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Semileptonic $\eta^{(\prime)}$ decays in the Standard Model

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We performed a theoretical analysis of the semileptonic decays $\eta^{(\prime)} \to \pi^0\ell^+\ell^-$ and $\eta' \to \eta\ell^+\ell^-$, where $\ell = e, \mu$, via a charge-conjugation-conserving two-photon mechanism. The underlying form factors are modelled using vector-meson dominance, phenomenological input, and U(3) flavour symmetry. We considered both a monopole and a dipole model, the latter tailored such that the expected high-energy behaviour is ensured. Furthermore, we benchmarked the effect of S-wave rescattering contributions to the decays. We inferred significant effects from the form factors neglected in the literature so far, still finding branching ratios of the various decays well below the current experimental upper limits.[†]

The 11th International Workshop on Chiral Dynamics (CD2024)
26-30 August 2024
Ruhr University Bochum, Germany

[†] These proceedings borrow heavily from Ref. [1].

[*] Speaker

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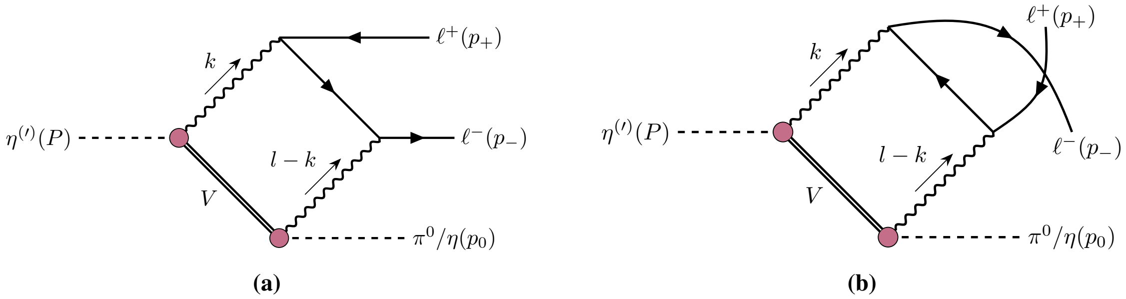

Figure 1: The t- (a) and u-channel (b) diagrams that contribute to $\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-$ under the assumption that the underlying two-photon amplitudes are dominated by the exchange of the vector mesons $V = \rho, \omega, \phi$.

1. Introduction

One of the fundamental open questions in physics concerns the matter–antimatter asymmetry in the Universe, which is related to the question of C and CP violation [2]. As mechanisms present in the standard model (SM) do not sufficiently explain the observed asymmetry, processes where potential C- and CP-violating interactions could be measured are interesting in the search for physics beyond the SM (BSM). $\eta^{(\prime)}$ mesons offer a good opportunity as they are eigenstates of C and P [3].

The semileptonic decay channels $\eta^{(\prime)} \to \pi^0\ell^+\ell^-$ and $\eta' \to \eta\ell^+\ell^-$, $\ell \in \{e, \mu\}$, are of special interest because they are forbidden in the SM as tree-level processes: the one-photon intermediate state violates C and CP such that the leading-order contribution comes from the two-photon intermediate state at one-loop order. A potential BSM contribution to this process could occur at tree-level and therefore would be enhanced compared to the SM decay rate [4].

In order to singularise such BSM contributions, one compares SM results with experimentally obtained (differential) decay rates. As these semileptonic decays are rare, they are hard to measure; to this day, only upper limits exist on the respective branching ratios, which are of $O(10^{-6})$ for $\eta \to \pi^0\ell^+\ell^-$, $O(10^{-3})$ for $\eta' \to [\pi^0/\eta]e^+e^-$, and $O(10^{-5})$ for $\eta' \to [\pi^0/\eta]\mu^+\mu^-$ at 90% confidence level [5–7]. There is the prospect of improved experimental results by the REDTOP collaboration [8].

Since the 1960s, a number of calculations have been performed for the $\eta \to \pi^0\ell^+\ell^-$ decay channels [9–13]. While the first calculations relied on *ad-hoc* form factors or reconstructed the amplitude dispersively from its imaginary part with a finite energy cut-off, most authors employed a vector-meson dominance (VMD) model, which introduces a vector-meson intermediate state in the crossed channels, see Fig. 1. The additional propagator from this left-hand cut dampens the high-energy behaviour of the integrand such that the loop integral is convergent.

With today's understanding of the two-photon decays in the framework of chiral perturbation theory (ChPT) [14], more experimental input on the hadronic couplings, and improved computational resources, an updated calculation is possible. In ChPT, the $\eta \to \pi^0\gamma\gamma$ amplitude is suppressed and the dominant contribution arises from counter terms at $O(p^6)$ [15, 16], whose size can be estimated by resonance saturation in terms of vector-meson exchanges. The resulting predictions agree with the data [17, 18] rather well [19], and rescattering corrections in the scalar channel [20] are moderate in size [21]. Similarly, vector-meson exchanges dominate the decays $\eta' \to \pi^0\gamma\gamma$ and

$\eta' \to \eta\gamma\gamma$ [22], with only minor S-wave corrections to the $\gamma\gamma$ spectra.

The most recent theoretical calculation [23], which also includes the η' decays, employs this modern knowledge to a large extent. What has still not been implemented, though, is the dependence on the photon virtualities, *i.e.*, the vector-to-pseudoscalar transition form factors [24, 25], for which also the behaviour for asymptotically large momentum transfers is known [26–28]. The main advance of our calculation is therefore that we provide a realistic model for $\eta^{(\prime)} \to [\pi^0/\eta]\gamma^*\gamma^*$, including the dependence on the photon virtualities, and hence are able to give a more reliable prediction for the rates of the corresponding dilepton decays in the SM. Furthermore, by lifting the (somewhat artificial) dependence of the loop regularisation on the left-hand cuts, we can, for the first time, also test the effect of S-wave rescattering contributions. Varying the form-factor models allows us to assess the remaining theoretical uncertainties of our predictions.

2. Amplitudes

The construction of the C-even decay amplitudes for $\eta^{(\prime)}(P) \to \pi^0(p_0)\ell^+(p_+)\ell^-(p_-)$ and $\eta'(P) \to \eta(p_0)\ell^+(p_+)\ell^-(p_-)$, where $\ell = e, \mu$, is based on the assumption that the underlying $\eta^{(\prime)} \to [\pi^0/\eta]\gamma^*\gamma^*$ amplitudes are dominated by the exchange of the vector mesons $V = \rho^0(770), \omega(782), \phi(1020)$; see Fig. 1. For our analysis, we define the Mandelstam variables $s = (p_+ + p_-)^2$, $t = (p_- + p_0)^2$, and $u = (p_+ + p_0)^2$, which fulfil the relation $\Sigma = s + t + u = M^2_{\eta^{(\prime)}} + M^2_{\pi^0/\eta} + 2m^2_\ell$. The relevant vector-to-pseudoscalar transition form factors $\mathcal{F}_{VP}(q^2)$ are defined according to

$$\langle P(p)|j_\mu(0)|V(p_V)\rangle = e\,\epsilon_{\mu\nu\alpha\beta}\epsilon^\nu(p_V)p^\alpha q^\beta \mathcal{F}_{VP}(q^2), \tag{1}$$

where $j_\mu = e(2\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d - \bar{s}\gamma_\mu s)/3$ denotes the electromagnetic current and $q = p_V - p$. The normalisations $|\mathcal{F}_{VP}(0)|$ at the real-photon point can be derived from phenomenological input [29] in a straightforward manner.

Using Eq. (1) and summing over the t- and u-channel diagrams shown in Fig. 1 as well as $V = \rho, \omega, \phi$, we find the amplitude $\mathcal{M} \equiv \mathcal{M}(\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-)$ to be

$$\begin{aligned}\mathcal{M} = \mathrm{i}\frac{\alpha^2}{\pi^2}\sum_V \int \mathrm{d}^4k\, g^{\beta\tilde{\beta}}\epsilon_{\mu\nu\alpha\beta}\epsilon_{\tilde{\mu}\tilde{\nu}\tilde{\alpha}\tilde{\beta}}P^\alpha k^\mu\big(P^{\tilde{\alpha}}k^{\tilde{\mu}} - P^{\tilde{\alpha}}l^{\tilde{\mu}} + k^{\tilde{\alpha}}l^{\tilde{\mu}}\big)P_V^{\mathrm{BW}}\big((P-k)^2\big)P_\gamma\big((l-k)^2\big)\\ P_\gamma(k^2)\mathcal{F}_{V\eta^{(\prime)}}(k^2)\mathcal{F}_{V[\pi^0/\eta]}\big((l-k)^2\big)\bar{u}_s\left[\gamma^{\tilde{\nu}}\frac{\not{k} - \not{p}_+ + m_\ell}{(k-p_+)^2 - m_\ell^2}\gamma^\nu + \gamma^\nu\frac{\not{p}_- - \not{k} + m_\ell}{(p_- - k)^2 - m_\ell^2}\gamma^{\tilde{\nu}}\right]v_r,\end{aligned} \tag{2}$$

with $\bar{u}_s \equiv \bar{u}_s(p_-)$ and $v_r \equiv v_r(p_+)$. Here, we defined $l = p_+ + p_-$ and the [Breit–Wigner (BW)] propagators

$$P_V^{\mathrm{BW}}(q^2) = \frac{1}{q^2 - M_V^2 + \mathrm{i}M_V\Gamma_V}, \quad P_\gamma(q^2) = \frac{1}{q^2 + \mathrm{i}\epsilon}, \tag{3}$$

where M_V is the mass of the respective vector meson and Γ_V its width. Due to their narrowness, a constant-width approximation is well justified for the ω and ϕ, whereas the broad ρ meson necessitates an energy-dependent width to avoid sizable unphysical imaginary parts below threshold.

The branching ratios of the semileptonic decays are commonly normalised to the two-photon analogues, the amplitudes of which are calculated in complete analogy.

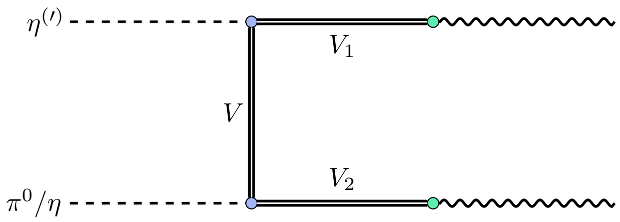

Figure 2: Modelling of the two-photon decay mechanism in the VMD framework via vector mesons V_1, V_2.

3. Form factors

In order to parameterise the form factors $\mathcal{F}_{VP}(q^2)$, we use the VMD framework. As a consequence, the photon couplings at the $VP\gamma^*$ vertices of the diagrams in Fig. 1 are mediated via two intermediate vector mesons V_1 and V_2; see Fig. 2. We construct two distinct such models: a monopole (MP) parameterisation with $V_i = \rho, \omega, \phi$ and a dipole (DP) ansatz with $V_i = \rho^{(\prime)}, \omega^{(\prime)}, \phi^{(\prime)}$, $\rho' \equiv \rho^0(1450)$, $\omega' \equiv \omega(1420)$, and $\phi' \equiv \phi(1680)$, that ensures the expected high-energy behaviour of the form factors [26–28]. For reference, we also include a model calculation with constant form factors, *i.e.*, a point-like (PL) interaction, which closely resembles the parameterisation of Ref. [23].

The conservation of isospin—and thus G parity combined with C—imposes constraints on V_1 and V_2 in dependence on the initial and final states as well as the t- or u-channel vector meson V, such that only specific combinations are allowed. U(3) flavour symmetry and ideal mixing of the vector-meson multiplets, as well as the inclusion of the ϕ coupling constants in spite of these constraints, fix the relative signs of the coupling constants.

The MP model only takes the lowest-lying vector mesons ρ, ω, and ϕ into account, so that the form factors are parameterised according to

$$\mathcal{F}_{VP}(q^2) = C_{VP\gamma} M_{V_i}^2 P_{V_i}^{\mathrm{BW}}(q^2), \quad V_i \in \{\rho, \omega, \phi\}. \tag{4}$$

Given that the asymptotic behaviour of the vector-to-pseudoscalar transition form factors is expected to be $\mathcal{F}_{VP}(q^2) \propto q^{-4}$ [26–28], we can additionally include the next-higher multiplet of vector mesons, ρ', ω', and ϕ', to achieve this property by tuning a free parameter ϵ_V. For the DP model, we thus make the ansatz

$$\widetilde{\mathcal{F}}_{VP}(q^2) = C_{VP\gamma}\big[(1-\epsilon_{V_i}) M_{V_i}^2 P_{V_i}^{\mathrm{BW}}(q^2) + \epsilon_{V_i} M_{V_i'}^2 P_{V_i'}^{\mathrm{BW}}(q^2)\big], \tag{5}$$

where we assume the excited vector states to couple according to the exact same symmetry restrictions as the ground-state multiplet. Due to the large widths of the excited vector mesons, a constant-width approximation leads to a rather poor description of these mesons, however. We will therefore, analogously to the ρ, construct dispersively improved BW propagators for ρ', ω', and ϕ' based on energy-dependent widths in Sec. 3.1, leading to replacements of the kind $P_{V'}^{\mathrm{BW}}(q^2) \to P_{V'}^{\mathrm{disp}}(q^2)$. Our results will be quoted for both the MP and the DP in the variant CW, a simple approximate description with constant widths for all vector mesons, and the variant VW where constant widths for the ω and ϕ but energy-dependent ones for $\rho^{(\prime)}$, ω', and ϕ' are used.

3.1 Spectral representation

We construct energy-dependent widths and to ensure the correct analytic properties when inserting the form factors into the amplitude, Eq. (2), we furthermore introduce dispersively improved variants [30] of the form factors that contain a $\rho^{(\prime)}$-, ω'-, or ϕ'-meson propagator.

For the ρ meson, we use the energy-dependent width [31]

$$\Gamma_\rho(q^2) = \theta(q^2 - 4M_{\pi^\pm}^2)\frac{\gamma_{\rho\to\pi^+\pi^-}(q^2)}{\gamma_{\rho\to\pi^+\pi^-}(M_\rho^2)} f(q^2)\Gamma_\rho, \quad \text{with} \quad \gamma_{\rho\to\pi^+\pi^-}(q^2) = \frac{(q^2 - 4M_{\pi^\pm}^2)^{3/2}}{q^2} \tag{6}$$

and the so-called barrier factor $f(q^2)$. We calculate the dispersive ρ propagator via

$$P_V^{\text{disp}}(q^2) = -\frac{1}{\pi}\int_{s_{\text{thr}}}^{\infty} \mathrm{d}x\, \frac{\mathrm{Im}\,[P_V^{\text{BW}}(x)]}{q^2 - x + \mathrm{i}\epsilon}, \qquad \mathrm{Im}\,[P_V^{\text{BW}}(x)] = \frac{-\sqrt{x}\,\Gamma_V(x)}{(x - M_V^2)^2 + x\Gamma_V(x)^2}, \tag{7}$$

where $s_{\text{thr}} = 4M_{\pi^\pm}^2$ is the threshold for $\rho \to \pi^+\pi^-$. The spectral representations of the form factors $\mathcal{F}_{VP}(q^2)$ for $VP \in \{\rho\eta^{(\prime)}, \omega\pi^0, \phi\pi^0\}$ are thus given by

$$\widehat{\mathcal{F}}_{VP}(q^2) = \frac{C_{VP\gamma}}{N_\rho} M_\rho^2 P_\rho^{\text{disp}}(q^2), \qquad N_\rho = -M_\rho^2 P_\rho^{\text{disp}}(0) \approx 0.898. \tag{8}$$

For the dipole variant, we model the energy-dependent widths and dispersive propagators of ρ', ω', and ϕ' similar to the ρ case, taking into account the respective dominant decay channels [31].

4. Observables

The phenomenological analysis is performed in terms of doubly- and singly-differential decay widths as well as integrated branching ratios. We define $\nu = t - u$ for the Mandelstam variables t and u, in terms of which the twofold differential decay width $\mathrm{d}\Gamma \equiv \mathrm{d}\Gamma(\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-)$ is given by [29] $\mathrm{d}\Gamma = \frac{1}{64(2\pi)^3} M_{\eta^{(\prime)}}^{-3} |\overline{\mathcal{M}}|^2 \mathrm{d}s\, \mathrm{d}\nu$. The singly-differential decay width $\mathrm{d}\Gamma/\mathrm{d}s$ follows from an integration of $\mathrm{d}\Gamma$ over ν and the branching ratio $\mathcal{B}(\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-) = \Gamma/\Gamma_{\eta^{(\prime)}}$ is obtained after performing the full three-body phase-space integration. In order to calculate $|\overline{\mathcal{M}}|^2$, we perform a Passarino–Veltman (PV) decomposition of Eq. (2) with *FeynCalc* [32] after inserting explicit expressions for the form factors.

We also consider the normalised semileptonic branching ratios $\widehat{\mathcal{B}}(\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-) = \mathcal{B}(\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-)/\mathcal{B}(\eta^{(\prime)} \to [\pi^0/\eta]\gamma\gamma)$, which are particularly useful from the theoretical point of view, since they reduce the effect of the uncertainties from the coupling constants; the two-photon branching ratios $\mathcal{B}(\eta^{(\prime)} \to [\pi^0/\eta]\gamma\gamma)$ are calculated analogously to the two-lepton ones. We perform the phase-space integrations of the differential decay widths numerically with the *Cuhre* and *Vegas* algorithm from the *Cuba* library [33]. For the numerical evaluation of the PV functions contained in the amplitudes, we use *Collier* [34].

5. Scalar rescattering contributions

While there are good reasons to assume that the VMD model captures the most significant contributions to the semileptonic $\eta^{(\prime)}$ decays, we assess scalar rescattering contributions explicitly

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by calculating them for the $\eta \to \pi^0\ell^+\ell^-$ channels. For the η' channels, the vector mesons have sufficient energy to go quasi on-shell, so that an even stronger dominance of the VMD mechanism is expected. Starting again from the two-photon amplitude, we consider the s-channel S-wave contribution and express it in terms of helicity amplitudes $H_{\lambda\lambda'}$,

$$\langle\gamma(q_1,\lambda)\gamma(q_2,\lambda')|\,S\,|\eta(P)\pi^0(p_0)\rangle = \mathrm{i}(4\pi\alpha)(2\pi)^4\delta^{(4)}(P+p_0-q_1-q_2)\,e^{\mathrm{i}(\lambda-\lambda')\varphi}H_{\lambda\lambda'}. \quad (9)$$

We neglect D- and higher waves, which means that only $H_{++}(s)$ contributes and that it is proportional to the S-wave amplitude $h^0_{++}(s)$. One can set up a dispersion relation for h^0_{++} with a coupled-channel ($\pi^0\eta$ and $(K\bar{K})_{I=1}$) formalism to find a representation in terms of the Omnès solution [21]. After subtracting the VMD contribution that is already taken care of in our model, we obtain the S-wave rescattering contribution $\widetilde{\mathcal{M}}(s)$.

6. Results and discussion

6.1 Differential decay widths

The most prominent features visible in the doubly- and singly-differential distributions of the semileptonic decays (Fig. 3) are the differences between the decays with electrons and muons in the final state. While the majority of the distribution for the electron channels is contained in a small fraction close to the threshold in the invariant lepton mass, the decays with muons in the final state display a spread-out distribution that covers large parts of the available phase space.[1] These differences are in correspondence with the observation that for $m_\ell \approx 0$, the threshold in s approximately collapses to the threshold of the two-photon intermediate state, $s = 0$, where the two-photon cut induces a behaviour $\propto \log(s)$ [11]. Hence, for the electron final state, this logarithmic divergence manifests itself as a peak close to the threshold in s, regularised by a phase-space factor and forced to zero at $s = 4m_\ell^2$, whereas the muon channels have a much higher threshold, far from the logarithmic divergence.

For all decay channels, the obtained Dalitz plots do not follow a flat distribution, which was assumed for the experimental analysis of $\eta \to \pi^0 e^+e^-$ in Ref. [5];[2] we therefore propose a reevaluation of the experimental data and a reassessment of the reported upper limit.

6.2 Branching ratios in the different models

The sensitivity of the semileptonic decays to the different form-factor parameterisations can be probed by comparing the results for the branching ratios and normalised branching ratios collected in Table 1. Due to partial cancellations in the latter, the quoted uncertainties are reduced drastically, however with the caveat that they are likely to be underestimated. At the same time, potential corrections to the semileptonic branching ratios that are not included in the plain VMD model, *e.g.*, the a_2 resonance, are assumed to partially cancel as well because they emerge in the hadronic part of the amplitudes that is shared with the photonic decays.

[1]For the electron final state, in particular, it is important to take account of the region close to the threshold in the invariant lepton mass both when integrating over the phase space and when performing a measurement, as significant parts of the decay width are readily missed otherwise.

[2]This assumption is justified for a potential C-violating contribution [35] but inaccurate for the standard-model result.

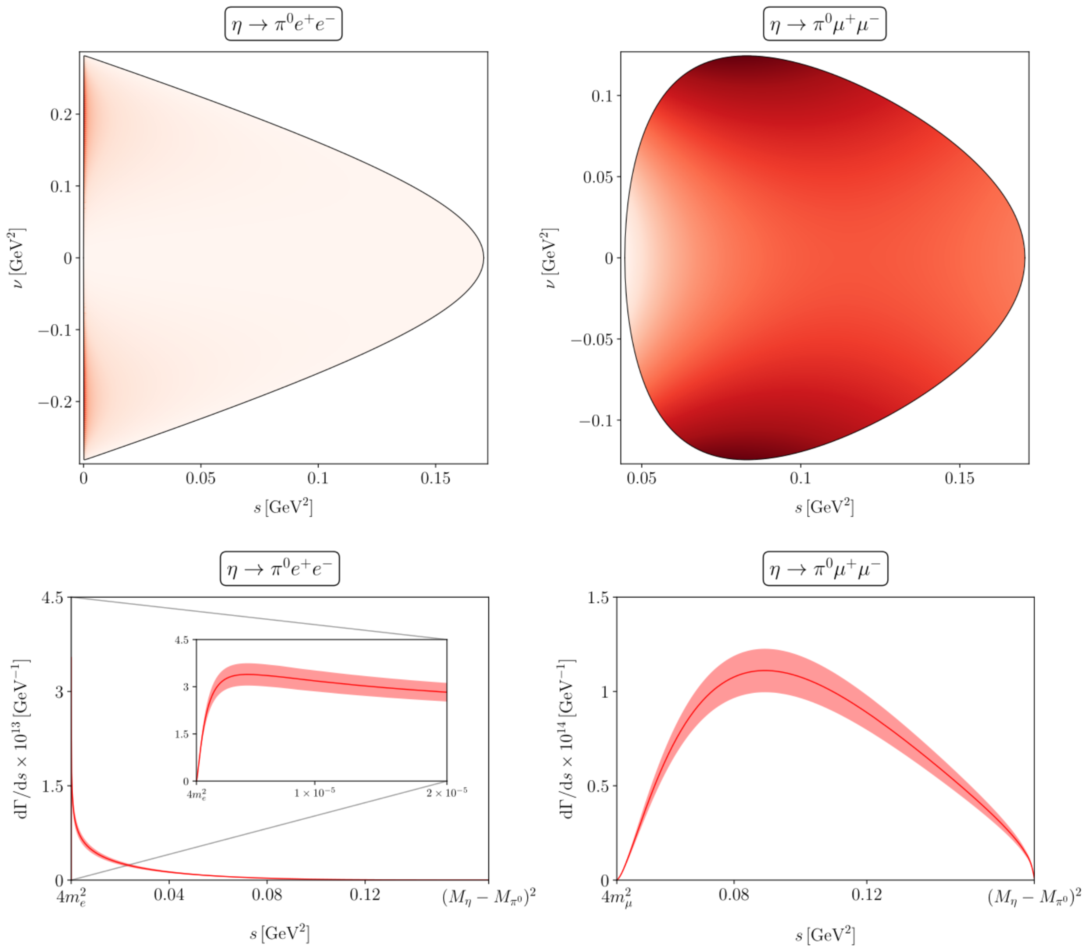

Figure 3: Dalitz plots (normalised to the maximum value of the respective channel) and singly-differential decay widths (uncertainty due to the dominant phenomenological uncertainty of $|\mathcal{F}_{VP}(0)|$) for $\eta \to \pi^0 \ell^+ \ell^-$ for the MP model in the variant CW. For the other channels, see Ref. [1]; figure adapted from *ibid.*

Our results for the decays $\eta \to \pi^0 \ell^+ \ell^-$ obtained with constant form factors and widths are compatible with the results of Ref. [23]; for the η' decays, we find significant disagreement, maybe due to numerical difficulties with their Feynman parameterisation. Implementing non-trivial form factors leads to a significant decrease of the branching ratio for all decays, with the muon channels being subject to a larger reduction than the electron channels and the η' decays to less reduction than the η decays. This gives strong indication that the photon virtualities cannot be neglected in the analysed processes, since constant form factors are likely to overestimate the decay widths. The additional effect of the dipole form factors, featuring the expected high-energy behaviour, is negligible and using spectral representations to implement energy-dependent widths for the broad vector mesons leads to changes at the level of 5 – 10% compared to the CW variant. As all these variations are small compared to the difference between the results in the PL model and any other model and mostly even small compared to the phenomenological uncertainties, we infer the semileptonic decays to be rather insensitive to the precise parameterisation of the photon virtualities in the form factors.

		Branching ratio/10^{-9}				Norm.Branching ratio/10^{-6}		
		PL	MP	DP	Ref. [23]	PL	MP	DP
$\eta \to \pi^0 e^+ e^-$	CW	2.10(23)	1.35(15)	1.33(15)	2.0(2)	17.422(28)	11.197(11)	11.032(9)
	VW	2.06(22)	1.40(15)	1.36(15)		17.510(20)	11.855(7)	11.531(4)
$\eta \to \pi^0 \mu^+ \mu^-$	CW	1.37(15)	0.70(8)	0.66(7)	1.1(2)	11.371(20)	5.781(7)	5.450(6)
	VW	1.32(14)	0.71(8)	0.67(7)		11.197(25)	6.020(10)	5.647(5)
$\eta' \to \pi^0 e^+ e^-$	CW	3.82(33)	3.08(27)	3.14(27)	4.5(6)	1.37(7)	1.11(6)	1.13(6)
	VW	3.81(33)	3.30(28)	3.30(28)		1.36(7)	1.17(6)	1.18(6)
$\eta' \to \pi^0 \mu^+ \mu^-$	CW	2.57(23)	1.69(15)	1.68(15)	1.7(3)	0.92(5)	0.610(35)	0.603(35)
	VW	2.53(23)	1.81(16)	1.81(16)		0.90(5)	0.64(4)	0.65(4)
$\eta' \to \eta e^+ e^-$	CW	0.53(4)	0.48(4)	0.49(4)	0.4(2)	4.77(7)	4.38(6)	4.41(6)
	VW	0.51(4)	0.50(4)	0.50(4)		4.65(7)	4.56(7)	4.56(7)
$\eta' \to \eta \mu^+ \mu^-$	CW	0.287(26)	0.213(18)	0.207(18)	0.15(5)	2.60(6)	1.93(4)	1.88(4)
	VW	0.280(25)	0.225(20)	0.240(21)		2.54(5)	2.05(4)	2.18(4)

Table 1: The branching ratios and normalised branching ratios for the models PL, MP, and DP in both variants CW and VW (uncertainty due to the dominant experimental uncertainty of $|\mathcal{F}_{VP}(0)|$), and the corresponding results for the branching ratios from Ref. [23] for reference (uncertainties added in quadrature).

We have calculated the scalar rescattering contributions exemplarily for the $\eta \to \pi^0 \ell^+ \ell^-$ decay channels. Adding these to the VMD amplitude leads to two additional terms on the level of the squared amplitude in the branching ratio, one pure rescattering term and one term mixing rescattering and VMD effects. For $\eta \to \pi^0 e^+ e^-$, both the rescattering and the mixed contribution are of $\mathcal{O}(10^{-4})$ compared to the VMD result. This seems plausible, given that a spin flip is necessary to couple a scalar resonance to two leptons, resulting in an amplitude proportional to m_ℓ. For $\eta \to \pi^0 \mu^+ \mu^-$, the rescattering and mixed contributions are at the level of 5% in comparison to the VMD contributions, still notably below the uncertainties of the latter, and have opposite signs, such that they largely cancel, leading to a suppression of $\mathcal{O}(10^{-3})$.

7. Conclusion

We have reanalysed the standard-model contribution to the semileptonic decays $\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-$. As an improvement to previous calculations, we have implemented a realistic dependence of the hadronic sub-processes on the photon virtualities via vector-to-pseudoscalar transition form factors and assessed the sensitivity to the chosen parameterisations by comparing three different schemes. The observables are mostly insensitive to the details of the parameterisation at the level of uncertainty induced by the phenomenological coupling constants and all predicted branching ratios are, as expected, well below the current experimental upper limits. We have shown for $\eta \to \pi^0\ell^+\ell^-$ that scalar rescattering effects are indeed small. With improved experimental sensitivities in the future, our theoretical branching ratios of these rare $\eta^{(\prime)}$ decays can hopefully be compared to experiment and thus help cast a light on possible symmetry violations and physics beyond the SM in the light-meson sector.

Acknowledgments

Financial support by the DFG through the funds provided to the Sino–German Collaborative Research Center TRR110 "Symmetries and the Emergence of Structure in QCD" (DFG Project-ID 196253076 – TRR 110) and by the MKW NRW under the funding code NW21-024-A is gratefully acknowledged.

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