

Transversity: Theory/Phenomenology Overview

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In this review, I will discuss the main properties of transversity, the only chiral-odd collinear parton distribution function at leading twist. I will list the possible channels that can be explored to extract transversity from experimental data. I will describe our current knowledge of transversity with particular emphasis on its first Mellin moment, the tensor charge, and the puzzling comparison between phenomenological results and lattice computations. I will conclude by reminding the new useful data that still need to be included in phenomenological analyses, and by exploring the impact of future experimental setups.

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1. Transversity properties

In the framework of leading-twist collinear factorization [1], Parton Distribution Functions (PDFs) describe combinations of number densities of quarks and gluons carrying a fraction x of the momentum of their fast-moving parent hadron. For a spin- $\frac{1}{2}$ hadron like the nucleon, its partonic structure is completely described by three species of PDFs: the unpolarized distribution f_1 , for an unpolarized parton in an unpolarized nucleon; the helicity distribution g_1 , for the net amount of partons polarized along or opposite a longitudinally polarized nucleon; the transversity distribution h_1 , for the net amount of partons polarized along or opposite a transversely polarized nucleon. As such, PDFs encode the properties of partons when confined inside hadrons, hence they are a fundamental ingredient in describing the physics of hadrons. Ultimately, the accuracy of this description crucially depends on the accuracy of PDFs. Viceversa, the uncertainty on PDFs limits the precision of this description, especially when searching for signals of new physics in very precise comparisons between experimental data and Standard Model (SM) predictions.

This applies in particular to the transversity h_1 which is the least known among the three PDF. In fact, by involving transverse polarizations it is connected to processes that flip the parton helicity and that are suppressed in perturbative QCD [2]. Since at leading twist helicity and chirality are the same [3], transversity is usually named a chiral-odd function. As such, it can be measured only in processes where another chiral-odd object enters the cross section (see next section). In canonical field theory, PDFs are defined in terms of matrix elements of bilocal operators; the transversity is connected to a tensor operator that does not appear in the SM Lagrangian at leading order. All these arguments make h_1 an elusive object but, at the same time, very interesting as it could represent a doorway to explore new physics beyond SM (BSM).

For example, high-precision low-energy measurements of β -decays or rare meson decays may expose BSM effects generated at TeV scales through effective Lagrangians that describe new semi-leptonic transitions, involving four-fermion contact terms or scalar/pseudo-scalar/tensor/($V + A$) interactions with operators up to dimension six (for a review, see Ref. [4]). The scalar and tensor operators contribute linearly to β -decay through their interference with SM operators, therefore they are more easily detectable. The transition amplitude is proportional to the product of the BSM coupling and of the corresponding hadronic charge. For tensor interactions, the hadronic charge is connected to the isovector component of the so-called tensor charge, g_T ,

$$g_T = \delta^u(Q^2) - \delta^d(Q^2), \quad \delta^q(Q^2) = \int_0^1 dx \left[h_1^q(x, Q^2) - h_1^{\bar{q}}(x, Q^2) \right] \equiv \int_0^1 dx h_1^{q-\bar{q}}(x, Q^2), \quad (1)$$

namely the isovector combination of first Mellin moments of the transversity for up and down quarks in the nucleon. The experimental measurements have reached nowadays a per-mil precision level, and will perform better in the near future [4]. Therefore, it is important to determine g_T with the largest possible precision in order to deduce reliable information on the unknown BSM coupling [5].

A further example is represented by SM Effective Field Theories (SMEFT) that try to justify the observed matter-antimatter asymmetry in terms of violation of the strong CP symmetry, by relating it to the phenomenon of permanent Electric Dipole Moment (EDM) in fermions like the neutron. The neutron EDM results as a linear combination of quark EDM's whose coefficients are

represented by the quark tensor charges δ^q [6]. Stringent constraints from experiments on neutron EDM could in turn constrain the amount of CP violation provided that quark tensor charges are known with great accuracy.

The transversity h_1 is an interesting object by itself since it significantly differs from the helicity g_1 . All PDFs are defined in the Infinite Momentum Frame, where the parent hadron is boosted to a deeply inelastic kinematic regime. In the non-relativistic limit where boosts and Galilean rotations commute, h_1 and g_1 would be connected by a simple rotation, *i.e.*, they would have the same partonic content. Hence, any observed difference between the two informs us about the relativistic nature of quark dynamics. Indeed, while the first Mellin moment of transversity, the tensor charge, is connected to a C-odd structure (see Eq. (1)), the first Mellin moment of helicity, the axial charge g_A^q , is connected to the C-even structure $g_1^{q+\bar{q}}(x, Q^2)$. Moreover, the axial charge is a true charge, *i.e.*, it is a constant, because in the evolution equations of g_1 the anomalous dimension $\Delta\gamma^{(1)} = 0$. On the contrary, the chiral-odd anomalous dimension $\delta\gamma^{(1)} = -C_F/2$, with $C_F = (N_c^2 - 1)/(2N_c)$ and N_c the number of colors: the tensor ‘‘charge’’ scales with Q^2 . There is one more very important difference. In spin- $\frac{1}{2}$ hadrons like the nucleon, there is no gluon transversity because the maximum hadron helicity flip $\Delta\lambda_h = 1$ cannot match the vector boson helicity flip $\Delta\lambda_g = 2$ of the gluon ($\lambda_g = 1$). This implies that the transversity evolves with the hard scale Q^2 as a non-singlet function, while both quarks and gluons contribute to the (singlet and non-singlet) evolution of the helicity g_1 . In spin-1 hadrons like the deuteron, the gluon transversity is possible because of the hadron transverse tensor polarization [7], but it does not exist in the collinear limit: it explicitly depends on the gluon intrinsic transverse momentum, *i.e.*, it is a transverse-momentum dependent parton distribution (TMD PDF) and it vanishes upon integrating this transverse momentum [8].

Finally, while both transversity h_1 and helicity g_1 must satisfy the positivity constraint with respect to the unpolarized PDF f_1 , namely $|g_1| \leq f_1$ and $|h_1| \leq f_1$ for any (x, Q^2) , the transversity must satisfy also the so-called Soffer bound $|h_1| \leq (f_1 + g_1)/2$ for any (x, Q^2) [9].

2. Extraction from data

The accuracy and precision of a PDF extraction largely depend on the size of the utilized data set. As it is evident in Fig.1 of Ref. [10], the situation is remarkably different across the three PDF species: while there are thousands of measurements available to determine the unpolarized PDF f_1 , and only hundreds of them for the helicity g_1 , the transversity h_1 is limited by its chiral-odd nature to tens of possible measurements where another chiral-odd object appears in the cross section, thus covering a restricted portion of the (x, Q^2) phase space.

Ideally, the simplest situation is represented by the Drell-Yan process $p^\uparrow p^\uparrow \rightarrow \ell + \bar{\ell} + X$, namely by the inclusive production of a lepton-antilepton pair from the collision of two transversely polarized spin- $\frac{1}{2}$ hadrons, like the proton p . In this case, the leading-twist cross section contains an asymmetric term in the angles of the lepton-antilepton pair that includes the combination $h_1^q h_1^{\bar{q}}$, summed upon all flavors q ; the latter could be extracted through a transverse double-spin asymmetry (DSA). However, the transversity $h_1^{\bar{q}}$ of an antiquark in the proton is likely very small; moreover, it must fulfill the Soffer bound. Numerical simulations have suggested that at the collision energies of RHIC (the only hadronic collider currently running transversely polarized hadron beams) the transverse DSA is too small to be measured with the current available precision [11]. A possible

alternative could be the $p^\uparrow \bar{p}^\uparrow \rightarrow \ell + \bar{\ell} + X$ process, where valence degrees of freedom are involved in both sides of the collision [12]. But the technology for accelerating transversely polarized antiprotons while keeping their degree of polarization is not yet available.

Historically, transversity was extracted for the first time from the semi-inclusive deep-inelastic scattering (SIDIS) process $\ell + p^\uparrow \rightarrow \ell' + \pi + X$, where a spin-0 hadron (a pion, in this case) is inclusively measured [13]. If the pion is detected collinear to some reference direction (typically, the spatial momentum transferred to the target) the leading-twist cross section does not contain h_1 . But if the transverse momentum of the pion is measured, then the correlation $\mathbf{S}_T \cdot \mathbf{k} \times \mathbf{P}_{hT}$ among the quark fragmenting momentum \mathbf{k} , its transverse polarization \mathbf{S}_T and the pion transverse momentum \mathbf{P}_{hT} , produces a distortion in the azimuthal distribution of final pions, the so-called Collins effect [14]. The distortion is weighted by the combination $h_1^q \otimes H_1^{\perp q}$, where the chiral-odd $H_1^{\perp q}$ describes the probability for a transversely polarized quark q to fragment into the observed pion with measured transverse momentum \mathbf{P}_{hT} . The \otimes operator symbolizes a complicate convolution on the transverse momenta of the detected pion and the initial transversely polarized quark inside the target proton. The Collins function H_1^{\perp} can be extracted from a specific asymmetry in the azimuthal distribution of inclusively produced pions in e^+e^- annihilations, created by the elementary process $e^+e^- \rightarrow q^\uparrow \bar{q}^\downarrow$ [15]. Therefore, through the Collins effect the h_1 can be extracted from a transverse single-spin asymmetry (SSA) only as a TMD PDF. The TMD factorization framework introduces many complications with respect to the collinear framework but, most importantly, cannot be applied to the inclusive hadron production in hadronic collisions, where factorization is explicitly broken [16]. However, TMD factorization can be demonstrated for the specific class of hadronic collisions represented by the Drell-Yan process [17]. And for the case of collisions of pions on transversely polarized protons the polarized part of the cross section contains the convolution $h_{1\pi}^{\perp \bar{q}} \otimes h_{1p}^q$ [18], where $h_{1\pi}^{\perp \bar{q}}$ is the so-called Boer-Mulders function and the convolution involves valence components in both the pion and the proton. This channel poses the problem how to determine the unknown chiral-odd Boer-Mulders function in the pion.

The first example where transversity could be extracted as a PDF in a collinear framework, is represented by the SIDIS process $\ell + p^\uparrow \rightarrow \ell' + (\pi^+, \pi^-) + X$, where the expression (π^+, π^-) denotes the direct fragmentation of a quark into a pair of charged pions, and in general of spin-0 hadrons [19]. The correlation is now of the type $\mathbf{S}_T \cdot \mathbf{P}_{\pi^+} \times \mathbf{P}_{\pi^-} = \mathbf{S}_T \cdot \mathbf{P}_{\text{tot}} \times \mathbf{R}_T$, where $P_{\text{tot}} = P_{\pi^+} + P_{\pi^-}$ and $R = (P_{\pi^+} - P_{\pi^-})/2$. The correlation produces a distortion in the azimuthal distribution of the pion pair that survives the integration upon $\mathbf{P}_{\text{tot}T}$, namely upon quark transverse momenta or, equivalently, in the collinear framework. The first advantage is that the distortion is weighted by the simple product $h_1^q H_1^{\perp q}$ [20, 21], such that h_1^q can be extracted from a transverse SSA as a collinear PDF. The chiral-odd $H_1^{\perp q}$ (sometimes called interference fragmentation function) is the analogue of the Collins function but for the fragmentation into a dihadron, it does not depend on $\mathbf{P}_{\text{tot}T}$ but on R_T^2 , which is related to the di-hadron invariant mass $M_{\pi^+\pi^-}^2$ [22]. Similarly, it can be extracted from the e^+e^- annihilation process through the elementary $e^+e^- \rightarrow q^\uparrow \bar{q}^\downarrow$ mechanism but inclusively producing dihadron pairs [23]. The second advantage is that collinear factorization holds also for all hadronic collisions. Hence, data from the $p + p^\uparrow \rightarrow (\pi^+, \pi^-) + X$ process can be usefully included in phenomenological analyses to attempt a kind of global fit similar to what is the standard for unpolarized and helicity PDFs [24].

The transversity h_1 can be extracted as a collinear PDF also in the SIDIS process $\ell + p^\uparrow \rightarrow$

$\ell' + \Lambda^\uparrow(\bar{\Lambda}^\uparrow) + X$, *i.e.*, for the inclusive production of Λ ($\bar{\Lambda}$), and in general of spin- $\frac{1}{2}$ hadrons [25]. In this case, the correlation is represented by $\mathbf{S}_T \cdot \mathbf{k} \times \mathbf{S}_\Lambda$ and describes a spin transfer from the quark transverse polarization \mathbf{S}_T to the Λ transverse polarization \mathbf{S}_Λ (and similarly for $\bar{\Lambda}$). The induced distortion of the azimuthal distribution of Λ ($\bar{\Lambda}$) is weighted by the product $h_1^q H_1^q$, where the chiral-odd fragmentation function H_1^q can be extracted again from the e^+e^- annihilation process but inclusively producing $(\Lambda, \bar{\Lambda})$ pairs [26, 27]. As before, hadronic collisions like $p+p^\uparrow \rightarrow \Lambda^\uparrow + X$ can be included in phenomenological analyses [54], although experimental data sets are not as large as in the case of di-hadron inclusive production. The COMPASS collaboration measured the transverse SSA in the SIDIS $\ell + p^\uparrow \rightarrow \ell' + \Lambda^\uparrow(\bar{\Lambda}^\uparrow) + X$ process and extracted the strange component of transversity, h_1^s , assuming either that the Λ^\uparrow ($\bar{\Lambda}^\uparrow$) polarization is carried exclusively by the strange quark, or describing the spin transfer in the quark-diquark model [29]. In both cases, the h_1^s turns out to be very small and compatible with zero for most of the x bins explored.

Finally, the transversity PDF can be extracted also in hadronic collisions like $p+p^\uparrow \rightarrow \text{jet}(\pi)+X$, *i.e.*, for leading spin-0 hadrons inside an inclusively produced jet. The hard scale of this process is usually identified with the transverse momentum $P_{\text{jet}T}$ of the jet with respect to the collision direction. If the transverse momentum j_T of the hadron with respect to the jet axis is much smaller than $P_{\text{jet}T}$, then a hybrid factorization was demonstrated where the initial state can be described in a collinear framework and the final hadronic state can be described in the TMD framework [30]. For the polarized collision, the transverse polarization of the fragmenting quark can generate a sort of ‘‘hadron-in-jet Collins effect’’ such that the azimuthal distribution of the hadron inside the jet is distorted by the factor $f_1^q h_1^q [C \otimes H_1^{\perp q}]$, where C are perturbatively calculable coefficients and the hadron-in-jet Collins function depends not only on j_T but also on $P_{\text{jet}T} r$, with r the jet radius. In principle, the same mechanism is active also in the SIDIS process but no data are currently available. The hadron-in-jet Collins effect has been measured at RHIC [31, 32] at both collision energies $\sqrt{s} = 200, 500$ GeV and compared with theoretical predictions based on the parton model [33] or in the CSS formalism including TMD evolution effects [34].

	Framework	e+e-	SIDIS	A_N	Lattice	Soffer bound
Anselmino 2015 P.R. D92 (15) 114023	parton model	✓	✓	✗	✗	✓
Kang et al. 2016 P.R. D93 (16) 014009	TMD / CSS	✓	✓	✗	✗	✓
Lin et al. 2018 P.R.L. 120 (18) 152502	parton model	✗	✓	✗	✓ _{gr}	✗
D'Alesio et al. 2020 (CA) P.L. B803 (20) 135347	parton model	✓	✓	✗	✗	✗, ✓
JAM3D-20 P.R. D102 (20) 054002	parton model	✓	✓	✓	✗	✗
JAM3D-22 P.R. D106 (22) 034014	parton model	✓	✓	✓	✓ _{gr}	✓ $\leq \Delta f_1, \Delta g_1$
Boglione et al. 2024 (TO) P.L. B854 (24) 138712	parton model	✓	✓	✓ reweighing	✗	✓ a posteriori

Figure 1: Most recent extractions of transversity through the Collins effect.

3. Current knowledge

The most recent transversity extractions using the Collins effect are listed in Fig. 1. From left to right, the columns indicate the reference paper, the theoretical framework adopted in the phenomenological analysis (either the naïve parton model or the TMD factorization framework in the Collins–Soper–Sterman (CSS) formalism [35]), the type of process included in the analysis (e^+e^- annihilation to determine the Collins function, SIDIS process, A_N asymmetry in $p + p^\uparrow \rightarrow h + X$ process), the inclusion of lattice results for the isovector tensor charge g_T as priors constraining the fit, and finally the inclusion of the Soffer Bound. In the last entry of Ref. [36], the impact of A_N data has been studied through the reweighting technique, and the Soffer bound has been introduced a posteriori, *i.e.*, letting the fit explore all the possible phase space of parameters and then excluding those solutions that violate such bound. In the JAM3D-22 of Ref. [37], the Soffer bound was implemented including also the uncertainties of the adopted extractions of the f_1 and g_1 PDFs. Transversity was determined with the Collins effect also through a point-by-point direct extraction from experimental data for the SIDIS process [38].

	e+e- unpol. $d\sigma^0$	e+e- asymmetry	SIDIS	p-p collisions	Lattice	Soffer bound
Radici & Bacchetta 2018 P.R.L. 120 (18) 192001	PYTHIA (separately)	✓ (separately)	✓	✓	✗	✓
Benel et al. 2020 E.P.J. C80 (20) 5	PYTHIA (separately)	✓ (separately)	✓	✗	✗	✓ $\leq \Delta f_1, \Delta g_1$
JAMDIFF 2024 P.R.L. 132 (24) 091901	✓	✓	✓	✓	✓ $\delta u, \delta d$	✓ $\leq \Delta f_1, \Delta g_1$

Figure 2: Most recent extractions of transversity through the di-hadron mechanism.

In Fig. 2, we list the most recent extractions of the transversity PDF using the mechanism of inclusive di-hadron production. Again, from left to right the columns indicate the reference paper, the type of process included in the analysis (for the e^+e^- annihilation into di-hadron fragmentation, distinction is made between the unpolarized cross section and the azimuthal asymmetry leading to the extraction of H_1^ζ), the inclusion of lattice results for the tensor charge as constraining priors, and the inclusion of the Soffer bound. For the first two entries of Refs. [24, 39], experimental data for the unpolarized cross section of the $e^+e^- \rightarrow (\pi^+, \pi^-) + X$ process were not available at the time of these analyses and they have been replaced by Monte Carlo simulation of inclusive production of hadron pairs with the PYTHIA code. These Monte Carlo data and the data for the azimuthal asymmetry were separately fitted to extract the unpolarized and interference di-hadron fragmentation functions (DiFFs) D_1 and H_1^ζ , respectively, which in turn were used as input to the analysis of SIDIS (and $p - p$ collision) data for the extraction of transversity. The last entry of Ref. [40] used also lattice results for the tensor charges $\delta u, \delta d$ and g_T as priors to constrain the fit. Finally, in this work and in the work of Ref. [39] the Soffer bound was implemented including also the uncertainties of the adopted extractions of the f_1 and g_1 PDFs.

The transversity of the valence up quark turns out to be positive along all the range of explored x values. The valence down component suffers from large statistical uncertainties. For all extractions,

the central value of the distribution is negative for all x , but the spread of possible solutions included in the band at 68% confidence level is so large that it is difficult to make a clear statement about the sign of the valence down transversity. In the JAM3D-22 of Ref. [37], the attempt to extract also the sea-quark components produced very small numbers with large uncertainties such that with current experimental data we must conclude that $h_1^{\bar{u},\bar{d}}(x)$ are compatible with zero. All extractions agree that for $x \rightarrow 0$ all valence components $h_1(x)^{q_v}$ must diverge less rapidly than $1/x$, since in this limit $x h_1(x)^{q_v} \rightarrow 0$. More importantly, there is a general agreement within statistical uncertainties among various extractions using different data sets and different approaches, except for those ones that include lattice results for the tensor charge as priors to constrain the phenomenological fit: in this case, the h_1^u turns out definitely larger than for other extractions (see also Sec. 3.1). A similar result is obtained by JAM3D-20 of Ref. [41] and CA-20 of Ref. [42] when the Soffer bound is not applied, neither a priori nor a posteriori. Direct calculation of the x -dependence of transversity is also possible on lattice using the LaMET theory [43], and pioneering results are in fair agreement with some of the phenomenological extractions (see, *e.g.*, Ref. [44]).

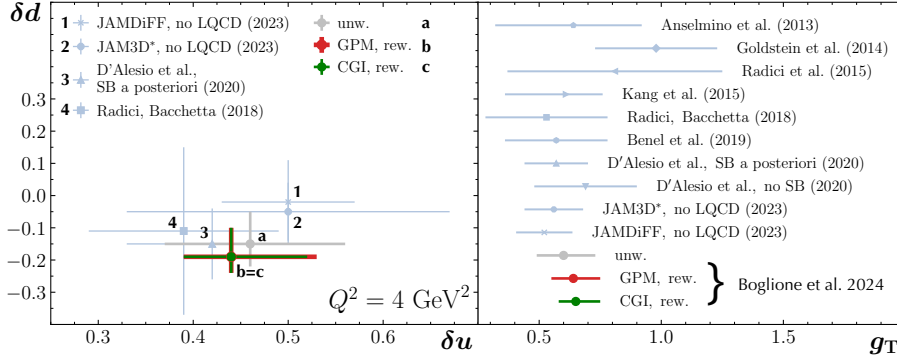


Figure 3: Phenomenological extractions of tensor charge δd vs. δu at $Q^2 = 4 \text{ GeV}^2$ (left panel) and of isovector tensor charge $g_T = \delta u - \delta d$ at the same scale (right panel).

3.1 Tensor charge

The observed approximate agreement between extractions of h_1 that do not use lattice results for tensor charges as priors to the fit, reflects also in the calculated first Mellin moment of it, namely the tensor charge of Eq. (1). In Fig. 3 (see Ref. [45] in these proceedings), one can appreciate the general consistency among different phenomenological extractions of g_T , δu , δd , across very different approaches applied to different experimental data sets. It is also worth noting that the precision of the extractions has significantly increased with time because of data sets with better quality and of more sophisticated theoretical frameworks. However, it must be acknowledged that experimental data approximately cover the x range [0.008, 0.35]; therefore, all phenomenological results for tensor charges should include a systematic error due to extrapolation outside this range that is difficult to quantify.

Surprisingly, the phenomenological results for g_T are in marked disagreement (at least, by $2 - 3\sigma$) with lattice results, as it is evident in the right panel of Fig. 4 (adapted from Ref. [40]) by comparing results of PNDME and ETMC lattice collaborations with other results. This trend can be generalized also to other lattice computations of g_T (see, *e.g.*, Ref. [46]). Inspection of the

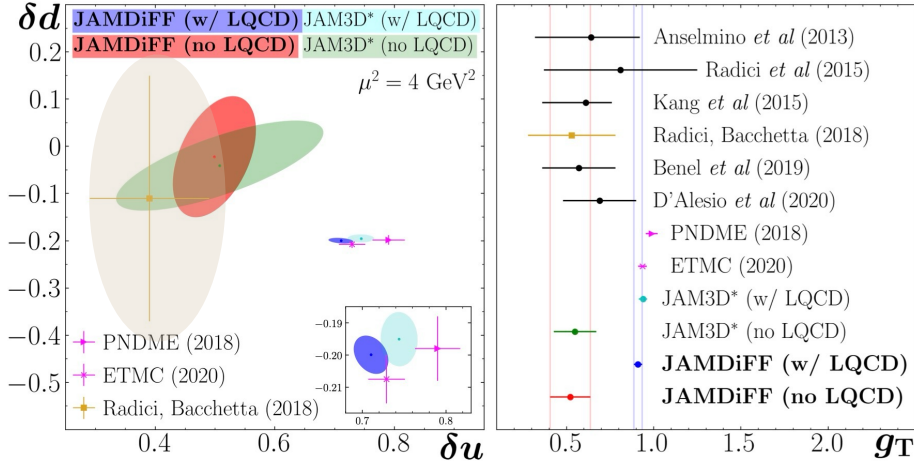


Figure 4: Phenomenological extractions and lattice calculations (PNDME and ETMC) of tensor charge δd vs. δu at $Q^2 = 4 \text{ GeV}^2$ (left panel) and of isovector tensor charge $g_T = \delta u - \delta d$ at the same scale (right panel).

left panel suggests that the discrepancy is driven by δu because of the large uncertainties in the phenomenological extractions of δd that make it compatible with very precise lattice results (see also Figs. 7-54 and 7-56 of Ref. [47]). This is a well known problem that was discussed for the first time in Ref. [48].

As we noted in the previous section, there are phenomenological results that have been obtained by constraining the fit with lattice results for tensor charges as priors, both for the Collins effect (JAM3D-22 of Ref. [37]) and the di-hadron mechanism (JAMDIF-24 of Ref. [40]). In these cases, the h_1^u results bigger than other extractions and, consequently, δu is larger, as it can be realized by comparing in the left panel the light blue blob with the green one (for the Collins effect) and the dark blue blob with the red one and the yellowish one (for the di-hadron mechanism). The remarkable agreement for δu and δd between the light and dark blue blobs and the magenta points from lattice (see the inserted zoom in the left panel) reflects in the right panel for g_T , allowing authors of JAM3D-22 and JAMDIF-24 extractions to claim that lattice and phenomenology results for tensor charges are statistically compatible.

However, several caveats should be put forward about the JAM3D-22 and JAMDIF-24 extractions:

- the reduced χ^2 on some lattice results is very large; for example, for δu of PNDME the JAMDIF-24 fit obtains $\chi_{\text{red}}^2 = 8.68$. Nevertheless, the statistical weight of the lattice results for δu , δd , g_T , is irrelevant; for example, the JAMDIF-24 extraction used almost 1500 data points, and the total χ^2 is not altered by adding 6 points (3 results for the two PNDME and ETMC lattice collaborations)
- introducing lattice results as priors deteriorates the quality of the fit for some of the data sets; for example, in the JAMDIF-24 extraction the quality of the fit for the COMPASS SIDIS data set with proton target changes from $\chi_{\text{red}}^2 = 0.65$ to 1.98; similarly, for the pseudorapidity binning of STAR data at $\sqrt{s} = 500 \text{ GeV}$ the $\chi_{\text{red}}^2 = 1.83$ increases to 2.97

- most of the data used in global fits are insensitive to tensor charges, hence they should not be considered in the comparison between lattice and phenomenology. For example, for the JAMDIFF-24 analysis the 1277 points measured by the BELLE collaboration for the di-hadron production in e^+e^- annihilations are used to extract the DiFFs D_1 and H_1^x ; if we neglect them, the fit with no lattice priors has a total $\chi^2 \sim 203$, while including the lattice priors it increases by 20% to $\chi^2 \sim 238$.

In conclusion, the issue of the compatibility between lattice and phenomenological results for the tensor charge is not solved and it should be considered with great care. The topic will be discussed in this workshop during a dedicated round table.

4. New data and future developments

New data have been recently made available that could improve our knowledge of transversity and help in solving current issues. For example, the COMPASS collaboration released new measurements of transverse SSA on transversely polarized deuteron target in SIDIS production of single hadrons with the Collins effect [49] and of unidentified hadron pairs [50]. These data combined with similar data on proton target are extremely useful to disentangle the valence flavors of transversity in the proton. They contribute to reduce the current large uncertainty in the extraction of the down component h_1^d , as already shown in preliminary impact studies (see pag. 157 in Ref. [48]). Also HERMES data for the SIDIS transverse SSA have been updated [51] and have already been used in the analysis of T0-24 of Ref. [36] (see last entry in Fig. 1).

Several new data relevant for transversity have been recently released also about transversely polarized hadronic collisions. The COMPASS collaboration has added new measurements for the pion-induced Drell-Yan process $\pi + p^\uparrow \rightarrow \ell^+ + \ell^- + X$ [52] where the transverse SSA is proportional to $h_{1\pi}^{\perp\bar{q}} \otimes h_{1p}^q$. We remind, however, that for taking full advantage of this channel the unknown Boer-Mulders function in the pion, $h_{1\pi}^+$, needs to be independently determined from another process. The STAR collaboration has presented new preliminary data for the hadron-in-jet Collins effect by measuring transverse SSA in the $p + p^\uparrow \rightarrow \text{jet}(\pi^\pm) + X$ process at $\sqrt{s} = 500$ GeV [53]. The collaboration also published new data for the spin transfer in $p + p^\uparrow \rightarrow \Lambda^\uparrow + X$ at $\sqrt{s} = 200$ GeV [54]; in this case, the transverse DSA is proportional to $f_1 \otimes h_1 \otimes d\hat{\sigma} \otimes H_1$, where the convolution relates the internal partonic variables to the pseudorapidity, longitudinal and transverse momenta of the detected final hadrons produced by the Λ decay. Finally, the STAR collaboration presented also new more precise results for inclusive di-hadron production in the process $p + p^\uparrow \rightarrow (\pi^+, \pi^-) + X$ at both $\sqrt{s} = 200$ and 500 GeV [55], including measurements not only for the transverse SSA but also for the di-hadron multiplicity, necessary to constrain the unpolarized DiFF D_1 that enters the SSA denominator.

Future developments are promising a huge impact on the current uncertainty of the extracted transversity distribution. The SoLID collaboration recently presented projected errors for the measurement of transverse SSA in the SIDIS process $\ell + p^\uparrow \rightarrow \ell' + h + X$ in the Hall A of Jefferson Lab with upgraded electron beam energy of 12 GeV [56]. The expected uncertainties on δu and δd are drastically reduced and could become comparable or even smaller than current lattice error bars. Similarly, impact studies for the future Electron-Ion Collider (EIC) show that for

both the Collins effect and the dihadron mechanism in $e - p^\uparrow$ collisions with luminosity $\mathcal{L} = 10 \text{ fb}^{-1}$ the statistics accumulated at various collision energies also allows to drastically reduce the uncertainty on all valence components of transversity [47], reaching very small error bars on the extracted tensor charges that could enlighten the puzzling comparison with corresponding lattice results. The capability of clearly identifying jets at the EIC will also allow to study azimuthal correlations between the electron beam and leading hadrons inside produced jets, opening up the channel of hadron-in-jet Collins effect for the extraction of transversity [47]. Moreover, the abundant production of heavy flavors makes the EIC a suitable machine to study “Collins-like” azimuthal asymmetries that give access to the gluon transversity, and could help in enlightening the tensor structure of the deuteron [47].

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