astrophys. J.* **185** (1973) 649

[14] D. Christodoulou, *The formation of black holes in General Relativity*, in *12th Marcel Grossmann Meeting on General Relativity*, pp. 24–34, May, 2008, DOI [0805.3880]

[15] J. Luk and S.-J. Oh, *Proof of linear instability of the Reissner–Nordström Cauchy horizon under scalar perturbations*, *Duke Math. J.* **166** (2017) 437 [1501.04598]

[16] M. Dafermos and I. Rodnianski, *The wave equation on Schwarzschild-de Sitter spacetimes*, 0709.2766

[17] M. Dafermos and Y. Shlapentokh-Rothman, *Rough initial data and the strength of the blue-shift instability on cosmological black holes with $\Lambda > 0$*, *Class. Quant. Grav.* **35** (2018) 195010 [1805.08764]

[18] P. Hintz and A. Vasy, *Analysis of linear waves near the Cauchy horizon of cosmological black holes*, *J. Math. Phys.* **58** (2017) 081509 [1512.08004]

[19] J.L. Costa and A.T. Franzen, *Bounded energy waves on the black hole interior of Reissner-Nordström-de Sitter*, *Annales Henri Poincare* **18** (2017) 3371 [1607.01018]

[20] R.A. Konoplya and A. Zhidenko, *Charged scalar field instability between the event and cosmological horizons*, *Phys. Rev. D* **90** (2014) 064048 [1406.0019]

[21] P.A. González, E. Papantonopoulos, J. Saavedra and Y. Vásquez, *Quasinormal modes for massive charged scalar fields in Reissner-Nordström dS black holes: Anomalous decay rate*, *JHEP* **06** (2022) 150 [2204.01570]